ECEN689: Special Topics in High-Speed Links Circuits and Systems
Spring 2011

Lecture 6: Channel Pulse Model

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Agenda

• ISI
• Channel pulse model
• Peak distortion analysis
• Reference material for this lecture
  • Peak distortion analysis paper by Casper (posted on web)
  • Notes from H. Song, Arizona State
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI).
- ISI is caused by:
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

\[ y^{(1)}(t) = c^{(1)}(t) \ast h(t) \]
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated “1”s and “0”s

Data = “1000101”

\[ c_k^{(1)}(t) = u(t - kT) - u(t - (k + 1)T) \]

\[ c_k^{(0)}(t) = -c_k^{(1)}(t) \]

where \( u(t) \equiv \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \)
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s

\[ V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t) \]
Channel Response to NRZ Data

- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses

\[ V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H(c_k^{(d_k)}(t)) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT) \]
Channel Pulse Response

\[ y^{(d_k)}(t) = c^{(d_k)}(t) \ast h(t) \]

\[ y^{(1)}(t) \text{ sampled relative to pulse peak:} \]

\[ \ldots 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 \ldots \]

\[ k = \ldots -2 1 0 1 2 3 4 5 6 \ldots \]

By Linearity: \[ y^{(0)}(t) = -1 \cdot y^{(1)}(t) \]
Channel Data Stream Response

**Input Data Stream**

![Graph showing input data stream with voltage (V) over time (UI).]

**Pulse Responses**

![Graph showing pulse responses with voltage (V) over time (UI).]

**Channel Response**

![Graph showing channel response with voltage (V) over time (UI).]
Channel “FIR” Model

\[ c_0^{(1)}(t) \rightarrow H \rightarrow H(c_0^{(1)}(t)) = y_0^{(1)}(t) \]

\( c_0^{(1)}(t) \rightarrow Z^{(D-1)} \rightarrow Z^{-1} \rightarrow \cdots \)

D is the delay from the channel input to the output pulse peak.

\[ H(c_0^{(1)}(t)) = y_0^{(1)}(t) \]

\( y^{(1)}(t) \) sampled relative to pulse peak:

\[ […] 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 […] \]

\( a = [ \ldots a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad \ldots ] \)
Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response
- Worst-case “1” is summation of a “1” pulse with all negative non k=0 pulse responses
  \[
  s_1(t) = y_0^{(1)}(t) + \sum_{k=-\infty, k\neq 0}^{\infty} y^{(d_k)}(t-kT) \left|_{y(t-kT)<0} \right.
  \]
- Worst-case “0” is summation of a “0” pulse with all positive non k=0 pulse responses
  \[
  s_0(t) = y_0^{(0)}(t) + \sum_{k=-\infty, k\neq 0}^{\infty} y^{(d_k)}(t-kT) \left|_{y(t-kT)>0} \right.
  \]
Peak Distortion Analysis

- Worst-case eye height is \( s_1(t) - s_0(t) \)

\[
s(t) = s_1(t) - s_0(t) = \left( y_0^{(1)}(t) - y_0^{(0)}(t) \right) + \left\{ \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT) \right\}_{y(t-kT) < 0} - \left\{ \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT) \right\}_{y(t-kT) > 0}
\]

Because \( y_0^{(0)}(t) = -1(y_0^{(1)}(t)) \)

\[
s(t) = 2 \left\{ \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \right\}_{y(t-kT) < 0} - \left\{ \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \right\}_{y(t-kT) > 0}
\]

“1” pulse worst-case “1” edge

“1” pulse worst-case “0” edge

- If symmetric “1” and “0” pulses (linearity), then only positive pulse response is needed
Peak Distortion Analysis Example 1

\[ y_0^{(1)}(t) = 0.540 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t-kT) \bigg|_{y(t-kT)<0} = -0.007 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t-kT) \bigg|_{y(t-kT)>0} = 0.389 \]

\[ s(t) = 2(0.540 - 0.007 - 0.389) = 0.288 \]
Worst-Case Bit Pattern

- Pulse response can be used to find the worst-case bit pattern
  \[
  \text{Pulse } a = [... \ a_{-2} \ a_{-1} \ a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ ...]
  \]

- Flip pulse matrix about cursor \(a_0\) and the bits are the inverted sign of the pulse ISI
  \[
  [... \ -\text{sign}(a_6) \ -\text{sign}(a_5) \ -\text{sign}(a_4) \ -\text{sign}(a_3) \ -\text{sign}(a_2) \ -\text{sign}(a_1) \ 1 \ -\text{sign}(a_{-1}) \ -\text{sign}(a_{-2}) \ ...]
  \]
Peak Distortion Analysis Example 2

\[ y_0^{(1)}(t) = 0.426 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT)<0} = -0.053 \]

\[ \sum_{k=-\infty}^{\infty} y^{(1)}(t - kT) \Big|_{y(t-kT)>0} = 0.542 \]

\[ s(t) = 2(0.426 - 0.053 - 0.542) = -0.338 \]
Next Time

• Modulation schemes
  • Binary, PAM-2, NRZ
  • PAM-4
  • Duo-binary

• Link Circuits
  • Termination structures
  • Drivers
  • Receivers