ECEN689: Special Topics in High-Speed Links Circuits and Systems
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Lecture 3: Transmission Lines

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Announcements

• HW1 due 1/28
  • One page summary of recent link design paper

• Lecture Reference Material
  • Dally, Chapter 3.1 – 3.4
Agenda

• Transmission Lines
  • Propagation constant
  • Characteristic impedance
  • Loss
  • Reflections
  • Termination examples
  • Differential transmission lines
Wire Models

- Model Types
  - Ideal
  - Lumped C, R, L
  - RC transmission line
  - LC transmission line
  - RLGC transmission line

- Condition for LC or RLGC model (vs RC)

\[ f_0 \geq \frac{R}{2\pi L} \]

<table>
<thead>
<tr>
<th>Wire</th>
<th>R</th>
<th>L</th>
<th>C</th>
<th>&gt;f (LC wire)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWG24 Twisted Pair</td>
<td>0.08Ω/m</td>
<td>400nH/m</td>
<td>40pF/m</td>
<td>32kHz</td>
</tr>
<tr>
<td>PCB Trace</td>
<td>5Ω/m</td>
<td>300nH/m</td>
<td>100pF/m</td>
<td>2.7MHz</td>
</tr>
<tr>
<td>On-Chip Min. Width M6 (0.18µm CMOS node)</td>
<td>40kΩ/m</td>
<td>4µH/m</td>
<td>300pF/m</td>
<td>1.6GHz</td>
</tr>
</tbody>
</table>
As $dx \to 0$

\[ \frac{\partial V(x,t)}{\partial x} = -RI(x,t) - L \frac{\partial I(x,t)}{\partial t} \quad (1) \]

\[ \frac{\partial I(x,t)}{\partial x} = -GV(x,t) - C \frac{\partial V(x,t)}{\partial t} \quad (2) \]
Time-Harmonic Transmission Line Eqs.

- Assuming a traveling sinusoidal wave with angular frequency, $\omega$

\[
\frac{dV(x)}{dx} = -(R + j\omega L)I(x) \quad (3)
\]

\[
\frac{dI(x)}{dx} = -(G + j\omega C)V(x) \quad (4)
\]

- Differentiating (3) and plugging in (4) (and vice versa)

\[
\frac{d^2 V(x)}{dx^2} = \gamma^2 V(x) \quad (5)
\]

\[
\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x) \quad (6)
\]

- where $\gamma$ is the propagation constant

\[
\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \text{ (m}^{-1}\text{)}
\]
Transmission Line Propagation Constant

- Solutions to the Time-Harmonic Line Equations:

\[ V(x) = V_f(x) + V_r(x) = V_{f0}e^{-\gamma x} + V_{r0}e^{\gamma x} \]

\[ I(x) = I_f(x) + I_r(x) = I_{f0}e^{-\gamma x} + I_{r0}e^{\gamma x} \]

where

\[ \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (m^{-1}) \]

- What does the propagation constant tell us?
  - Real part (\( \alpha \)) determines attenuation/distance (Np/m)
  - Imaginary part (\( \beta \)) determines phase shift/distance (rad/m)
  - **Signal phase velocity**

\[ \nu = \omega / \beta \quad (m/s) \]
Transmission Line Impedance, $Z_0$

- For an infinitely long line, the voltage/current ratio is $Z_0$
- From time-harmonic transmission line eqs. (3) and (4)

$$Z_0 = \frac{V(x)}{I(x)} = \sqrt{\frac{R + j \omega L}{G + j \omega C}} \quad (\Omega)$$

- Driving a line terminated by $Z_0$ is the same as driving an infinitely long line
Lossless LC Transmission Lines

• If $R_{dx}=G_{dx}=0$

\[
\gamma = \alpha + j\beta = j\omega\sqrt{LC}
\]

\[
\alpha = 0 \quad \text{No Loss!}
\]

\[
\beta = \omega\sqrt{LC}
\]

