Section 3

1) Active Filters, Part I

2) Active Filters, Part II
Active Filters

1) The transfer function

Common frequency response:


Practical filters: Approximation of the brick-wall magnitude filters

Active filters: no inductance implementation, providing gain, excellent for below MHz applications such as audio and instrumentation applications

\[ H(s) : \text{The circuit transfer function} \]

\[ H(s) = H_0 \frac{(s - z_1)(s - z_2) \ldots (s - z_m)}{(s - p_1)(s - p_2) \ldots (s - p_n)} \]

\( z_1, z_2, \ldots, z_m \text{: Zeros} \)
\( p_1, p_2, \ldots, p_n \text{: Poles} \)

Roots: They are characteristic frequencies. They can be real or complex (conjugate Pairs)
Active Filters

**H (s) and stability:**

An stable system has a bounded input to any bounded output. The response of a L.T.I system to an impulse input (h(t)) is determined by the poles of the circuit (Natural Response).

RLC circuits are always stable. If circuit contains dependent circuits such as op amps its pole may split into right half of the complex plane (RHP) and we have an unstable Circuit.

**H (s) and frequency response:**

The complex response; \(x_o(t)\), consists of transient and steady-state responses. If all poles of the system are in LHP, the transient response dies, leaving the steady State component.

\[
x_i(t) = X_{im} \cos(\omega t + \theta_i)
\]

Input

\[
x_o(t) = X_{om} \cos(\omega t + \theta_o)
\]

Steady State Output

\[
X_{om} = |H(j\omega)| \cdot X_{im}
\]

\[
\theta_o = \phi H(j\omega) + \theta_i
\]
Active Filters

2) First order active filters

**Differentiator**

\[ H(s) = \frac{V_o}{V_i} = -(RC)s, \]

\[ H(j\omega) = -j \cdot \left(\frac{\omega}{\omega_0}\right), \quad \omega_0 = \frac{1}{RC} \]

**Miller integrator (inverting)**

\[ H(s) = \frac{V_o}{V_i} = -\frac{1}{(RC)s}, \]

\[ H(j\omega) = \left[\frac{-1}{j \cdot \left(\frac{\omega}{\omega_0}\right)}\right], \quad \omega_0 = \frac{1}{RC} \]

**Deboo integrator (non-inverting)**

\[ H(s) = \frac{V_o}{V_i} = \frac{1}{(RC)s}, \]

We use the Howland current pump circuit.
**Active Filters**

**Low-pass filter with gain**

\[ H(s) = \left( -\frac{R_2}{R_1} \right) \cdot \left( \frac{1}{R_2Cs + 1} \right) \]

\[ H(j\omega) = \frac{H_0}{1 + j \cdot \left( \frac{\omega}{\omega_0} \right)} , \quad \omega_0 = \frac{1}{R_2C} \]

Crude approximation of the brick-wall filter

**High-pass filter with gain**

\[ H(s) = \left( -\frac{R_2}{R_1} \right) \cdot \left( \frac{R_1Cs}{R_1Cs + 1} \right) \]

\[ H(j\omega) = H_0 \cdot \left( \frac{j \cdot \left( \frac{\omega}{\omega_0} \right)}{1 + j \cdot \left( \frac{\omega}{\omega_0} \right)} \right) \]

\[ \omega_0 = \frac{1}{R_1C} \]
Active Filters

Band-pass filter (wide band)

Good for audio applications

\[ H(s) = \frac{(-R_2/R_1) \cdot [(R_1C_1s) / ((R_1C_1s + 1) \cdot (R_2Cs + 1))]}{1 + j \cdot (\omega / \omega_L)} \cdot (1 + j \cdot (\omega / \omega_H)) \]
\[ \omega_L = 1 / (R_1C_1), \quad \omega_H = 1 / (R_2C_2) \]

Phase shifter

\[ H(s) = \frac{(-RCs + 1)}{(RCs + 1)} \]
\[ H(j\omega) = \frac{(1 - j \cdot (\omega / \omega_0))}{(1 + j \cdot (\omega / \omega_0))} \]
\[ \omega_0 = 1 / (RC) \]
3 ) **Standard second order response (Biquadratic function)**

\[ H_{\text{LP}}(j\omega) = \frac{1}{1 + j(\omega/\omega_0)/Q - (\omega/\omega_0)^2} \]

