Announcements & Agenda

- HW5 (Preliminary Project Report) Due Apr. 18
- Two Stage Miller OTA
- OpAmp Characterization
Multi-Stage Amplifiers

- Single-stage amplifiers typically have to trade-off gain and swing range
- Multi-stage amplifiers allow for higher gain without sacrificing swing range
- The major challenge with multi-stage amplifiers is achieving adequate phase margin to insure stability in a feedback configuration
Two Stage Miller OTA

\[
\begin{align*}
\text{DC Gain } A_{VDC} &= A_{v1}A_{v2} = \left( -\frac{g_{m2}}{g_{o2} + g_{o4}} \right) \left( -\frac{g_{m8}}{g_{o8} + g_{o7}} \right) = \frac{g_{m2}g_{m8}}{(g_{o2} + g_{o4})(g_{o8} + g_{o7})} \\
A_{VDC} &= G_m R_{out} \\
R_{out} &= \frac{1}{g_{o8} + g_{o7}} \\
G_m &= -g_{m8}A_{v1} = \frac{g_{m8}g_{m2}}{g_{o2} + g_{o4}}
\end{align*}
\]
• Stage 1 is a differential amplifier with an active load
• Stage 2 is a common-source amplifier with a large Miller capacitor
• Using a Thevenin equivalent for Stage 1, we can use the common-source equations from Lecture 8
The amplifier should be designed to yield one dominant pole, so we use the dominant pole approximation equations:

Neglecting Transistor Capacitances

\[ |\omega_{p1}| = \frac{1}{R_{out1}(1 + g_{m8}R_{out2})C_C + R_{out2}(C_C + C_L)} \approx \frac{1}{R_{out1}g_{m8}R_{out2}C_C} \]

\[ |\omega_{p2}| = \frac{R_{out1}(1 + g_{m8}R_{out2})C_C + R_{out2}(C_C + C_L)}{R_{out1}R_{out2}C_CC_L} \approx \frac{g_{m8}}{C_L} \]

where \( R_{out1} = r_{O2}r_{O4} \) and \( R_{out2} = r_{O7}r_{O8} \)
Frequency Response – No Compensation

Main equations

\[ A_{VDC} = -\frac{g_{m1} g_{m3}}{g_1 g_L} \]

\[ \omega_{p1} = -\frac{g_1}{C_1} \quad \text{(LHP)} \]

\[ \omega_{p2} = -\frac{g_L}{C_L} \quad \text{(LHP)} \]

\[ GBW = (A_{VDC}) \ast \left( \min(\omega_{p1}, \omega_{p2}) \right) \quad \text{(if dominant pole system, valid?)} \]

\[ \text{Phase Margin} = 180 - \tan^{-1}\left( \frac{\omega_u}{\omega_{p1}} \right) - \tan^{-1}\left( \frac{\omega_u}{\omega_{p2}} \right) \]

Phase Margin < 45 degrees

\[ A(s) = \frac{A_{VDC}}{1 + \frac{s}{\omega_{p1}} \left( 1 + \frac{s}{\omega_{p2}} \right)} \]
Frequency Response – Miller Compensation (Ignoring $\omega_z$)

Phase compensation $\Rightarrow$ Pole splitting techniques!!

$$A_{VDC} = \frac{-g_{m1} g_{m3}}{g_1 g_L}$$

$$\omega_{p1} = -\frac{g_1}{C_1 + \frac{g_{m3} C_M}{g_L}} \quad \text{(LHP)}$$

$$\omega_{p2} = \frac{-g_{m3}}{C_1 + C_L} \quad \text{(LHP)}$$

$$GBW' = (A_{VDC})^*|\omega_{p1}| \approx \frac{g_{m1}}{C_M}$$

$$Phase\_ margin = 180 - \tan^{-1}\left(\frac{GBW'}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{GBW'}{\omega_{p2}}\right)$$

After compensation $\Rightarrow$ Phase Margin > 45 degrees $\Rightarrow$ Bandwidth is reduced!!!
Frequency Response – Miller Compensation (Considering $\omega_z$)

\[ A_{VDC} = -\frac{g_{m1} g_{m3}}{g_1 g_L} \]  
\[ \omega_{p1} = -\frac{g_1}{C_1 + \frac{g_{m3}}{g_L} C_M} \] (LHP)  
\[ \omega_{p2} = -\frac{g_{m3}}{C_1 + C_L} \] (LHP)  
\[ GBW' = (A_{VDC})' \left| \omega_{p1} \right| \approx \frac{g_{m1}}{C_M} \]  
\[ \text{Phase margin} = 180 - \tan^{-1}\left(\frac{GBW'}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{GBW'}{\omega_{p2}}\right) - \tan^{-1}\left(\frac{GBW'}{\omega_{ZERO}}\right) \]  

 Parasitic (bad) RHP zero!!

\[ \omega_{ZERO} = +\frac{g_{m3}}{C_M} \] (RHP)

After compensation
- Phase Margin > 45 degrees
- Bandwidth is reduced!!
Parasitic (bad) RHP zero!!
Can be catastrophic if close or below wu!

\[ \omega_{\text{ZERO}} = \frac{g_{m3}}{C_M} \]

Phase margin = \(180 - \tan^{-1}\left(\frac{\omega_u}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p2}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{\text{ZERO}}}\right)\)

After compensation
- Phase Margin \(<< 45\) degrees
- Phase is equivalent to having 3 poles below unity gain frequency \(\Rightarrow\) Unstable!

