Announcements

• Lab 5 Report and Prelab 6 due Apr. 3

• Reference Material
  • Jitter application notes posted on website
  • Majority of today’s material from Hall reference
Agenda

- Jitter Definitions
- Jitter Categories
- Dual Dirac Jitter Model
- System Jitter Budgeting
• Links must have margin in both the voltage AND timing domain for proper operation
• For independent design (interoperability) of TX and RX, a spec eye mask is used

Eye at RX sampler

RX clock timing noise or jitter (random noise only here)
Jitter Definitions

• Jitter can be defined as “the short-term variation of a signal with respect to its ideal position in time”

• Jitter measurements
  • Period Jitter ($J_{\text{PER}}$)
    • Time difference between measured period and ideal period
  • Cycle to Cycle Jitter ($J_{\text{CC}}$)
    • Time difference between two adjacent clock periods
    • Important for budgeting on-chip digital circuits cycle time
  • Accumulated Jitter ($J_{\text{AC}}$)
    • Time difference between measured clock and ideal trigger clock
    • Jitter measurement most relative to high-speed link systems
Jitter Statistical Parameters

- **Mean Value**
  - Can be interpreted as a fixed timing offset or “skew”
  - Generally not important, as usually can be corrected for

- **RMS Jitter**
  - Useful for characterizing the random component of jitter

- **Peak-to-Peak Jitter**
  - Function of both deterministic (bounded) and random (unbounded) jitter components
  - Must be quoted at a given BER to account for random (unbounded) jitter
Jitter Calculation Examples

\[ J_{CC1} = J_{PER2} - J_{PER1} \]
\[ J_{CC2} = J_{PER3} - J_{PER2} \]
\[ J_{CC3} = J_{PER4} - J_{PER3} \]

\( J_{PER} \) = time difference between measured period and ideal period
\( J_{CC} \) = time difference between two adjacent clock periods
\( J_{AC} \) = time difference between measured clock and ideal trigger clock

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Mean</th>
<th>RMS</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{PER} )</td>
<td>-0.06</td>
<td>0.02</td>
<td>-0.06</td>
<td>0.12</td>
<td>0.005</td>
<td>0.085</td>
<td>0.18</td>
</tr>
<tr>
<td>( J_{CC} )</td>
<td>0.08</td>
<td>-0.08</td>
<td>0.18</td>
<td>-</td>
<td>0.06</td>
<td>0.131</td>
<td>0.26</td>
</tr>
<tr>
<td>( J_{AC} )</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-0.11</td>
<td>0.01</td>
<td>-0.055</td>
<td>0.05</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Jitter Histogram

- Used to extract the jitter PDF
- Consists of both deterministic and random components
  - Need to decompose these components to accurately estimate jitter at a given BER
Jitter Categories

Total Jitter

Random Jitter (RJ)
- Unbounded, Gaussian distributed
- Key parameters: $\mu = 0$, $\sigma_{\text{RMS}}$
- Sources: Device noise (shot, flicker, thermal)

Deterministic Jitter (DJ)
- Bounded, peak-to-peak
- Key parameters: Maximum pk-pk jitter
- Sources: Losses, reflections, $t_f/t_i$ mismatch, spread spectrum clocking, crosstalk

Sinusoidal Jitter (SJ)

Data Dependent Jitter (DDJ)

Intersymbol Interference (ISI)

Duty Cycle Distortion (DCD)

Bounded Uncorrelated Jitter (Crosstalk)
Random Jitter (RJ)

- Unbounded and modeled with a gaussian distribution
  - Assumed to have zero mean value
  - Characterized by the rms value, $\sigma_{RJ}$
  - Peak-to-peak value must be quoted at a given BER
- Originates from device noise
  - Thermal, shot, flicker noise

$$RJ(t) = \frac{1}{\sqrt{2\pi\sigma_{RJ}^2}} e^{-\frac{t^2}{2\sigma_{RJ}^2}}$$
Deterministic Jitter (DJ)

- Bounded with a peak-to-peak value that can be predicted
- Caused by transmission-line losses, duty-cycle distortion, spread-spectrum clocking, crosstalk
- Categories
  - Sinusoidal Jitter (SJ or PJ)
  - Data Dependent Jitter (DDJ)
  - Intersymbol Interference (ISI)
  - Duty Cycle Distortion (DCD)
  - Bounded Uncoirrelated Jitter (BUJ)
Sinusoidal or Periodic Jitter (SJ or PJ)

- Repeats at a fixed frequency due to modulating effects
  - Spread spectrum clocking
  - PLL reference clock feedthrough
- Can be decomposed into a Fourier series of sinusoids
  \[ SJ(t) = \sum_i A_i \cos(\omega_i t + \theta_i) \]
- The jitter produced by an individual sinusoid is
  \[ PDF_{SJ}(t) = \begin{cases} 
  \frac{1}{\pi \sqrt{A^2 - t^2}} & A > |t| \\
  0 & A \leq |t| 
\end{cases} \]
Data Dependent Jitter (DDJ)

