Texas A&M University
Department of Electrical and Computer Engineering

ECEN 620 – Network Theory (Broadband Circuit Design)

Fall 2014

Exam #1

Instructor: Sam Palermo

- Please write your name in the space provided below
- Please verify that there are 6 pages in your exam
- You may use one double-sided page of notes and equations for the exam
- Good Luck!

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Problem 1 (50 points)
For the PLL block diagram shown below, assume that the phase detector gain is $K_{PD}$ and the VCO gain is $K_{VCO}$.

![Block Diagram](image)

(a) Find the expressions for the total loop gain, $LG(s)$, the closed-loop transfer function $H(s)=\phi_{out}(s)/\phi_{in}(s)$, and the phase error transfer function, $E(s)=\phi_e(s)/\phi_{in}(s)$. (25 points)

\[
F(s) = R + \frac{1}{sC} \\
LG(s) = \frac{K_{PD}K_{VCO}R(s + \frac{1}{RC})}{s^2} \\
H(s) = \frac{LG(s)}{1 + LG(s)} = \frac{K_{PD}K_{VCO}R(s + \frac{1}{RC})}{s^2 + sK_{PD}K_{VCO}R + \frac{K_{PD}K_{VCO}}{C}} \\
E(s) = \frac{1}{1 + LG(s)} = \frac{s^2}{s^2 + sK_{PD}K_{VCO}R + \frac{K_{PD}K_{VCO}}{C}}
\]

\[
LG(s) = \frac{K_{PD}K_{VCO}R(s + \frac{1}{RC})}{s^2} \\
H(s)=\phi_{out}(s)/\phi_{in}(s)= \frac{K_{PD}K_{VCO}R(s + \frac{1}{RC})}{s^2 + sK_{PD}K_{VCO}R + \frac{K_{PD}K_{VCO}}{C}} \\
E(s)=\phi_e(s)/\phi_{in}(s)= \frac{s^2}{s^2 + sK_{PD}K_{VCO}R + \frac{K_{PD}K_{VCO}}{C}}
\]
b) Assume that \( K_{PD} = 10\mu A/(2\pi) \) and \( K_{VCO} = 2\pi \times (100\text{MHz}/V) \). Give the loop filter elements (R and C) for a \( \zeta = 1 \) and steady-state phase error of \( 10^{-2} \) rad. with a \( 10^{11} \text{rad/s}^2 \) frequency ramp input. (10 points)

\[
E(s) = \frac{s^2}{s^2 + sK_{PD}K_{VCO} R + \frac{K_{PD}K_{VCO}}{C}}
\]

\[
\Phi_{\text{ss}} = \lim_{s \to 0} \frac{s^3}{s^2} \frac{\Lambda}{s^2} = \frac{\Lambda}{K_{PD}K_{VCO} C} \Rightarrow C = \frac{\Phi_{\text{ss}} K_{PD}K_{VCO}}{\Lambda}
\]

\[
\Lambda = \frac{\omega_d}{2} R = \frac{K_{C}}{2} \sqrt{K_{PD}K_{VCO} C} = \frac{K_{C}}{2} \sqrt{K_{PD}K_{VCO} C}
\]

\[
R = \frac{\sqrt{2} \Lambda}{K_{PD}K_{VCO} C} = \frac{\sqrt{2} (1)}{\sqrt{(\frac{\omega_d}{2\pi})(\frac{10\mu A}{2\pi})(100\text{MHz})(100\mu F)}} = 6.32 \text{ kQ}
\]

\[
C = 100 \mu F
\]

\[
R = 6.32 \text{ kQ}
\]

\[
C = 100 \mu F
\]

c) Now assume that the loop filter capacitor C is modified to relax the steady-state phase error to 0.1 rad. with a \( 10^{11} \text{ rad/s}^2 \) frequency ramp input. Using the same R numerical value from part(b), what is the new \( \zeta \)? (10 points)

For 0.1 rad phase error \( \Rightarrow \zeta_{\text{new}} = 10 \text{Cold} \)

\[
\Lambda = \frac{K_{C}}{2} \sqrt{K_{PD}K_{VCO} C_{\text{new}}} = \sqrt{10} \zeta_{\text{old}} \zeta_{\text{old}} = \sqrt{10} \zeta_{\text{old}}
\]

\[
\zeta = \sqrt{10}
\]

d) Qualitatively comment on how the accuracy of the steady-state phase error with a frequency ramp impacts the PLL stability. (5 points)

If the PLL is designed with a smaller steady-state phase error the stability will be worse (lower \( \zeta \)) if the loop filter's zero (resistor) is not adjusted to a lower (higher R) value.
Problem 2 (50 points)
This problem analyzes a PLL with a voltage-controlled delay line (VCDL) in the feedback path. The VCDL can be modeled as adding phase in the feedback path with a certain gain factor $K_{VCDL}$ (rad/V).

