

Generalized Pairwise Complementary Codes With Set-Wise Uniform Interference-Free Windows

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Abstract—This paper introduces an approach to generate *generalized pairwise complementary* (GPC) codes, which offer a uniform interference free windows (IFWs) across the entire code set. The GPC codes work in pairs and can fit extremely power efficient quadrature carrier modems. The characteristic features of the GPC codes include: the set size is $2K$, the processing gain is $4NK$, and the IFW's width is $8N$ identically for all codes in a set, where K is the times to perform Walsh–Hadamard expansions and N is element code length of seed complementary codes. Therefore, by using different N , the IFW width of a GPC code set can be adjusted with its set size unchanged. Each GPC code set consists of two code groups, with each having K codes, and they have sparsely and uniformly distributed autocorrelation side lobes and cross-correlation levels outside the IFWs.

Index Terms—Code-division multiple-access (CDMA), complementary sequences, interference-free window (IFW).

I. INTRODUCTION

IN GENERAL, there are two types of code-division multiple-access (CDMA) codes. One is *unitary* codes, which work in a one-code-per-user basis. They can be further classified into two subgroups, one being quasi-orthogonal codes (such as m -sequences [1], Gold codes [2], Kasami codes [3], etc.) and the other being orthogonal codes (such as Walsh–Hadamard sequences [4], OVFS codes, etc.). Other less well-known unitary codes include GMW codes [5], No codes [6], Bent codes [7], etc.

Another type of CDMA codes is complementary codes, which were first studied by Golay and Turyn [8], [9] in early

1960s for their possible applications in radar systems. The complementary codes work on one-flock-per-user basis. It was shown in [10] that joint application of orthogonal complementary (OC) codes and offset stacking (OS) spreading can improve bandwidth-efficiency of a CDMA system in addition to other desirable properties, such as isotropic multiple-access interference (MAI) free operation, agility to implement rate matching and suitability for burst traffic applications and so on. It is noted that if the flock size is limited to two, complementary codes become pairwise complementary codes.

It has been shown in [11] that a unitary code set can never give ideal cross-correlation and autocorrelation properties. In other words, interference-free windows (IFWs) for any unitary codes will never be equal to its code length. Other works have also been reported on the design of a unitary CDMA code set with an IFW or zero-correlation-zone (ZCZ) [12], [13]. An example to exploit the IFWs of a CDMA code set is TD-LAS system [12] proposed by LinkAir, Inc. In fact, the TD-LAS uses pairwise complementary codes called LS codes with their element codes named as C and S sections. Unfortunately, the LS codes can offer an IFW with its width reduced as the number of active codes increases. Therefore, its IFW is not uniform across the entire code set. On the other hand, the ZCZ codes proposed in [13], [14] can offer a relatively small ZCZ, let alone the fact that most of them are nonbinary codes, which make it extremely difficult to be used in a real system. In addition, those ZCZ codes usually have uncontrollable autocorrelation functions (ACFs) and cross-correlation functions (CCFs) outside the ZCZ.

This paper tries to combine the advantages of unitary codes (implementation simplicity) and OC codes (ideal correlation properties). A generation approach based on complete complementary (CC) codes [9], [10] and generalized even shift orthogonal sequences [15] is proposed to generate generalized pairwise complementary (GPC) codes with a desirable IFWs that are uniform across the entire code set. The GPC codes work in pairs, allowing the use of extremely power-efficient quadrature phase-shift keying (QPSK) modulation. The GPC code sets possess sparsely distributed ACF side-lobes and CCF levels outside the IFW and, thus, controllable ACF and CCF levels can be ensured even outside the IFWs. The IFWs of the GPC codes spans a width of $8N$, where N is the element code length of the CC codes used to generate the GPC codes. Therefore, using different N we can change the width of IFWs of GPC code sets. If only half of the GPC code set is used in a CDMA system, the width of CCF IFWs can be extended to cover the code length, ensuring a MAI-free operation.

