Lecture 10: Voltage-Controlled Oscillators
Announcements & Agenda

- HW3 is due Friday Oct 17
- VCO Fundamentals
- VCO Examples
- VCO Noise
Charge-Pump PLL Circuits

- Phase Detector
- Charge-Pump
- Loop Filter
- VCO
- Divider
Voltage-Controlled Oscillator

- Time-domain phase relationship

\[ \omega_{out}(t) = \omega_0 + \Delta \omega_{out}(t) = \omega_0 + K_{VCO} v_c(t) \]

\[ \phi_{out}(t) = \int \Delta \omega_{out}(t) dt = K_{VCO} \int v_c(t) dt \]
Voltage-Controlled Oscillators (VCO)

- **Ring Oscillator**
  - Easy to integrate
  - Wide tuning range (5x)
  - Higher phase noise

- **LC Oscillator**
  - Large area
  - Narrow tuning range (20-30%)
  - Lower phase noise
Barkhausen’s Oscillation Criteria

• Sustained oscillation occurs if
  \[ H(j\omega) = 1 \]

• 2 conditions:
  • Gain = 1 at oscillation frequency \( \omega_0 \)
  • Total phase shift around loop is \( n360^\circ \) at oscillation frequency \( \omega_0 \)

Closed-loop transfer function:

\[
\frac{H(j\omega)}{1 - H(j\omega)}
\]
Ring Oscillator Example

Three-stage ring oscillator

\[ H(s) = -\frac{A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3} \]

\[ \omega_{osc} = \sqrt{3}\omega_0 \]

\[ \tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = 60^\circ \]

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{-A_0^3}{\left(1 + s/\omega_0\right)^3} = \frac{-A_0^3}{\frac{A_0^3}{(1 + s/\omega_0)^3} + A_0^3} \]

\[ \sqrt{1 + \left(\frac{\omega_{osc}}{\omega_0}\right)^2} = 1 \]

\[ A_0 = 2 \]
Ring Oscillator Example

• 4-stage oscillator – work this one out yourself
  • \( A_0 = \sqrt{2} \)
  • Phase shift = 45°

• Easier to make a larger-stage oscillator oscillate, as it requires less gain and phase shift per stage, but it will oscillate at a lower frequency
LC Oscillator Example

- Oscillation phase shift condition satisfied at the frequency when the LC (and R) tank load displays a purely real impedance, i.e. 0° phase shift

\[
Z_{eq}(s) = \frac{R_s + L_1s}{1 + L_1C_1s^2 + R_sC_1s}
\]

\[
|Z_{eq}(s = j\omega)|^2 = \frac{R_s^2 + L_1^2\omega^2}{(1 - L_1C_1\omega^2)^2 + R_s^2C_1^2\omega^2}
\]
LC Oscillator Example

- Transforming the series loss resistor of the inductor to an equivalent parallel resistance

\[ L_P = L_1 \left(1 + \frac{R_S^2}{L_1^2 \omega^2}\right), \quad C_P = C_1, \quad R_P \approx \frac{L_2^2 \omega^2}{R_S} \]

\[ \omega_1 = \frac{1}{\sqrt{L_P C_P}} \]

[Razavi]
LC Oscillator Example

- Phase condition satisfied at

\[ \frac{1}{\sqrt{L_p C_p}} \]

- Gain condition satisfied when

\[ \left( g_m R_p \right)^2 \geq 1 \]

- Can also view this circuit as a parallel combination of a tank with loss resistance 2R_p and negative resistance of 2/g_m

- Oscillation is satisfied when

\[ \frac{1}{g_m} \leq R_p \]
CMOS Inverter Ring Oscillator

- Noise in the system will initiate oscillation, with the signals eventually exhibiting rail-to-rail swings
- While the small-signal transistor parameters \( g_m, g_o, C_g \), etc... can be used to predict the initial oscillations during small-signal start-up, these parameters can vary dramatically during large-signal operation
For this large-signal oscillator, the frequency is set by the stage delay, $T_D$