- \( Q \) is a measure of filter selectivity
- In practice; 0.5 < \( Q \) < 100

\[ H_{\text{HP}}(j\omega) = \frac{-(\omega/\omega_0)^2}{1 + j(\omega/\omega_0)/Q - (\omega/\omega_0)^2} \]

\[ j(\omega/\omega_0) \rightarrow 1/j(\omega/\omega_0) \]

- Besides the pole – pair, the high pass response has a double zero at \( \omega = 0 \)
Active Filters

- The band pass response (pole pairs + one zero at origin + one zero at infinity)

\[ H_{BP}(j\omega) = \frac{(j\omega/\omega_0)/Q}{1 + j(\omega/\omega_0)/Q - (\omega/\omega_0)^2} \]

\[ Q = \omega_0 / BW \]

\[ \omega_L = \omega_0 \left( 1 + \frac{1}{4Q^2} \right)^{0.5} - \frac{1}{2Q} \]

\[ \omega_H = \omega_0 \left[ 1 + \frac{1}{4Q^2} \right]^{0.5} + \frac{1}{2Q} \]

\[ BW = \omega_H - \omega_L \]

\[ \omega_0 = \left( \omega_L \omega_H \right)^{0.5} \]

- The notch response (pole pairs + zero pairs at imaginary axis)

\[ H_N(j\omega) = \frac{1 - (\omega/\omega_0)^2}{1 + j(\omega/\omega_0)/Q - (\omega/\omega_0)^2} \]

\[ H_N = H_{LP} + H_{HP} = 1 - H_{BP} \]

- The all pass response (2 poles + 2 zeroes)

\[ H_{AP}(j\omega) = \frac{1 - j(\omega/\omega_0)/Q - (\omega/\omega_0)^2}{1 + j(\omega/\omega_0)/Q - (\omega/\omega_0)^2} \]

\[ H_{AP} = H_{LP} - H_{BP} + H_{HP} = 1 - 2H_{BP} \]
Active Filters

4) Single op amp second order active filters

**KRC or Sallen-Key filters**

The second order passive low pass filter has an asymptotic Slope of 40 dB / dec at high frequencies, but does not have Sufficient flexibility to control the magnitude of the filter Around $\omega = \omega_0$

- We use positive feedback to bolster the magnitude of the filter response around $\omega_0$

[(Diagram of KRC or Sallen-Key filters)]
Active Filters

**Low-pass KRC filter**

- $K$ and $Q$ (or BW) are dependent to component ratios
- $\omega_0$ is dependent to component products

**Equal component KRC circuit:** $R_1 = R_2 = R$, $C_1 = C_2 = C$

Design Equations: $RC = 1/\omega_0$, $K = 3 - (1/Q)$

**Unity gain KRC circuit:** $K = 1$, $R_1 = mR$, $R_2 = R$, $C_1 = nC$, $C_2 = C$

Design Equations: $\omega_0 = 1 / [(mn)^{1/2} \cdot RC]$, $Q = (mn)^{1/2} / (m+1)$
Active Filters

- At high frequencies, the equal component design becomes too sensitive to the tolerance of the \( K \) when \( K = 2 \). A slight mismatch, causing intolerable departure of \( Q \) from the desired value and filter oscillation.

- KRC filters are used for \( Qs \) below 10

**High-pass KRC filter**

\[
H(s) = \frac{s^2K}{s^2 + \left( \frac{1}{R_4C_3} + \frac{1}{R_4C_1} + \frac{1-K}{R_2C_1} \right) s + \frac{1}{R_2R_4C_1C_3}}
\]

\[
\omega_0^2 = \frac{1}{R_2R_4C_1C_3}
\]

\[
H(\omega = \omega_0) = K
\]

\[
H(\omega = \infty) = K
\]

\[
BW = \frac{1}{R_4C_3} + \frac{1}{R_4C_1} + \frac{1-K}{R_2C_1}
\]

Again, two specific designs are equal component and unity gain design.
**Active Filters**

**Band-pass KRC filter**

\[ H(s) = \frac{sK\left(\frac{1}{R_1C_5}\right)}{s^2 + \left(\frac{1}{R_1C_5} + \frac{1}{R_4C_5} + \frac{1}{R_4C_3} + \frac{1-K}{R_2C_5}\right)s + \frac{1}{R_4C_3C_5}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \]

\[ BW = \frac{1}{R_1C_5} + \frac{1}{R_4C_5} + \frac{1}{R_4C_3} + \frac{1-K}{R_2C_5} \]

\[ H(\omega = \omega_0) = \frac{K\left(\frac{1}{R_1C_5}\right)}{1 + \frac{1}{R_1C_5} + \frac{1}{R_4C_5} + \frac{1}{R_4C_3} + \frac{1-K}{R_2C_5}} \]