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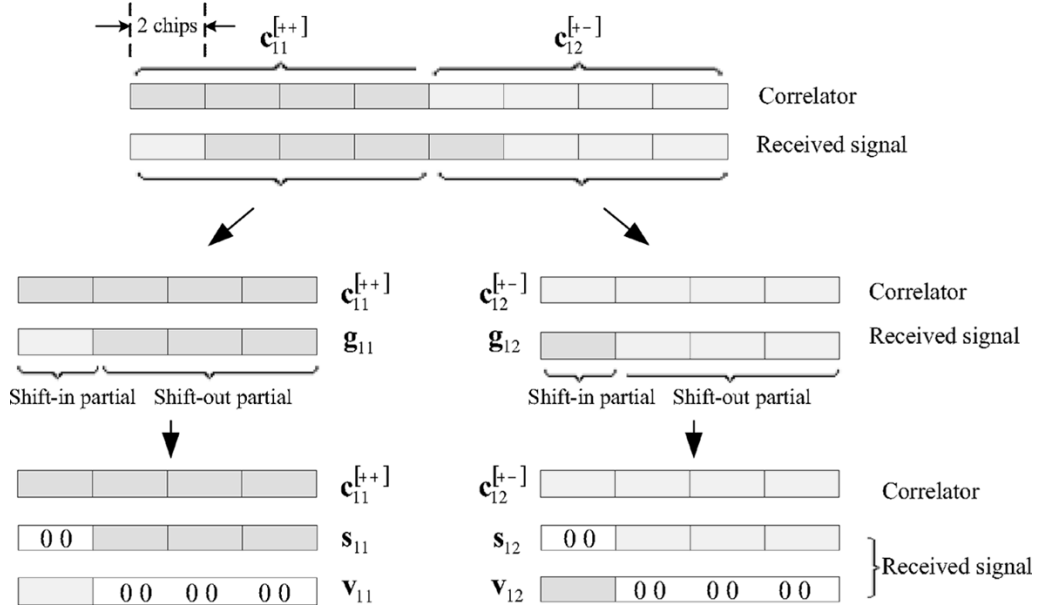


Fig. 1. Illustration of ideal aperiodic ACF for a GESO sequence $\mathbf{c}_1 = [c_{11}^{++}, c_{12}^{+-}]$, whose aperiodic ACF can be decomposed into two components: shift-in partial ACF ($c_{11}^{++}\mathbf{v}_{11}^*$, $c_{12}^{+-}\mathbf{v}_{12}^*$) and shift-out partial ACF ($c_{11}^{++}\mathbf{s}_{11}^*$, $c_{12}^{+-}\mathbf{s}_{12}^*$), respectively.

The rest of this paper is outlined as follows. Section II will introduce generalized even shift orthogonal (GESO) sequences, which are the basis of GPC codes. Section III will explain how to use Walsh–Hadamard matrix expansion to convert a CC code set into a unitary code set. Section IV will describe the *odd shift correlation elimination* algorithm to remove nonzero odd-shift correlation levels to obtain the GPC codes. The correlation properties of the GPC codes will be shown in Section V, followed by the conclusion. Two appendices are given to show flockwise orthogonality and chipwise orthogonality of the CC codes.

II. GENERALIZED EVEN SHIFT ORTHOGONAL SEQUENCES

Let us consider a simple CC code set with its element code length and flock size being $N = 4$ and $M = 2$, respectively. The first flock of this CC code set is $\mathbf{c}_{11} = [c_{11}^{(1)}, c_{11}^{(2)}, c_{11}^{(3)}, c_{11}^{(4)}]$ and $\mathbf{c}_{12} = [c_{12}^{(1)}, c_{12}^{(2)}, c_{12}^{(3)}, c_{12}^{(4)}]$; the second flock is $\mathbf{c}_{21} = [c_{21}^{(1)}, c_{21}^{(2)}, c_{21}^{(3)}, c_{21}^{(4)}]$ and $\mathbf{c}_{22} = [c_{22}^{(1)}, c_{22}^{(2)}, c_{22}^{(3)}, c_{22}^{(4)}]$. Now, consider only the flock of \mathbf{c}_{11} and \mathbf{c}_{12} . It can be easily shown that, if first spreading each chip in \mathbf{c}_{11} and \mathbf{c}_{12} by $[1, 1]$ and $[1, -1]$, respectively, we have

