Announcements

• Lab
  • Lab 1 begins on 9/14
  • Check the updated Lab 1 on website

• TA Noah Yang’s office hours are on Tuesday 10AM-12PM

• HW1 is posted on the website and due 9/18
BJT Circuit Symbols

• BJTs are 3 terminal devices
  • Collector, Base, & Emitter

• 2 complementary BJT devices: NPN & PNP
MOSFET Circuit Symbols

- MOSFETs are 4-terminal devices
  - Drain, Gate, Source, & Body
- Body terminal generally has small impact in normal operation modes, thus device is generally considered a 3-terminal device
  - Drain, Gate, and Source are respectively similar to the Collector, Base, and Emitter of the BJT
- 2 complementary MOSFETS: NMOS, PMOS
For analog applications, we generally desire the BJT to operate in the "Active" region

$$V_{CE} \geq V_{CE, sat} = 0.3V$$
NPN “Large-Signal” (DC) Model

- Exact equation

$$V_{CE} \geq V_{CE,\text{sat}}$$

$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$

$$i_C = I_S e^{v_{BE}/V_T}$$

- However, $V_{BE}$ doesn’t change much over large values of $I_C$. So, for hand calculations we assume a fixed $V_{BE}$ and use the following

$$V_{BE} = 0.7 \text{ V}$$

$$I_C = \beta I_B$$

$$I_E = I_C + I_B$$
For analog applications, we generally desire the MOSFET to operate in the “Saturation” region

\[ V_{DS} \geq V_{OV} = V_{GS} - V_{in} \]
NMOS “Large-Signal” (DC) Model

\[ V_{DS} \geq V_{ov} \Rightarrow \text{Saturation} \]

\[ V_{ov} = V_{GS} - V_{tn} \]

\[ I_G = 0 \]

\[ I_D = \frac{k_n'}{2} \frac{W}{L} V_{ov}^2 \]

where \( k_n' = \mu_n C_{ox} \)
PNP “Large-Signal” (DC) Output Characteristic

- Similar operation to NPN transistor, except
  - $I_C$ flows out of the device
  - Flip the terminal voltages in calculations
- For analog applications, we generally desire the BJT to operate in the “Active” region
  \[ V_{EC} \geq V_{EC_{sat}} = 0.3V \]
PNP “Large-Signal” (DC) Model

- Exact equation

\[ V_{EC} \geq V_{EC, sat} \]

\[ i_B = \frac{I_S e^{v_{EB}/V_T}}{\beta} \]

\[ i_C = I_S e^{v_{EB}/V_T} \]

- However, \( V_{EB} \) doesn’t change much over large values of \( I_C \). So, for hand calculations we assume a fixed \( V_{EB} \) and use the following

\[ V_{EB} = 0.7 \text{ V} \]

\[ I_C = \beta I_B \]

\[ I_E = I_C + I_B \]
PMOS “Large-Signal” (DC) Output Characteristic

- Similar operation to NMOS transistor, except
  - $I_D$ flows out of the device
  - Flip the terminal voltages in calculations

- For analog applications, we generally desire the BJT to operate in the “Saturation” region

$$V_{SD} \geq V_{OV} = V_{SG} - |V_{tp}|$$
PMOS “Large-Signal” (DC) Model

\[ V_{SD} \geq V_{ov} \Rightarrow \text{Saturation} \]

\[ V_{ov} = V_{SG} - |V_{tp}| \]

\[ I_G = 0 \]

\[ I_D = \frac{k'_p}{2} \frac{W}{L} V_{ov}^2 \]

where \( k'_p = \mu_p C_{ox} \)
Large-Signal “DC” Response

\[ v_O = v_{DC} = V_{DD} - I_{DS}R \]
For small-signal analysis, we “linearize” the response about the DC operating point.