After compensation
- Phase Margin > 45 degrees
- Bandwidth is reduced!!!
Adding a series resistance

\[ A(s) = \frac{A_{VD}}{s} \left( 1 + \frac{s}{\omega_p} \right) \left( 1 + \frac{s}{\omega_{p1}} \right) \left( 1 + \frac{s}{\omega_{p2}} \right) \left( 1 + \frac{s}{\omega_{p3}} \right) \]

\[ \omega_{p3} \approx -\frac{1}{R_Z C_1} \] (Generally high frequency & can be ignored)

\[ \omega_z = \frac{1}{\left( \frac{1}{g_m} - R_Z \right) C_M} \]

Can design \( R_Z \) to improve phase margin

Non-zero \( R_Z \) will push RHP to a higher frequency (initially)

\[ R_Z = \frac{1}{g_m} \] pushes the RHP zero to infinity

\[ R_Z > \frac{1}{g_m} \] pushes zero from RHP to LFP

\[ R_Z = \frac{C_L + C_M + C_1}{g_m C_M} \] can cancel \( \omega_{p2} \)
Two Stage Miller OTA Noise

Output - Referred Noise Current PSD

\[
\frac{i_o^2}{\Delta f} = \frac{8kT}{3} \left[ g_{m8} + g_{m7} + 2g_{m2} \left( \frac{g_{m8}}{g_{o2} + g_{o4}} \right)^2 + 2g_{m4} \left( \frac{g_{m8}}{g_{o2} + g_{o4}} \right)^2 \right]
\]

Input - Referred Noise Voltage PSD

\[
\frac{v_i^2}{\Delta f} = \frac{i_o^2}{\Delta f} \left( \frac{1}{G_m^2} \right) = \frac{8kT}{3g_m} \left[ 2 + 2g_{m4} + \frac{g_{m8} + g_{m7}}{g_{m2} \left( \frac{g_{m8}}{g_{o2} + g_{o4}} \right)^2} \right]
\]
OPAMP Characterization

Main parameters to be measured:

• DC gain \((10^4-10^6) \text{ V/V}\)
• Frequency Limitations
  • Bandwidth (Few Hertz~1kHz)
  • Gain-Bandwidth product (1~100 Mhz)
• Output resistance
• Input Impedance
• Signal Swing
  • Common-mode input range
  • Output swing
• Stability
• DC Offset
• Slew-rate
• CMRR
• PSRR

For this section, see:
CMOS Analog design, Allen & Holberg
OPAMP Characterization

DC gain \((10^4-10^6) \text{ V/V})\ :

- Very difficult to measure in open-loop due to DC offsets.

\[
\begin{align*}
\begin{array}{c}
- \\
\end{array}
\begin{array}{c}
+ \\
\end{array}
\end{align*}
\]

\[V_0 \neq 0\]

How to measure/characterize it?

- Stabilize for DC

\[
\begin{align*}
\begin{array}{c}
V_i \\
\end{array}
\begin{array}{c}
+ \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
- \\
\end{array}
\begin{array}{c}
V_0 \\
\end{array}
\end{align*}
\]

- For DC, the OPAMP operates in closed loop!!
- For frequencies higher than \(1/R_C C_C\), the OPAMP operates in open-loop with a grounded load given by \(R_C\).
OPAMP: DC Characterization

How to measure/characterize it?

- At DC
  \[ \frac{v_o}{v_i} = \frac{A(s)}{1 + A(s)} \approx 1 \]

- If \( A(s)B(s) \ll 1 \) then the measured gain is dominated by the OPAMP transfer function!

\[ \frac{v_o}{v_i} \approx A(s) \]

Make sure to set \( R_C C_C \) such that

\[ \omega_{p1} > [A_{DC}] \left( \frac{1}{R_C C_C} \right) \]
OPAMP Characterization

DC Offset
OPAMP Characterization: GBW and stability

\[ V_{in} \rightarrow + \quad A(s) \rightarrow V_0 \]

\[ A_{VDC} \]

\[ \omega \rightarrow \omega_{p1} \rightarrow GBW \]

\[ V_{in} \rightarrow + \quad A(s) \rightarrow V_0 \]

\[ V_i \rightarrow t \]

Enough phase margin
OPAMP Characterization: GBW and stability

OPAMP (open-loop)

Not enough phase margin
OPAMP Characterization: Slew-Rate (max speed)

\[ V_{in} \quad \rightarrow \quad A(s) \quad \rightarrow \quad V_0 \]

Max output level

Slew-Rate

\[ \max \frac{d}{dt} V_o(t) \]

Min output level

VSS

VDD
Use a slow triangular input signal such that the raising and falling edges are not determined by slew rate limitations.
OPAMP Characterization: Input and Output impedance

\[ \omega \]

\[ R_C \text{ and } C_C \text{ as large as possible!!} \]

At Low frequencies:
\[ Z_{\text{measured}} \neq Z_o \quad \text{(Why?????)} \]

At medium frequencies:
\[ Z_{\text{measured}} = Z_o || R_C \]

Be sure that the OPAMP (all internal transistors) is properly biased during characterization!!
Next Time

- OpAmp Feedback & Stability
- Common-Mode Feedback Techniques