• Waves propagate w/o distortion
  • Velocity and impedance independent of frequency
  • Impedance is purely real

\[
\nu = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}
\]

\[
Z_0 = \sqrt{\frac{L}{C}}
\]

\[\text{Distance } (Y - X)\]

\[\text{At point } X, \text{ the step is still of size } V, \text{ but delayed}\]

\[\text{At point } Y, \text{ step is delayed even more}\]

\[\text{Time delay } t_1 - t_0 = (Y - X)\sqrt{LC}\]

\[\text{A step of } V \text{ volts propagates down the transmission line}\]
Low-Loss LRC Transmission Lines

- If $\frac{R}{\omega L}$ and $\frac{G}{\omega C} << 1$
  - Behave similar to ideal LC transmission line, but ...
  - Experience resistive and dielectric loss
  - Frequency dependent propagation velocity results in dispersion
    - Fast step, followed by slow DC tail

\[
\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}
\]
\[
\approx j\omega \sqrt{LC} \left(1 - j \frac{RC + GL}{\omega LC}\right)^{\frac{1}{2}}
\]
\[
\approx \frac{R}{2Z_0} + \frac{GZ_0}{2} + j\omega \sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{R}{\omega L}\right)^2 + \frac{1}{8} \left(\frac{G}{\omega C}\right)^2\right]
\]
\[
= \alpha_R + \alpha_D + j\beta
\]

\begin{align*}
\alpha_R &\approx \frac{R}{2Z_0} & \text{Resistive Loss} \\
\alpha_D &\approx \frac{GZ_0}{2} & \text{Dielectric Loss}
\end{align*}

\[
\beta \approx \omega \sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{R}{\omega L}\right)^2 + \frac{1}{8} \left(\frac{G}{\omega C}\right)^2\right]
\]

\[
\nu \approx \left(\sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{R}{\omega L}\right)^2 + \frac{1}{8} \left(\frac{G}{\omega C}\right)^2\right]\right)^{-1}
\]
Skin Effect (Resistive Loss)

- High-frequency current density falls off exponentially from conductor surface
- Skin depth, $\delta$, is where current falls by $e^{-1}$ relative to full conductor
  - Decreases proportional to $\sqrt{\text{frequency}}$
- Relevant at critical frequency $f_s$ where skin depth equals half conductor height (or radius)
  - Above $f_s$ resistance/loss increases proportional to $\sqrt{\text{frequency}}$

\[ J = e^{-\frac{d}{\delta}} \]
\[ \delta = \left( \pi \mu \sigma \right)^{\frac{1}{2}} \]

For rectangular conductor:

\[ f_s = \frac{\rho}{\pi \mu \left( \frac{h}{2} \right)^2} \]
\[ R(f) = R_{DC} \left( \frac{f}{f_s} \right)^{\frac{1}{2}} \]
\[ \alpha_R = \frac{R_{DC}}{2Z_0} \left( \frac{f}{f_s} \right)^{\frac{1}{2}} \]
Skin Effect (Resistive Loss)

5-mil Stripguide
\[ R_{DC} = 7 \Omega/m, \; f_s = 43\text{MHz} \]

30 AWG Pair
\[ R_{DC} = 0.08 \Omega/m, \; f_s = 67\text{kHz} \]

\[ \alpha_R = \frac{R_{DC}}{2Z_0} \left( \frac{f}{f_s} \right) \]

[Dally]
Dielectric Absorption (Loss)

- An alternating electric field causes dielectric atoms to rotate and absorb signal energy in the form of heat.
- Dielectric loss is expressed in terms of the loss tangent.
- Loss increases directly proportional to frequency.

