Please write your name in the space provided below
- Please verify that there are 5 pages in your exam
- You may use one double-sided page of notes and equations for the exam
- Good Luck!

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
<th>Max Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Name:  

UIN:  

1
Problem 1 (35 points)

For the oscillator shown below, assume that at the frequency of oscillation the inductor $Q = \frac{\omega_{osc} L_1}{R_s} = 3$, all transistors are operating in saturation, and you can ignore any transistor device capacitors. Also, use the following NMOS parameters

$$K_{PN} = \mu_n C_{ox} = 600 \mu A/V^2, \quad V_{TN} = 0.35 V, \quad \lambda_N = 0 V^{-1}$$

a) Give the frequency of oscillation.
b) What is the minimum $I_{TAIL}$ necessary for oscillation?
c) What is the maximum $I_{TAIL}$, such that all transistors remain in saturation throughout the oscillation period?

\[\alpha. \text{ Need to convert } L_1 \Rightarrow L_P\]

\[L_P = L_1 \left(1 + \left(\frac{1}{Q}\right)^2\right) = L_1 \left(1 + \frac{1}{3}\right) = 1.11 L_1\]

\[\omega_{osc} = \frac{1}{\sqrt{L_P C_P}} = \frac{1}{\sqrt{(1.11 L_1) (100 \mu F)}} = 30 \text{ Grad/s}\]

\[\omega_{osc} = 30 \text{ Grad/s} \Rightarrow 4.78 \text{ GHz}\]

\[b. \text{ For oscillation: } \quad g_m = \frac{1}{K_P} = \frac{R_s}{L_1 \omega_{osc}^2} = \frac{L_1 \omega_{osc}}{Q L_1 \omega_{osc}} = \frac{1}{3 (L_1 \omega_{osc})} = 11.1 \text{ mA/V}\]

Using $g_m = -\sqrt{K_P \frac{W}{L} \geq I_0}$ where $I_0 = \frac{I_{TAIL}}{2}$ during initial startup

\[I_{TAIL} = \frac{g_m^2}{K_P \frac{W}{L}} = \frac{(11.1 \text{ mA/V})^2}{(60 \mu A/\mu m) (100)} = 2.05 \text{ mA}\]

\[I_{TAIL} \geq 2.05 \text{ mA}\]

C. The differential swing amplitude is the transistor $V_{60}$, which is $I_{TAIL} R_P$. For saturation

\[I_{TAIL} \leq \frac{V_{TN}}{R_P} = \frac{0.35 V}{90 \Omega}\]

\[I_{TAIL} \leq 0.00389 mA\]
Problem 2 (35 points)
For the CDR shown below, assume that the incoming data has a transition density $TD = 0.25$. Assume that the only source of noise in the CDR below is from the VCO, which has $\kappa = 5\times10^{-9}\sqrt{s}$.

![CDR Circuit Diagram]

a) What is the necessary CDR loop bandwidth to satisfy a jitter generation of $1\text{ps}_{\text{rms}}$?

b) Design the loop filter components to yield a $\zeta = 1$. Note, for $\zeta = 1$, $\omega_{3\text{dB}} = 2.48\omega_n$.

c. In a PLL, the VCO will accumulate jitter equal to a value of

$$\sigma_{\text{rms}} = K\sqrt{\frac{1}{\omega_{3\text{dB}}^2}}$$

$$\omega_{3\text{dB}} = \frac{K^2 \omega_n^2}{\sigma_{\text{rms}}^2} = \frac{(5\times10^{-9}\sqrt{s})^2}{(1\text{ps})^2} = 25\text{MHz}$$

b. $\frac{\omega_n}{k} = 1 \Rightarrow \omega_{3\text{dB}} = 2.48\omega_n \Rightarrow \omega_n = \frac{\omega_{3\text{dB}}}{2.48} = 10.1\text{MHz}$

$$\omega_n = \sqrt{\frac{K_p\rho I_{cp} K_{VCO}}{C_p}} \Rightarrow C_p = \frac{K_p I_{cp} K_{VCO}}{\omega_n^2}$$

For the Hagen PD above, $K_p = \frac{1}{4\pi (7\text{pF})} = \frac{1}{4\pi}$

$$C_p = \frac{(100\mu\text{A})(2\pi 16\text{kHz})}{4\pi (10.1\text{MHz})^2} \approx 0.07\text{pF}$$

$$R_p = \frac{2\kappa}{\omega_n C_p} = \frac{2 (1)}{(10.1\text{MHz})(0.07\text{pF})^3} = 404\Omega$$
Problem 3 (30 points)
For the TIA shown below, assume that all transistors are operating in saturation with $r_o=\infty$. Obtain expressions for the following:

a) Low-Frequency Transimpedance.
b) The TIA’s input bandwidth. Note, it’s OK to neglect the transistor capacitors here.

\[
\begin{align*}
R_T &= R_1 \\
R_{in} &= \frac{1}{g_{m1}(1+g_{m2}R_2)} \\
\text{Input Bandwidth} &= \frac{g_{m1}(1+g_{m2}R_2)}{C_{\text{in}}} 
\end{align*}
\]
Scratch Paper