$T_D$ is a function of the nonlinear current drive and capacitances of each stage

As an “edge” has to propagate twice around the loop

$$f_{osc} = \frac{1}{6T_D}, \text{ or } \frac{1}{2NT_D}$$

where N is the oscillator stage number
Supply-Tuned Ring Oscillator

\[ T_{VCO} = 2nT_D \approx \frac{2nC_{stage}}{\beta(V_c - V_{th})} \]

\[ K_{VCO} = \frac{\partial f_{VCO}}{\partial V_c} = \frac{\beta}{2nC_{stage}} \]
Current-Starved Ring Oscillator

Current-starved VCO.
Capacitive-Tuned Ring Oscillator

\[ C_{\text{eff}} = \frac{C}{1 + sCR} \]
Symmetric Load Ring Oscillator

- Symmetric load provides frequency tuning at excellent supply noise rejection
- See Maneatis papers for self-biased techniques to obtain constant damping factor and loop bandwidth (% of ref clk)
LC Oscillator

• A variable capacitor (varactor) is often used to adjust oscillation frequency

• Total capacitance includes both tuning capacitance and fixed capacitances which reduce the tuning range

\[
\omega_{osc} = \frac{1}{\sqrt{L_P C_P}} = \frac{1}{\sqrt{L_P (C_{tune} + C_{fixed})}}
\]
Varactors

- **pn junction varactor**
  - Avoid forward bias region to prevent oscillator nonlinearity

- **MOS varactor**
  - Accumulation-mode devices have better Q than inversion-mode
Oscillator Noise

![Graphs of Jitter, Phase Noise, and Other Related Functions]

[McNeill] 20
Oscillator Phase Noise Model

\[
L(\Delta f) = 10 \log \left( \frac{\text{Noise Spectral Density}}{\text{Carrier Power}} \right) \quad \text{(dBc/Hz)}
\]

Leeson’s Model: \[
L(\Delta f) = 10 \log \left( \frac{2FkT}{P_{\text{sig}}} \left( 1 + \left( \frac{1}{2Q \Delta f} \right)^2 \right) \left( 1 + \frac{\Delta f_{1/f^3}}{\Delta f} \right) \right)
\]

- For improved model see Hajimiri papers
Open-Loop VCO Jitter

- Measure distribution of clock threshold crossings
- Plot $\sigma$ as a function of delay $\Delta T$
Open-Loop VCO Jitter

- Jitter $\sigma$ is proportional to $\sqrt{\Delta T}$
- $\kappa$ is VCO time domain figure of merit

$\sigma_{\Delta T(OL)}(\Delta T) \approx \kappa \sqrt{\Delta T}$
VCO in Closed-Loop PLL Jitter

- PLL limits $\sigma$ for delays longer than loop bandwidth $\tau_L$

\[
\tau_L = \frac{1}{2\pi f_L}
\]

[McNeill]
Ref Clk-Referenced vs Self-Referenced

Generally, we care about the jitter w.r.t. the ref. clock ($\sigma_x$)

However, may be easier to measure w.r.t. delayed version of output clk

- Due to noise on both edges, this will be increased by a $\sqrt{2}$ factor relative to the reference clock-referred jitter
Converting Phase Noise to Jitter

- RMS jitter for $\Delta T$ accumulation
  \[ \sigma^2_{\Delta T} = \frac{8}{\omega_o} \int_0^\infty S_\phi(f) \sin^2(\pi f \Delta T) df \]

- As $\Delta T$ goes to $\infty$
  \[ \sigma^2_T = \frac{2}{\omega_o^2} R_\phi(0) = \frac{4}{\omega_o} \int_0^\infty S_\phi(f) df \]

- Integration range depends on application bandwidth
  - $f_{\min}$ set by standard
    - Ex. Assumed CDR tracking bandwidth
  - Usually stop integration at $f_o/2$ or $f_o$ due to measurement limitations and aliasing components
Next Time

- VCO Noise (cont.)
- Divider Circuits