$$\begin{aligned} \mathbf{c}_{11}^{++} &= [c_{11}^{(1)}, c_{11}^{(1)}, c_{11}^{(2)}, c_{11}^{(2)}, c_{11}^{(3)}, c_{11}^{(3)}, c_{11}^{(4)}, c_{11}^{(4)}] \\ &= [\mathbf{c}_{11}^{(1)}, \mathbf{c}_{11}^{(2)}, \mathbf{c}_{11}^{(3)}, \mathbf{c}_{11}^{(4)}] \end{aligned} \quad (1)$$

$$\begin{aligned} \mathbf{c}_{12}^{+-} &= [c_{12}^{(1)}, -c_{12}^{(1)}, c_{12}^{(2)}, -c_{12}^{(2)}, c_{12}^{(3)}, -c_{12}^{(3)}, c_{12}^{(4)}, -c_{12}^{(4)}] \\ &= [\mathbf{c}_{12}^{(1)}, \mathbf{c}_{12}^{(2)}, \mathbf{c}_{12}^{(3)}, \mathbf{c}_{12}^{(4)}] \end{aligned} \quad (2)$$

and then combining \mathbf{c}_{11}^{+-} and \mathbf{c}_{12}^{+-} to form a unitary sequence or

$$\begin{aligned} \mathbf{c}_1 &= [\mathbf{c}_{11}^{++}, \mathbf{c}_{12}^{+-}] \\ &= [\mathbf{c}_{11}^{(1)}, \mathbf{c}_{11}^{(2)}, \mathbf{c}_{11}^{(3)}, \mathbf{c}_{11}^{(4)}, \mathbf{c}_{12}^{(1)}, \mathbf{c}_{12}^{(2)}, \mathbf{c}_{12}^{(3)}, \mathbf{c}_{12}^{(4)}]. \end{aligned} \quad (3)$$

We will prove all even shifted ACF side lobes of \mathbf{c}_1 are zero. For any odd time shift i and j , we always have

$$\begin{aligned} &c_{11}^{++}(i)c_{12}^{+-}(j) + c_{11}^{++}(i+1)c_{12}^{+-}(j+1) \\ &= c_{11}^{\frac{(i+1)}{2}} \left[c_{12}^{\frac{(j+1)}{2}} \right]^H \\ &= c_{11}^{(i)} c_{12}^{(j)} [1, 1][1, -1]^H \\ &= 0 \end{aligned} \quad (4)$$

where \mathbf{x}^H denotes the Hermitian transpose of a vector \mathbf{x} , and $\text{mod}(i, 2) = \text{mod}(j, 2) = 1$. Here, $\text{mod}(x, 2)$ stands for the residual of x after being divided by 2. The property given in (4) will be used to show ideal even shifted ACF of a GESO sequence, as shown in Fig. 1, which illustrates ACF between \mathbf{c}_1 and its two-chip shifted version and can be calculated (note that the same result will be obtained for any other even relative shifts) as follows:

$$\begin{aligned} \mathbf{c}_1[\mathbf{g}_{11}, \mathbf{g}_{12}]^* &= \mathbf{c}_1[\mathbf{s}_{11}, \mathbf{s}_{12}]^* + \mathbf{c}_1[\mathbf{v}_{11}, \mathbf{v}_{12}]^* \\ &= [\mathbf{c}_{11}^{++}, \mathbf{c}_{12}^{+-}] [\mathbf{s}_{11}, \mathbf{s}_{12}]^* \\ &\quad + [\mathbf{c}_{11}^{++}, \mathbf{c}_{12}^{+-}] [\mathbf{v}_{11}, \mathbf{v}_{12}]^* \\ &= \mathbf{c}_{11}^{++}\mathbf{s}_{11}^* + \mathbf{c}_{12}^{+-}\mathbf{s}_{12}^* \\ &\quad + \mathbf{c}_{11}^{++}\mathbf{v}_{11}^* + \mathbf{c}_{12}^{+-}\mathbf{v}_{12}^*. \end{aligned} \quad (5)$$