\[ \tan \delta_D = \frac{G}{\omega C} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>( \varepsilon_r )</th>
<th>( \tan \delta_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woven glass, epoxy resin (“FR-4”)</td>
<td>4.7</td>
<td>0.035</td>
</tr>
<tr>
<td>Woven glass, polyimide resin</td>
<td>4.4</td>
<td>0.025</td>
</tr>
<tr>
<td>Woven glass, polyphenylene oxide resin (GETEK)</td>
<td>3.9</td>
<td>0.010</td>
</tr>
<tr>
<td>Woven glass, PTFE resin (Teflon)</td>
<td>2.55</td>
<td>0.005</td>
</tr>
<tr>
<td>Nonwoven glass, PTFE resin</td>
<td>2.25</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\[ \alpha_D = \frac{GZ_0}{2} = \frac{2\pi f C \tan \delta_D \sqrt{L/C}}{2} = \pi f \tan \delta_D \sqrt{LC} \]
Total Wire Loss

Dielectric Loss
Conductor Loss
Calculated Attenuation

Attenuation

Frequency

Measured Attenuation

[1 MHz, 10 MHz, 100 MHz, 1 GHz, 6 GHz]

[Dally]
**Reflections & Telegrapher’s Eq.**

- With a Thevenin-equivalent mode of the line:

  Termination Current: \( I_T = \frac{2V_i}{Z_0 + Z_T} \)

- KCL at Termination:

  \[
  I_r = I_f - I_T
  \]

  \[
  I_r = \frac{V_i}{Z_0} - \frac{2V_i}{Z_T + Z_0}
  \]

  \[
  I_r = \frac{V_i}{Z_0} \left( \frac{Z_T - Z_0}{Z_T + Z_0} \right)
  \]

  Telegrapher’s Equation or Reflection Coefficient

  \[
  k_r = \frac{I_r}{I_i} = \frac{V_r}{V_i} = \frac{Z_T - Z_0}{Z_T + Z_0}
  \]
Termination Examples - Ideal

\[ V_i = 1V \left( \frac{50}{50 + 50} \right) = 0.5V \]

\[ k_{rT} = \frac{50 - 50}{50 + 50} = 0 \]

\[ k_{rS} = \frac{50 - 50}{50 + 50} = 0 \]

**Parameters:**
- \( R_S = 50\Omega \)
- \( Z_0 = 50\Omega, t_d = 1\text{ns} \)
- \( R_T = 50\Omega \)

**Diagram:**
- The diagram shows a circuit with a source and a load, illustrating the termination at 1ns.
- The graph on the right plots the source signal with a step beginning at 1ns.
Termination Examples - Open

\[ V_i = 1V \left( \frac{50}{50 + 50} \right) = 0.5V \]

\[ k_{rT} = \frac{\infty - 50}{\infty + 50} = +1 \]

\[ k_{rS} = \frac{50 - 50}{50 + 50} = 0 \]

\[ R_S = 50\Omega \]

\[ Z_0 = 50\Omega, \ t_d = 1\text{ns} \]

\[ R_T \sim \infty \ (1\text{M}\Omega) \]
Termination Examples - Short

- \( R_S = 50\Omega \)
- \( Z_0 = 50\Omega, t_d = 1\text{ns} \)
- \( R_T = 0\Omega \)

\[
V_i = 1V \left( \frac{50}{50 + 50} \right) = 0.5V
\]

\[
k_{rT} = \frac{0 - 50}{0 + 50} = -1
\]

\[
k_{rS} = \frac{50 - 50}{50 + 50} = 0
\]
Arbitrary Termination Example

\[ V_i = 1V \left( \frac{50}{400 + 50} \right) = 0.111V \]

\[ k_{rT} = \frac{600 - 50}{600 + 50} = 0.846 \]

\[ k_{rS} = \frac{400 - 50}{400 + 50} = 0.778 \]

\[ R_s = 400\Omega \]

\[ Z_0 = 50\Omega, t_d = 1\text{ns} \]

\[ R_T = 600\Omega \]
Lattice Diagram

\[ R_S = 400\Omega \]
\[ Z_0 = 50\Omega, \ t_d = 1\text{ns} \]
\[ R_T = 600\Omega \]