If the signal is small enough, linearity holds and the complete response is the summation of the large-signal “DC” response and the small-signal “AC” response.
First, DC analysis is performed to determine operating point and obtain small-signal parameters. Second, sources are set to zero and small-signal model is used.
NPN & NMOS Small-Signal T-Models ($r_o = \infty$)

\[
\begin{align*}
re &= \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m} \\
g_m &= \frac{I_C}{V_T} \\
\end{align*}
\]

\[
\begin{align*}
g_m &= k'_n \frac{W}{L} V_{ov} = \sqrt{2k'_n \frac{W}{L} I_D} \\
\end{align*}
\]
Formal PNP & PMOS Small-Signal T-Models \((r_o=\infty)\)

- While these are the formal models, for both the PNP and PMOS device you can use the exact same model as the NPN and NMOS device
  - This is easier to remember
  - Obtained by flipping the current sources in the formal model

\[
\begin{align*}
  r_e &= \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m} \\
  g_m &= \frac{I_C}{V_T}
\end{align*}
\]
Input/Output Impedances

- The figure above shows the techniques of measuring input and output impedances.
- Small signal analysis is used.

\[ R_x = \frac{V_x}{I_x} \]
BJT Node AC Resistances \((r_o=\infty)\)

- Using T-model, base AC resistance is the resistors connected from base through the emitter to ground multiplied by \((\beta+1)\)

\[
R_{\text{base}} = (\beta + 1)(r_e + R_E)
\]

- If \(r_o\) is \(\infty\), then collector AC resistance is \(\infty\)

\[
R_{\text{collector}} = \infty
\]

- Using T-model, emitter AC resistance is \(r_e\) plus the base resistor divided by \((\beta+1)\)

\[
R_{\text{emitter}} = r_e + \frac{R_B}{\beta + 1}
\]
BJT Amplifiers AC Gain ($r_o=\infty$)

**CE Amp**

$$\frac{v_o}{v_i} = -\frac{\alpha R_C}{r_e}$$

**CC Amp**

$$\frac{v_o}{v_i} = \frac{R_E}{r_e + R_E}$$

**CB Amp**

$$\frac{v_o}{v_i} = \frac{\alpha R_C}{R_{emitter}}$$

**CE Amp w/ $R_E$**

$$\frac{v_o}{v_i} = -\frac{\alpha R_C}{r_e + R_E}$$
MOSFET Node AC Resistances \( (r_o=\infty) \)

- Gate input impedance is capacitive, which is assumed to be infinite resistance at low frequencies

\[ R_{gate} = \infty \]

- If \( r_o = \infty \), then drain AC resistance is \( \infty \)

\[ R_{drain} = \infty \]

- Using T-model, source AC resistance is the \( \frac{1}{g_m} \) resistor

\[ R_{source} = \frac{1}{g_m} \]
MOSFET Amplifiers AC Gain \((r_0=\infty)\)

\[
\frac{v_o}{v_i} = \frac{-R_D}{1/gm}
\]

\[
\frac{v_o}{v_i} = \frac{R_S}{1/gm + R_S}
\]

\[
\frac{v_o}{v_i} = \frac{R_D}{R_{\text{source}}}
\]

\[
\frac{v_o}{v_i} = \frac{-R_D}{1/gm + R_S}
\]
NPN Small-Signal $\pi$ Model & Introducing Finite $r_o$

Early Effect $\rightarrow$ $r_o$

$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{V_A}{I_C}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$V_T = \frac{kT}{q}$$
PNP Small-Signal π Model w/ Finite $r_o$

- While this is the formal model, **you can use the exact same model as the NPN device**
  - This is easier to remember
  - Obtained by flipping the small signal $v_{eb}$ and the current source in the formal model

$g_m = \frac{I_C}{V_T}$

$r_o = \frac{V_A}{I_C}$

$r_{\pi} = \frac{\beta}{g_m}$

$V_T = \frac{kT}{q}$
MOSFET – Impact of Body Voltage

• Before we go over the MOSFET $\pi$-model, let's consider the impact of the 4th terminal, the Body, on the drain current $I_D$.

