ELEN 683: Wireless Communications
Homework 1

- Date Assigned: January 25, 2006
- Date Due: Feb 2, 2006

**Reading Exercise**: Chapter 1, 2 from the book

**Homework Problems**: The purpose of this homework in part is to understand some issues in cellular planning. Although we did not cover some of this in class at all, I figured it will be a good exercise to understand some aspects of this through this homework.

1. Consider a cellular system with hexagonal cells, where \( R \) is the distance between the center of the cell and its corner, and \( D \) is the distance between two cells that use the same frequency. Let \( N \) be the number of possible frequency bands for the whole cellular system (called the reuse factor). Clearly \( N \) cannot take arbitrary values and it can take only integer values such that \( N = i^2 + j^2 + ij \), where \( i \) and \( j \) are non-negative integers. By using geometric arguments, show that the co-channel reuse factor for cellular deployments based on hexagonal cells is given by \( D/R = \sqrt{3}N \).

2. Consider a regular hexagonal cell deployment, where the MSs and BSs use omnidirectional antennas. Suppose that we are interested in the forward channel (BS to MS) performance and consider only the first tier of co-channel interferers as shown in Fig. 1. Ignore the effects of shadowing and multipath fading, and assume that the propagation path loss is described by the inverse law in \( P_r \propto P_t d^{-\gamma} \).

   a) Determine the worst case carrier-to-interference ratio \( \Lambda \), as a function of the reuse cluster size \( N \), for \( \gamma = 3, 3.5, \) and 4.

   b) What is the minimum cluster size that is needed if the radio receivers have \( \Lambda_{th} = 18 \) dB.

   c) Referring to Fig. 2, repeat (a) and (b) for the reverse channel (MS to BS).

![Figure 1: Co-channel interference on the forward channel at the desired MS. There are six interfering BSs.](image)

3. Problem 2.8 from Goldsmith

4. Problem 2.19 from Goldsmith (assume \( \gamma = 4 \))
Figure 2: Co-channel interference on the reverse channel at the desired BS. There are six interfering MSs.

5. Problem 2.21 from Goldsmith

6. In class we derived the autocorrelation function $A_{rIrI}(u)$ first and then determined the power spectrum $S_{rr}(f)$. Another way to derive $S_{rr}(f)$ and $A_{rIrI}(u)$ is to first derive $S_{rr}(f)$ as follows. Assume a $f_\theta(\theta)$ for the density function of the angle of incoming paths and let $G(\theta)$ be the net gain of the Tx and Rx antennas in the direction $\theta$. Compute the amount of power contained in a small band of frequencies between $f$ and $f + df$ in terms of $G(\theta)$ and $f_\theta(\theta)$ and from this compute $S_{rr}(f)$. Verify that for 2-D isotropic scattering with omni-directional antennas, we get the same result as that derived in class. Sometimes this is easier to do than directly computing $A_{rIrI}(u)$.

7. For microcells that are deployed in dense urban areas, the plane waves may be channelled by the buildings along the streets and arrive at the receiver antenna from just one direction, as shown in Fig. 3. Clearly, the scattering is non-isotropic. In this case, a variety of models may be used for distribution of arriving plane waves. One plausible distribution is

$$f_\theta(\theta) = \begin{cases} \pi \frac{\cos \left( \frac{\pi}{2} - \frac{\theta}{\theta_m} \right)}{4|\theta_m|}, & -\theta_m \leq \theta \leq \theta_m; \\ 0, & \text{elsewhere.} \end{cases}$$

The parameter $\theta_m$ determines the directivity of the incoming waves. Suppose that the pdf of arriving plane waves $f_\theta(\theta)$, is given by (1). Find the band-pass Doppler power spectrum $S_{rr}(f)$ for $\theta_m = 30^0, 60^0$, and $90^0$, find the autocorrelation and plot it.

Does a narrow angle of arrival imply more correlation in time or less correlation in time?
Figure 3: Angle of arrivals for microcells (non-isotropic) scattering