**Typical design approach for** \( Q > 0.4 \): \( R_1 = R_2 = R_4 = R, C_3 = C_5 = C \)

\[ \omega_0^2 = \frac{1}{R_4C_3C_5\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \]

\[ H_{\text{peak}} = \frac{K}{4-K} \]

\[ Q = \frac{\sqrt{2}}{4-K} \]
Active Filters

Band-reject KRC filter

Twin T-network provides alternative paths through which \( v_i \) can reach the amplifier’s Input:

Low-freq. path: \( R - R \)
High-freq. path: \( C - C \)

At intermediate frequencies the two paths provide Opposing phase angles, indicating a tendency For signals to cancel each other.

Design Equations: \( RC = \frac{1}{\omega_0} \), \( K = 4 - \frac{1}{Q} \)
Active Filters

Multiple feedback filters:

- For LP filter: $Y_1$ and $Y_3$ must be conductance and $Y_5$ must be capacitor
- For BP filter: $Y_1$ or $Y_3$ must be capacitors
- For HP filter: $Y_1$, $Y_3$ and either $Y_4$ or $Y_5$ must be capacitors as

\[
H(s) = \frac{-Y_1 Y_3}{Y_5 (Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4}
\]

Low-pass filter

\[
H(s) = \frac{-G_1 G_3}{s^2 C_5 C_2 + s C_5 (G_1 + G_3 + G_4) + G_3 G_4}
\]

\[
\omega_0^2 = \frac{1}{C_5 C_2 R_3 R_4}
\]

\[
BW = \frac{1}{(R_1 \parallel R_3 \parallel R_4) C_2}
\]
Active Filters

**Band-pass filter**

\[ H(s) = \frac{-sG_1C_3}{s^2C_3C_4 + sG_5(C_3 + C_4) + G_2G_5} \]

\[ \omega_0^2 = \frac{1}{C_5C_2R_2R_5} \]

\[ BW = \frac{1}{\left(\frac{C_3C_4}{C_3 + C_4}\right)R_5} \]

**High-pass filter**

\[ H(s) = \frac{-s^2C_1C_3}{s^2C_3C_4 + sG_5(C_1 + C_3 + C_4) + G_2G_5} \]

\[ \omega_0^2 = \frac{1}{C_3C_4R_2R_5} \]

\[ BW = \frac{1}{\left(\frac{C_3C_4}{C_1 + C_3 + C_4}\right)R_5} \]
Active Filters

4) **State Variable and Biquad filters**

Second order filters using one op amp use near minimum components.

*Drawbacks*: Wide component spreads, Awkward tuning capabilities. High sensitivity To component variation, particularly to gain, limit these filters to $Q < 10$.

*State Variable and Biquad* filters are less sensitive to passive component variables Easier to tune, and do not require extravagant component spreads. They provide More than one filter response, simultaneously.

**State Variable (KHN) filters:**

- Two integrators and a summing amplifier to provide the second – order low pass, band pass and high pass responses
- By using a forth op amp, we can synthesize the notch response or the all pass response
Active Filters

Inverting SV filter:

Typical design approach: \( R_5 = R_4 = R_3, R_6 = R_7 = R, C_1 = C_2 = C \)

\[
\omega_0 = \frac{1}{RC} \quad Q = (1/3) \left( 1 + \frac{R_2}{R_1} \right)
\]

\[
H_{0HP} = \left( -\frac{R_5}{R_3} \right)
\]

\[
H_{0BP} = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{R_3}{R_4} + \frac{R_3}{R_5} \right)
\]

\[
H_{0LP} = \left( -\frac{R_4}{R_3} \right)
\]
Active Filters

Non-inverting SV filter:

- The BP response is generated by integrating HP response and the LP response is generated by integrating the BP response.
- The SV filters can easily yield dependable Q’s in the range of hundreds.
Active Filters

The Biquad (Tow-Thomas) filter with notch response:

Three cases:

1) \( R_4 = \infty \), \( \omega_z = \omega_0 \), \( H_{0N} = -(R_5 / R_4) \)
2) Switch Left: \( \omega_z > \omega_0 \), \( H_{0N} = -(R_5\omega_z^2)/(R_2\omega_0^2) \)
3) Switch Right; \( \omega_z, \omega_0 \), \( H_{0N} = -(R_5 / R_2) \)

We will use the low – and high –pass notches to synthesize a class of higher order filters known as elliptic filters