• Data dependent jitter is correlated with either the transmitted data pattern or aggressor (crosstalk) data patterns

• Caused by phenomena such as phase errors in serialization clocks, channel filtering, and crosstalk

• Categories
  • Duty Cycle Distortion (DCD)
  • Intersymbol Interference (ISI)
  • Bounded Uncorrelated Jitter (BUJ)
Duty Cycle Distortion (DCD)

- Caused by duty cycle errors in TX serialization clocks and rise/fall delay mismatches in post-serialization buffers
- Resultant PDF from a peak-to-peak duty cycle distortion ($\alpha_{DCD}$) is the sum of two delta functions:

$$PDF_{DCD}(t) = \frac{1}{2} \left[ \delta\left( t - \frac{\alpha_{DCD}}{2} \right) + \delta\left( t + \frac{\alpha_{DCD}}{2} \right) \right]$$
Intersymbol Interference (ISI)

- Caused by channel loss, dispersion, and reflections
- Equalization can improve ISI jitter
Bounded Uncorrelated Jitter (BUJ)

- Not aligned in time with the data stream
- Most common source is crosstalk
- Classified as uncorrelated due to being correlated to the aggressor signals and not the victim signal or data stream
- While uncorrelated, still a bounded source with a quantifiable peak-to-peak value
Total Jitter (TJ)

- The total jitter PDF is produced by convolving the random and deterministic jitter PDFs

\[ PDF_{JT}(t) = PDF_{RJ}(t) \ast PDF_{DJ}(t) \]

where \[ PDF_{DJ}(t) = PDF_{SJ}(t) \ast PDF_{DCD}(t) \ast PDF_{ISI}(t) \ast PDF_{BUJ}(t) \]
**Jitter and Bit Error Rate**

- Jitter consists of both deterministic and random components.

- Total jitter must be quoted at a given BER:
  - At BER=10^{-12}, jitter ~1675ps and eye width margin ~200ps.
  - System can potentially achieve BER=10^{-18} before being jitter limited.
Dual Dirac Jitter Model

- For system-level jitter budgets, the dual Dirac model approximates the complex total jitter PDF and allows for the budgeting of deterministic and random jitter components.

\[
RJ(t) = \frac{1}{\sqrt{2\pi}\sigma_{RJ}} e^{-\frac{t^2}{2\sigma_{RJ}^2}}
\]

\[
DJ(t) = \frac{\delta(t - DJ_{\delta\delta}/2)}{2} + \frac{\delta(t + DJ_{\delta\delta}/2)}{2}
\]

\[
JT(t) = RJ(t) \ast DJ(t) = \frac{1}{2\sqrt{2\pi}\sigma_{RJ}} \left[ e^{-\frac{t-DJ_{\delta\delta}/2}{2\sigma_{RJ}^2}} + e^{-\frac{t+DJ_{\delta\delta}/2}{2\sigma_{RJ}^2}} \right]
\]
Dual Dirac Jitter Model

- Jitter at a given BER is computed considering both leading and trailing edges

\[
\begin{align*}
\text{BER}_{\text{lead}}(t) &= 0.5 \left[ \text{erfc} \left( \frac{t - DJ_{\delta\delta} / 2}{\sqrt{2}\sigma_{RJ}} \right) - \text{erfc} \left( \frac{t + DJ_{\delta\delta} / 2}{\sqrt{2}\sigma_{RJ}} \right) \right], \\
\text{BER}_{\text{trail}}(t) &= 0.5 \left[ \text{erfc} \left( \frac{UI - t - DJ_{\delta\delta} / 2}{\sqrt{2}\sigma_{RJ}} \right) - \text{erfc} \left( \frac{UI - t + DJ_{\delta\delta} / 2}{\sqrt{2}\sigma_{RJ}} \right) \right]
\end{align*}
\]

where \( \text{erfc}(t) = \frac{2}{\sqrt{\pi}} \int_{t}^{\infty} e^{-x^2} \, dx \)
Dual Dirac Jitter Model Example

- Plot measured jitter PDF vs Q-scale

\[ Q_{BER}(BER) = \sqrt{2} \text{erf}^{-1} \left( 1 - \frac{BER}{\rho_T} \right) \]

where \( \rho_T \) is the transition density, typically 0.5

- Tails are used to extract \( \sigma_{RJ} \)

- Extrapolate to Q(0) to extract separation of dual-Dirac delta functions

\[ DJ_{\delta\delta} = \text{Extracted separation of dual-Dirac delta functions} \]
\[ DJ_{pp} = \text{Actual deterministic jitter peak-to-peak value} \]
Dual Dirac Jitter Model Example