- \( k_{r_S} = 0.778 \)
- \( x = 0 \)
- \( k_{r_T} = 0.846 \)
- \( x = \ell \)

\( t = 1\text{ns} \)
\( V_S = 0.111V \)
\( t = 2\text{ns} \)
\( V_T = 0.205V \)
\( t = 3\text{ns} \)
\( V_S = 0.278V \)
\( t = 4\text{ns} \)
\( V_T = 0.340V \)
\( t = 5\text{ns} \)
\( V_S = 0.388V \)
\( t = 6\text{ns} \)
\( V_T = 0.429V \)

\( t = 7\text{ns} \)
\( V_S = 0.461V \)

in (step begins at 1ns)

Rings up to 0.6V

( DC voltage division)
Termination Reflection Patterns

\( R_S = 25\Omega, RT = 25\Omega \)
\( kr_S \& kr_T < 0 \)
Voltages Converge

\( R_S = 25\Omega, RT = 100\Omega \)
\( kr_S < 0 \& kr_T > 0 \)
Voltages Oscillate

\( R_S = 100\Omega, RT = 25\Omega \)
\( kr_S > 0 \& kr_T < 0 \)
Voltages Oscillate

\( R_S = 100\Omega, RT = 100\Omega \)
\( kr_S > 0 \& kr_T > 0 \)
Voltages Ring Up
Termination Schemes

- **No Termination**
  - Little to absorb line energy
  - Can generate oscillating waveform
  - Line must be **very short** relative to signal transition time
    - $n = 4 - 6$
  - Limited off-chip use

- **Source Termination**
  - Source output takes 2 steps up
  - Used in moderate speed point-to-point connections

\[
t_r > nT_{\text{round-trip}} = 2nl\sqrt{LC}
\]

\[
t_{\text{porch}} \approx 2l\sqrt{LC}
\]
Termination Schemes

- **Receiver Termination**
  - No reflection from receiver
  - Watch out for intermediate impedance discontinuities
    - Little to absorb reflections at driver

- **Double Termination**
  - Best configuration for min reflections
    - Reflections absorbed at both driver and receiver
  - Get half the swing relative to single termination
  - Most common termination scheme for high performance serial links
Differential Transmission Lines

- Differential signaling advantages
  - Self-referenced
  - Common-mode noise rejection
  - Increased signal swing
  - Reduced self-induced power-supply noise

- Requires 2x the number of signaling pins relative to single-ended signaling
  - But, smaller ratio of supply/signal (return) pins
  - Total pin overhead is typically 1.3-1.8x (vs 2x)

- Even mode
  - When equal voltages drive both lines, only one mode propagates called even more

- Odd mode
  - When equal in magnitude, but out of phase, voltages drive both lines, only one mode propagates called odd mode
Balanced Transmission Lines

- **Even (common) mode excitation**
  - Effective $C = C_C$
  - Effective $L = L + M$

- **Odd (differential) mode excitation**
  - Effective $C = C_C + 2C_d$
  - Effective $L = L - M$

\[
Z_{DIFF} = 2Z_{odd}, \quad Z_{CM} = \frac{Z_{even}}{2}
\]
PI-Termination

Even Mode Equivalent

Odd Mode Equivalent

\[ Z_{\text{odd}} = R_1 \parallel R_2/2 = Z_{\text{even}} \parallel R_2/2 \]

\[ R_2 = 2 \left( \frac{Z_{\text{odd}} Z_{\text{even}}}{Z_{\text{even}} - Z_{\text{odd}}} \right) \]

\[ Z_{\text{even}} = R_1 \]
T-Termination

\[ Z_{\text{even}} = R_2 + 2R_1 \]

\[ Z_{\text{odd}} = R_2 \]

\[ R_1 = \frac{1}{2} \left( Z_{\text{even}} - Z_{\text{odd}} \right) \]
Next Time

• Channel modeling
  • Time domain reflectometer (TDR)
  • Network analysis