• The MOSFET $V_{tn}$ is a function of the Body-Source voltage $V_{BS}$.
  • If the threshold voltage changes, then so does $I_D$.

\[ I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{tn})^2 (1 + \lambda V_{DS}) \]

\[ V_{tn} = V_{tn0} + \gamma \left[ \sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F} \right] \Rightarrow V_{tn0} \left| V_{SB} = 0 \right. \]
Body Transconductance, $g_{mb}$

- The small-signal drain current changes with $V_{BS}$ modulation due to changes in $V_{tn}$

\[
g_{mb} = \left. \frac{\partial i_D}{\partial V_{bs}} \right|_Q \approx \mu C_{OX} \frac{W}{L_{eff}} [V_{GS} - V_{in}] \left( \frac{\partial V_{in}}{\partial V_{bs}} \right)_Q \approx \frac{\gamma g_m}{2\sqrt{2\phi_F + V_{SB}}}
\]

[Image of a graph showing the relationship between $I_D$ and $V_{BS}$ with a red wave pattern indicating modulation]

[Razavi]
NMOS Small-Signal $\pi$ Model w/ Finite $r_0$ & Body Transconductance $g_{mb}$

Channel Length Modulation $\rightarrow$ $r_0$

Body Effect

\[
V_m = V_{m0} + \gamma \left[ \sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F} \right] \Rightarrow V_{m0} \big|_{V_{SB}=0} \rightarrow g_{mb}
\]

\[
g_m = \sqrt{2k'_n \frac{W}{L}} I_D
\]

\[
r_0 = \frac{1}{\lambda_n I_D}
\]

\[
g_{mb} = \chi g_m
\]

where \( \chi = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}} \)

- Note, $g_{mb}$ is generally a weak effect ($\sim 0.1 g_m$). Thus, we often ignore it.
- In problems/assignments I’ll make it clear when we need to consider it
PMOS Small-Signal π Model w/ Finite $r_o$ & Body Transconductance $g_{mb}$

![Diagram of PMOS Small-Signal π Model]

- While this is the formal model, **you can use the exact same model as the NMOS device**
  - This is easier to remember
  - Obtained by flipping the small signal $v_{sg}$ and $v_{sb}$ and the current sources in the formal model

Channel Length Modulation $\rightarrow r_o$

Body Effect $V_{ip} = V_{ip0} + \gamma \left[ \sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F} \right] \Rightarrow V_{ip0}|_{V_{SB}=0} \rightarrow g_{mb}$

$$g_m = \sqrt{2k_p' \frac{W}{L} I_D}$$
$$r_o = \frac{1}{\lambda_p I_D}$$

$$g_{mb} = \chi g_m$$

where $\chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$

- Note, $g_{mb}$ is generally a weak effect ($\sim 0.1g_m$). Thus, we often ignore it.
- In problems/assignments I’ll make it clear when we need to consider it
Two-Port Modeling of Amplifiers

- It is often useful to model transistor amplifiers with a Norton-equivalent model consisting of a transconductance current source and parallel output resistance

\[
\frac{v_o}{v_i} = G_m R_n
\]
**Two-Port Modeling – Extracting $G_m$ and $R_n$**

- Short the output, apply an input voltage stimulus, and measure output current

$$G_m = \frac{i_{sc}}{v_i} \bigg|_{v_o=0}$$

- Short the input, apply an output voltage stimulus, and measure current into output node

$$R_n = \frac{v_o}{i_o} \bigg|_{v_i=0}$$
Two-Port Modeling – Multi-Stage Amplifiers

\[ \frac{v_o}{v_i} = \frac{v_1}{v_i} \cdot \frac{v_2}{v_1} \cdots \frac{v_o}{v_{N-1}} \]

\[ R_i = \frac{v_i}{i_i} \]

\[ R_o = \frac{v_o}{i_o} \]
Two-Port Modeling – Multi-Stage Amplifiers

- Note, R_n1 also includes the input resistance of Stage 2
Two-Port Modeling – Multi-Stage Amplifiers

- Repeat procedure stage by stage

\[ G_{m2} = \frac{i_{sc2}}{v_1} \]
\[ \frac{v_2}{v_1} = G_{m2}R_{n2} \]
Two-Port Modeling – Multi-Stage Amplifiers