Using the results obtained in Appendix B, the *shift-out partial ACFs* must be zero, as shown in (50) and (54)

$$\mathbf{c}_{11}^{++}\mathbf{s}_{11}^* + \mathbf{c}_{12}^{+-}\mathbf{s}_{12}^* = 0. \quad (6)$$

From the property shown in (4), we can also have

$$\mathbf{c}_{11}^{++}\mathbf{v}_{11}^* + \mathbf{c}_{12}^{+-}\mathbf{v}_{12}^* = 0. \quad (7)$$

Therefore, it has been proved that the aperiodic ACF for a GESO sequence is zero. Although the above simple proof was carried

out for a particular relative time shift (or two chips as illustrated in Fig. 1), the result is applicable to any other even shifts. A more rigorous proof can proceed as follows.

Assume that the length of \mathbf{c}_1 , as defined in (3), or \mathbf{c}_2 is P . The aperiodic ACF of code \mathbf{c}_1 with a relative time shift being $i \in (\text{mod}(i, 2) = 0) \cap (i < P/2)$ can be written as

$$\begin{aligned} \rho(\mathbf{c}_1; i) &= \frac{1}{P} \sum_{j=1}^{P-i} \mathbf{c}_1(j+i) \mathbf{c}_1^*(j) \\ &\quad \pm \underbrace{\frac{1}{P} \sum_{j=P-i+1}^P \mathbf{c}_1(j+i-P) \mathbf{c}_1^*(j)}_{\text{Shift-in partial ACF}} \\ &= \frac{1}{P} \underbrace{\sum_{j=1}^{\frac{P}{4}-\frac{i}{2}} \left[\mathbf{c}_{11}^{(j+\frac{i}{2})} \mathbf{c}_{11}^{*(j)} \right]}_{\text{Shift-out partial ACF}} \\ &\quad + \underbrace{\frac{1}{P} \sum_{j=\frac{P}{4}-\frac{i}{2}+1}^{\frac{P}{4}} \left[\mathbf{c}_{12}^{(j+\frac{i}{2}-\frac{P}{4})} \mathbf{c}_{11}^{*(j)} \right]}_{\text{Shift-in partial ACF}=0} \\ &\quad + \underbrace{\frac{1}{P} \sum_{j=\frac{P}{4}+1}^{\frac{P}{2}-\frac{i}{2}} \left[\mathbf{c}_{12}^{(j+\frac{i}{2}-\frac{P}{4})} \mathbf{c}_{12}^{*(j-\frac{P}{4})} \right]}_{\text{Shift-out partial ACF}} \\ &\quad \pm \underbrace{\frac{1}{P} \sum_{j=\frac{P}{2}-\frac{i}{2}+1}^{\frac{P}{2}} \left[\mathbf{c}_{11}^{(j+\frac{i}{2}-\frac{P}{2})} \mathbf{c}_{12}^{*(j-\frac{P}{4})} \right]}_{\text{Shift-in partial ACF}=0} \\ &= \frac{1}{P} \sum_{m=1}^2 \sum_{j=1}^{\frac{P}{4}-\frac{i}{2}} 2\mathbf{c}_{1m} \left(j + \frac{i}{2} \right) \mathbf{c}_{1m}^*(j) = 0 \quad (8) \end{aligned}$$

where we have made use of the ideal correlation properties for CC codes given in (49) in Appendix A.

