Texas A&M University  
Department of Electrical and Computer Engineering  

ECEN 620 – Network Theory (Broadband Circuit Design)  

Fall 2012  

Exam #2  

Instructor: Sam Palermo  

- Please write your name in the space provided below  
- Please verify that there are 5 pages in your exam  
- You may use one double-sided page of notes and equations for the exam  
- Good Luck!  

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
<th>Max Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Name: SAM PALERMO  

UIN:
Problem 1 (60 points)
For the PLL shown below, assume that the VCO gain is $K_{VCO}$ is positive, all transistors are operating in saturation with $r_o = \infty$ and you can ignore any transistor device capacitors.

![PLL Circuit Diagram]

a) Assume that the phase detector output UP is connected to charge pump inputs A and C, and that DN is connected to B and D. What is the effective loop filter transfer function

$$F(s) = \frac{V_{ctrl}(s)}{I_{CP}}$$

Will this yield the typical negative-feedback PLL system?

b) In the schematic above, connect (draw) the phase detector outputs to the charge pump inputs to realize the appropriate negative feedback proportional and integral control.

c) Draw the phase domain small signal model of the loop with the correct negative feedback connections.

d) Find the expressions for the loop gain, $LG(s)$, and determine the pole-zero locations of $LG(s)$.

e) In order to minimize the loop filter area, should $K$ be minimized or maximized? Why?

\[ a. \quad \frac{V_{ctrl}(s)}{I_{CP}} = \frac{K I_{CP}}{\frac{1}{g_{m2}}} + I_{CP} \left( \frac{1}{5c} \right) \left( -\frac{g_{m1}}{g_{m2}} \right) \]

\[ F(s) = \frac{V_{ctrl}(s)}{I_{CP}} = \frac{K}{g_{m2}} - \left( \frac{g_{m1}}{g_{m2}} \right) \left( \frac{1}{5c} \right) \]

The negative sign in the integral term will violate the typical negative-feedback PLL system.
c. 

\[ \phi_{\text{ref}} \rightarrow \begin{array}{c} + \\ \end{array} \rightarrow \frac{1}{2\pi} \rightarrow \begin{array}{c} + \\ \end{array} \rightarrow \frac{1}{g_{m2}} \rightarrow K I_{cp} \rightarrow \frac{g_{m1}}{g_{m2}} \frac{1}{sC} \rightarrow \frac{Kvco}{s} \rightarrow \phi_{\text{out}} \]

d. 

\[ L G(s) = \frac{I_{cp} \left[ K + \frac{g_{m1}}{g_{m2}} \left( \frac{1}{sC} \right) \right]}{s} \frac{Kvco}{s} \]

\[ = \frac{K I_{cp} K vco}{2\pi g_{m2}} \left( s + \frac{g_{m1}}{K C} \right) \]

2 poles at \( \pm \frac{g_{m1}}{K C} \)

1 zero at \( -\frac{g_{m1}}{K C} \)

e. The closed-loop transfer function will be

\[ \frac{K I_{cp} K vco}{2\pi g_{m2}} \left( s + \frac{g_{m1}}{K C} \right) \]

\[ \frac{s^2 + K I_{cp} K vco}{s} + \frac{I_{cp} K vco g_{m1}}{2\pi C g_{m2}} \]

\[ W_n = \sqrt{\frac{I_{cp} K vco g_{m1}}{2\pi C g_{m2}}} \]

\[ k = \frac{1}{2} \frac{W_n K C}{g_{m1}} \]

Maximizing \( k \) will allow a higher damping factor and better PM for a given capacitor size and won't affect \( W_n \).
Problem 2 (40 points)
A differential ring oscillator is shown below. Assume that all transistors are operating in saturation with $r_o=\infty$ and you can ignore any transistor device capacitors.

![Differential Ring Oscillator Diagram]

a) Find the frequency of oscillation if $R=1k\Omega$ and $C=100\text{pF}$.

b) What is the minimum transconductance ($g_m$) required for oscillation assuming that all stages are identical?

Each oscillator cell: $A_0 = \frac{s}{1 + \frac{s}{\omega_p}}$ where $A_0 = g_mR$

$$H(s) = -\frac{A_0^5}{(1 + \frac{s}{\omega_p})^5}$$

To oscillate each cell should contribute a phase shift of

$$\frac{360^\circ - 180^\circ}{5} = 36^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega_{osc}}{\omega_p}\right) = 36^\circ \Rightarrow \omega_{osc} = \tan(36^\circ) \omega_p$$

$$\boxed{\omega_{osc} = 7.276 \text{rad/s}}$$

b. $|H(\omega_{osc})| = 1 \Rightarrow \left(\frac{g_mR}{1 + (0.727)^2}\right)^5 = 1 \Rightarrow \left(\frac{g_mR}{1.24}\right)^5 = 1$

$$g_m \geq 1.24 \text{ mA/V}$$
Scratch Paper