• Repeat procedure stage by stage
• This procedure will be useful for analyzing multi-stage transistor amplifiers
BJT Amplifiers AC Gain ($r_o=\infty$)

**CE Amp**

\[ \frac{v_o}{v_i} = -g_m R_C = -\frac{\alpha R_C}{r_e} \]

\[ G_m = -g_m = -\frac{\alpha}{r_e} \]

\[ R_n = R_C \]

**CE Amp w/ $R_E$**

\[ \frac{v_o}{v_i} = -\frac{g_m R_C}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha R_C}{r_e + R_E} \]

\[ G_m = -\frac{g_m}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha}{r_e + R_E} \]

\[ R_n = R_C \]
BJT Amplifiers AC Gain \( (r_o = \infty) \)

**CC Amp**

\[
\frac{v_o}{v_i} = \frac{R_E}{r_e + R_E}
\]

\[
G_m = \frac{1}{r_e}
\]

\[
R_n = r_e || R_E
\]

**CB Amp**

\[
\frac{v_o}{v_i} = \frac{g_m R_C}{1 + \frac{g_m R_B}{\beta}} = \frac{\alpha R_C}{R_{emitter}}
\]

\[
G_m = \frac{\beta}{r_e (\beta + 1) + R_B} = \frac{\alpha}{R_{emitter}} = \frac{g_m}{1 + \frac{g_m R_B}{\beta}}
\]

\[
R_n = R_C
\]
MOSFET Amplifiers AC Gain \((r_o = \infty)\)

CS Amp

\[
\frac{v_o}{v_i} = -g_m R_D \\
G_m = -g_m \\
R_n = R_D
\]

CS Amp w/ \(R_S\)

\[
\frac{v_o}{v_i} = -\frac{g_m R_D}{1 + g_m R_S} \\
G_m = -\frac{g_m}{1 + g_m R_S} \\
R_n = R_D
\]
MOSFET Amplifiers AC Gain ($r_o = \infty$)

**CD Amp**

\[
\frac{v_o}{v_i} = \frac{g_m R_S}{1 + g_m R_S}
\]

\[G_m = g_m\]

\[R_n = \frac{1}{g_m} || R_S\]

**CG Amp**

\[
\frac{v_o}{v_i} = g_m R_D = \frac{R_D}{R_{source}}
\]

\[G_m = g_m = \frac{1}{R_{source}}\]

\[R_n = R_D\]
This multi-stage amplifier consists of common-emitter, common-base, and common-collector amplifier.

The first two common-emitter and common-base stages are commonly used together, and are called a “cascode amplifier”.

The cascode stage provides all the voltage gain of the circuit, while the output common-collector circuit allows driving of a low-resistance load.
3-Stage BJT Amplifier Example – 1st Stage Av

- Using the common-emitter amplifier equations, the gain from the input to $V_{E2}$ is

$$A_{v1} = \frac{v_{e2}}{v_{in}} = G_{m1}R_{n1}$$

$$G_{m1} = -\frac{g_{m1}}{1 + \frac{g_{m1}(R_E||R_G)}{r_{e1} + R_E||R_G}} = -\frac{\alpha}{r_{e1} + R_E||R_G}$$

$$R_{n1} = R_{i2} = r_{e2}$$

$$A_{v1} = -\frac{\alpha R_{i2}}{r_{e1} + R_E||R_G}$$

$$\frac{v_o}{v_i} = -\frac{g_m R_C}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha R_C}{r_e + R_E}$$

$$G_m = -\frac{g_m}{1 + \frac{g_m R_E}{\alpha}} = -\frac{\alpha}{r_e + R_E}$$

$$R_n = R_C$$
3-Stage BJT Amplifier Example – 2nd Stage Av

- Using the common-base amplifier equations, the gain from $V_{E2}$ to $V_{B3}$ is