- Extracted dual Dirac model matches well with measured jitter PDF

\[
\text{Dual - Dirac PDF} = \frac{1}{2\sqrt{2\pi}\sigma_{RJ}} \left[ e^{-\frac{t-DJ_{\delta}/2}{2\sigma_{RJ}^2}} + e^{-\frac{t+DJ_{\delta}/2}{2\sigma_{RJ}^2}} \right]
\]
System Jitter Budget

• For a system to achieve a minimum BER performance

\[ UI \geq DJ_{\delta\delta}(sys) + Q_{BER}\sigma_{RMS}(sys) \]

• The convolution of the individual deterministic jitter components is approximated by linear addition of the terms

\[ DJ_{\delta\delta}(sys) = \sum_{i} DJ_{\delta\delta}(i) \]

• The convolution of the individual random jitter components results in a root-sum-of-squares system rms value

\[ \sigma_{RMS}(sys) = \sqrt{\sum_{i} \sigma_{RMS}^2(i)} \]
Jitter Budget Example – PCI Express System

Architecture

Jitter Model

[Hall]
### Jitter Budget Example – PCI Express System

\[
DJ_{\delta\delta}(sys) = DJ_{\delta\delta}(TX) + DJ_{\delta\delta}(channel) + DJ_{\delta\delta}(RX) + DJ_{\delta\delta}(clock)
\]

\[
\sigma_{RMS}(sys) = \sqrt{\sigma_{RMS}^2(TX) + \sigma_{RMS}^2(channel) + \sigma_{RMS}^2(RX) + \sigma_{RMS}^2(clock)}
\]

#### TABLE 13-2. PCI Express 2.5-Gb/s Jitter Budget at 10^{-12} BER

<table>
<thead>
<tr>
<th>Component</th>
<th>Term</th>
<th>(\sigma_{RJ}) (ps)</th>
<th>(DJ_{\delta\delta}) (ps)</th>
<th>TJ (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference clock</td>
<td>TJ_{clock}</td>
<td>4.7</td>
<td>41.9</td>
<td>108</td>
</tr>
<tr>
<td>Transmitter</td>
<td>TJ_{TX}</td>
<td>2.8</td>
<td>60.6</td>
<td>100</td>
</tr>
<tr>
<td>Channel</td>
<td>TJ_{channel}</td>
<td>0</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Receiver</td>
<td>TJ_{Rx}</td>
<td>2.8</td>
<td>120.6</td>
<td>147</td>
</tr>
<tr>
<td>Linear TJ</td>
<td></td>
<td></td>
<td></td>
<td>458</td>
</tr>
<tr>
<td>RSS TJ</td>
<td></td>
<td></td>
<td></td>
<td>399.6</td>
</tr>
</tbody>
</table>

\[
6.15 \times 14.069 = 86.5
\]

#### TABLE 13-1. \(Q_{BER}\) as a Function of the Bit Error Rate

<table>
<thead>
<tr>
<th>BER</th>
<th>(Q_{BER})</th>
<th>BER</th>
<th>(Q_{BER})</th>
<th>BER</th>
<th>(Q_{BER})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \times 10^{-3}</td>
<td>6.180</td>
<td>1 \times 10^{-10}</td>
<td>12.723</td>
<td>1 \times 10^{-17}</td>
<td>16.987</td>
</tr>
<tr>
<td>1 \times 10^{-4}</td>
<td>7.438</td>
<td>1 \times 10^{-11}</td>
<td>13.412</td>
<td>1 \times 10^{-18}</td>
<td>17.514</td>
</tr>
<tr>
<td>1 \times 10^{-5}</td>
<td>8.530</td>
<td>1 \times 10^{-12}</td>
<td>14.069</td>
<td>1 \times 10^{-19}</td>
<td>18.026</td>
</tr>
<tr>
<td>1 \times 10^{-6}</td>
<td>9.507</td>
<td>1 \times 10^{-13}</td>
<td>14.698</td>
<td>1 \times 10^{-20}</td>
<td>18.524</td>
</tr>
<tr>
<td>1 \times 10^{-7}</td>
<td>10.399</td>
<td>1 \times 10^{-14}</td>
<td>15.301</td>
<td>1 \times 10^{-21}</td>
<td>19.010</td>
</tr>
<tr>
<td>1 \times 10^{-8}</td>
<td>11.224</td>
<td>1 \times 10^{-15}</td>
<td>15.882</td>
<td>1 \times 10^{-22}</td>
<td>19.484</td>
</tr>
<tr>
<td>1 \times 10^{-9}</td>
<td>11.996</td>
<td>1 \times 10^{-16}</td>
<td>16.444</td>
<td>7.7 \times 10^{-24}</td>
<td>20.000</td>
</tr>
</tbody>
</table>
Next Time

• Clocking Architectures