Similarly, for $i \in (\text{mod}(i, 2) = 0) \cap (i > P/2)$, we will have

$$\begin{aligned} \rho(\mathbf{c}_1; i) &= \frac{1}{P} \sum_{j=1}^{P-i} \mathbf{c}_1(j+i) \mathbf{c}_1^*(j) \\ &\quad \pm \underbrace{\frac{1}{P} \sum_{j=P-i+1}^P \mathbf{c}_1(j+i-P) \mathbf{c}_1^*(j)}_{\text{Shift-in partial ACF}} \\ &= \frac{1}{P} \underbrace{\sum_{j=1}^{\frac{P}{2}-\frac{i}{2}} \left[\mathbf{c}_{12}^{(j+\frac{i}{2}-\frac{P}{4})} \mathbf{c}_{11}^{*(j)} \right]}_{\text{Shift-in partial ACF}=0} \\ &\quad \pm \left\{ \underbrace{\frac{1}{P} \sum_{j=\frac{P}{2}-\frac{i}{2}+1}^{\frac{P}{2}} \left[\mathbf{c}_{11}^{(j+\frac{i}{2}-\frac{P}{2})} \mathbf{c}_{11}^{*(j)} \right]}_{\text{Shift-out partial ACF}} \right. \\ &\quad \left. + \underbrace{\frac{1}{P} \sum_{j=\frac{P}{4}-\frac{i}{2}+1}^{\frac{P}{4}} \left[\mathbf{c}_{11}^{(j+\frac{i}{2}-\frac{P}{2})} \mathbf{c}_{12}^{*(j-\frac{P}{4})} \right]}_{\text{Shift-in partial ACF}=0} \right\} \end{aligned}$$

$$\begin{aligned} &\quad \left. + \underbrace{\frac{1}{P} \sum_{j=\frac{P}{4}+1}^{\frac{P}{2}-\frac{i}{2}} \left[\mathbf{c}_{12}^{(j+\frac{i}{2}-\frac{P}{4})} \mathbf{c}_{12}^{*(j-\frac{P}{4})} \right]}_{\text{Shift-out partial ACF}} \right\} \\ &= \frac{1}{P} \sum_{m=1}^2 \sum_{j=1}^{\frac{P}{4}-\frac{i}{2}} 2\mathbf{c}_{1m} \left(j + \frac{i}{2} - \frac{P}{2} \right) \mathbf{c}_{1m}^*(j) = 0. \quad (9) \end{aligned}$$

In this way, it has been shown that the aperiodic ACF for a GESO sequence generated from a flock of CC codes, \mathbf{c}_{11} and \mathbf{c}_{12} , is zero for any even relative time shifts.

In fact, we can use another flock of the CC codes concerned here, or \mathbf{c}_{21} and \mathbf{c}_{22} , to generate another GESO sequence \mathbf{c}_2 in a way very similar to generate \mathbf{c}_1 , to yield

$$\mathbf{c}_2 = \begin{bmatrix} \mathbf{c}_{21}^{[++]} & \mathbf{c}_{22}^{[+-]} \end{bmatrix} \quad (10)$$

where we have

$$\mathbf{c}_{21}^{[++]} = \begin{bmatrix} c_{21}^{(1)}, c_{21}^{(1)}, c_{21}^{(2)}, c_{21}^{(2)}, c_{21}^{(3)}, c_{21}^{(3)}, c_{21}^{(4)}, c_{21}^{(4)} \end{bmatrix} \quad (11)$$

$$\mathbf{c}_{22}^{[+-]} = \begin{bmatrix} c_{22}^{(1)}, -c_{22}^{(1)}, c_{22}^{(2)}, -c_{22}^{(2)}, c_{22}^{(3)}, -c_{22}^{(3)}, c_{22}^{(4)}, -c_{22}^{(4)} \end{bmatrix}. \quad (12)$$

Using the orthogonality properties of a CC code set, as shown in Appendix A, it can be proven that the CCFs of \mathbf{c}_1 and \mathbf{c}_2 at zero and all even relative time shifts are always zero.

III. WALSH-HADAMARD MATRIX EXPANSION

In the previous section, we have demonstrated how to generate a GESO sequence set $\{\mathbf{c}_1, \mathbf{c}_2\}$ from a CC code set $\{++, +-, +-++, +---\}$ with its element code length, flock size, and set size being $N = 4$, $M = 2$, and $K = 2$, respectively. Obviously, any other CC codes can also be used to generate the GESO sequence sets with different lengths and set sizes. Unfortunately, a CC code set usually offers a small set size relative to its processing gain (PG). To increase the set size, we should use Walsh-Hadamard matrix expansion as follows.