![ PLL Diagram ]

a) Find the expressions for the total loop gain, $LG(s)$, the phase error transfer function, $E(s) = \phi_e(s)/\phi_{in}(s)$, and the closed-loop transfer function $H(s) = \phi_{out}(s)/\phi_{in}(s)$. (25 points)

$$LG(s) = \frac{Icp}{2\pi} \left( \frac{1}{sC} \right) \left[ \frac{K_{VCO}}{s} + K_{VCDL} \right] = \frac{Icp K_{VCDL}}{2\pi C} \left( \frac{s + \frac{K_{VCO}}{K_{VCDL}}} {s^2} \right)$$

$$E(s) = \frac{1}{1 + LG(s)} = \frac{s^2 + \frac{Icp K_{VCO}}{2\pi C} s + \frac{Icp K_{VCO}}{2\pi C}}{s^2 + \frac{Icp K_{VCO}}{2\pi C} s + \frac{Icp K_{VCO}}{2\pi C}}$$

$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{s^2 + \frac{Icp K_{VCO}}{2\pi C} s + \frac{Icp K_{VCO}}{2\pi C}}{s^2 + \frac{Icp K_{VCO}}{2\pi C} s + \frac{Icp K_{VCO}}{2\pi C}} \left( \frac{\frac{Icp}{2\pi} \left( \frac{1}{sC} \right) \left( K_{VCO} \right)}{s^2} \right)$$

$$LG(s) = \frac{\frac{Icp K_{VCDL}}{2\pi C} \left( \frac{s + \frac{K_{VCO}}{K_{VCDL}}} {s^2} \right)}{s^2 + \frac{Icp K_{VCO}}{2\pi C} s + \frac{Icp K_{VCO}}{2\pi C}}$$

$$E(s) = \frac{\phi_e(s)}{\phi_{in}(s)} = \frac{s^2}{s^2 + \frac{Icp K_{VCO}}{2\pi C} s + \frac{Icp K_{VCO}}{2\pi C}}$$

$$H(s) = \frac{\phi_{out}(s)}{\phi_{in}(s)} = \frac{Icp K_{VCO}}{2\pi C}$$
b) Assume that $K_{PD}=10 \mu A/(2\pi)$, $K_{VCO}=100\text{Mrad}/(s\text{V})$, and $K_{VCDL}=10\text{rad}/\text{V}$. Find the value of the loop filter capacitor $C$ for a 45° phase margin. (20 points)

$$PM = 180^\circ + \angle LG(j\omega)$$

$$180^\circ + \tan^{-1}\left(\frac{\omega K_{VCO}}{K_{VCO}}\right) - 180^\circ = 45^\circ$$

$$\tan^{-1}\left(\frac{K_{VCO}}{K_{VCO}}\right) = \frac{100\text{Mrad}/\text{V}}{10\text{rad}/\text{V}} = 10\text{Mrad}/\text{s}$$

$$\left|LG(j\omega)\right|^2 = 1$$

$$\frac{\left(\frac{10\text{mA}}{2\pi}\right)\left(10\text{rad}/\text{V}\right)}{(\sqrt{2}\ 10^7)} = 1$$

$$\left(10^7\right)^2$$

$$C = \left(\frac{\left(\frac{10\text{mA}}{2\pi}\right)\left(10\text{rad}/\text{V}\right)}{(\sqrt{2})} \right) = 2.25 \mu F$$

$$C = 2.25 \mu F$$

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c) Qualitatively comment on the main differences between this PLL with a VCDL in the feedback path and the conventional charge-pump PLL with a series RC loop filter. Which architecture provides more filtering to high-frequency input phase noise? (5 points)

While the loop gain and phase error transfer function are similar, the closed-loop output transfer function doesn't have a zero due to the proportional term appearing in the feedback. Thus, the PLL with VCDL in the feedback will filter high-frequency input phase noise by -40dB/dec, vs -20dB/dec with standard series RC loop filter.