$$A_{v2} = \frac{v_{b3}}{v_{e2}} = G_{m2}R_{n2}$$

$$G_{m2} = \frac{\beta}{r_{e2}(\beta + 1)} = \frac{\alpha}{R_{i2}} = g_{m2}$$

$$R_{n2} = R_C\|R_{i3} = R_C\|(\beta + 1)(r_{e3} + R_L)$$

$$A_{v2} = \frac{\alpha(R_C\|(\beta + 1)(r_{e3} + R_L))}{R_{i2}}$$

\[\text{CB Amp}\]

$$\frac{v_o}{v_i} = \frac{g_mR_C}{1 + \frac{g_mR_B}{\beta}} = \frac{\alpha R_C}{R_{emitter}}$$

$$G_m = \frac{\beta}{r_e(\beta + 1) + R_B} = \frac{\alpha}{R_{emitter}} = \frac{g_m}{1 + g_mR_B}$$

$$R_n = R_C$$
3-Stage BJT Amplifier Example – 3\textsuperscript{rd} Stage Av

- Using the common-collector amplifier equations, the gain from \( V_{B3} \) to \( V_{out} \) is

\[
A_{v3} = \frac{v_{out}}{v_{b3}} = G_{m3}R_{n3}
\]

\[
G_{m3} = \frac{1}{r_{e3}}
\]

\[
R_{n3} = r_{e3} || R_L
\]

\[
A_{v3} = \frac{R_L}{r_{e3} + R_L}
\]

\[
v_o = \frac{R_E}{r_e + R_E}
\]

\[
G_m = \frac{1}{r_e}
\]

\[
R_n = r_e || R_E
\]
The total gain is equal to the product of the individual stage gains

\[ A_{v,\text{tot}} = \frac{v_{\text{out}}}{v_{\text{in}}} = A_{v1} A_{v2} A_{v3} \]

\[ = \left( -\frac{\alpha R_{i2}}{r_{e1} + R_E \| R_G} \right) \left( \frac{\alpha (R_C \| R_{i3})}{R_{i2}} \right) \left( \frac{R_L}{r_{e3} + R_L} \right) \]

\[ \approx \left( -\frac{R_C \| R_{i3}}{r_{e1} + R_E \| R_G} \right) \left( \frac{R_L}{r_{e3} + R_L} \right) \]
The keys in solving this problem are recognizing the AC ground between $R_1$ and $R_2$, and Thevenin transformation of the input network.

Then the common-emitter amplifier equations can be used.

$C_1$ acts as an AC short.
Amplifier Example I

- Find the input Thevenin equivalent

\[ v_{Thev} = v_{in} \frac{R_1}{R_S + R_1} \quad R_{Thev} = R_S \parallel R_1 \]
Amplifier Example I

- Use the common-emitter amplifier equations

- As there is a base resistance (R_{Thev}), this must be reflected into the emitter by dividing by (\beta+1) to use the derived equation

\[
\frac{v_o}{v_i} = -\frac{\alpha \left( R_C // R_2 \right)}{R_1 // R_S + r_e + R_E} \cdot \frac{R_1}{R_1 + R_S} \frac{1}{\beta + 1} = -\frac{\alpha R_C}{1 + \frac{g_m R_E}{\alpha}} \frac{r_e + R_E}{r_e + R_E}
\]
Again, AC ground/short and Thevenin transformation are needed to transform the complex circuit into a simple stage with emitter degeneration.
Amplifier Example III

The key for solving this problem is first identifying $R_{eq}$, which is the impedance seen at the emitter of $Q_2$ in parallel with the infinite output impedance of an ideal current source.

$$R_{eq} = r_{e2} + \frac{R_1}{\beta + 1}$$

$$R_{in} = (\beta + 1)(r_{e1} + R_{eq}) = (\beta + 1)\left(r_{e1} + r_{e2} + \frac{R_1}{\beta + 1}\right) = r_{\pi 1} + r_{\pi 2} + R_1$$

$$\frac{v_o}{v_i} = -\frac{\alpha R_D}{r_{e1} + r_{e2} + \frac{R_1}{\beta + 1}}$$
Next Time

• Differential amplifiers
  • Razavi Chapter 10