After the first expansion, we obtain

$$\mathbf{H}_{1,2} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_1 \\ \mathbf{c}_1 & -\mathbf{c}_1 \end{bmatrix} \quad (13)$$

$$\mathbf{H}_{2,2} = \begin{bmatrix} \mathbf{c}_2 & \mathbf{c}_2 \\ \mathbf{c}_2 & -\mathbf{c}_2 \end{bmatrix} \quad (14)$$

where each row will be viewed as a sequence. In general, if matrix $\mathbf{H}_{1,K}$ can be expanded from matrix $\mathbf{H}_{1,K/2}$, we have

$$\mathbf{H}_{1,K} = \begin{bmatrix} \mathbf{H}_{1,K/2} & \mathbf{H}_{1,K/2} \\ \mathbf{H}_{1,K/2} & -\mathbf{H}_{1,K/2} \end{bmatrix} \quad (15)$$

$$\mathbf{H}_{2,K} = \begin{bmatrix} \mathbf{H}_{2,K/2} & \mathbf{H}_{2,K/2} \\ \mathbf{H}_{2,K/2} & -\mathbf{H}_{2,K/2} \end{bmatrix}. \quad (16)$$

The two matrices in (15) and (16) can be combined into one as

$$\mathbf{H}_K = \begin{bmatrix} \mathbf{H}_{1,K} \\ \mathbf{H}_{2,K} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1,K/2} & \mathbf{H}_{1,K/2} \\ \mathbf{H}_{1,K/2} & -\mathbf{H}_{1,K/2} \\ \mathbf{H}_{2,K/2} & \mathbf{H}_{2,K/2} \\ \mathbf{H}_{2,K/2} & -\mathbf{H}_{2,K/2} \end{bmatrix}_{2K \times KP}. \quad (17)$$

The sequences included in matrix \mathbf{H}_K can be divided into two groups, group 1 being included in $\mathbf{H}_{1,K}$ and group 2 being included in $\mathbf{H}_{2,K}$. It can be shown that intergroup even shift

CCFs are always zero. On the other hand, intragroup even shift CCFs will be nonzero at time shifts of nP , where $n = 1, \dots, \infty$, $P = 2N$, and N is the element code length of the CC codes.

It is seen from (17) that with Walsh–Hadamard matrix expansion we can expand the original GESO sequence set $\{\mathbf{c}_1, \mathbf{c}_2\}$ into a new set with its set size being $2K$ and its PG being $PK = 2NK$ after the K th expansion. However, we still have no way to control the odd shift CCFs of the codes given in (17).

IV. ODD SHIFT CORRELATION ELIMINATION

The *odd shift correlation elimination* algorithm will be used to remove all odd shift correlation levels of the sequences generated in the previous section. Let us define

$$\mathbf{H}_K = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_{2K} \end{bmatrix} \quad (18)$$

and a 2×2 Walsh–Hadamard matrix as

$$\tilde{\mathbf{D}} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix} \quad (19)$$

based on which we can proceed to generate two new matrices from \mathbf{H}_K and $\tilde{\mathbf{D}}$ as

$$\begin{cases} \mathbf{U}_I(p, q) = \mathbf{H}_K(p, q)\tilde{\mathbf{D}}(1, \varsigma) \\ \mathbf{U}_Q(p, q) = \mathbf{H}_K(p, q)\tilde{\mathbf{D}}(2, \varsigma) \end{cases} \quad (20)$$

where p , q , and ς are defined as

$$\begin{cases} p = 1 \dots \dots 2K \\ q = 1 \dots \dots PK \\ \varsigma = -\text{mod}(q, 2) + 2 \end{cases} \quad (21)$$

Thus, we can rewrite (20) into

$$\mathbf{U}_I(p, q) = \begin{bmatrix} \mathbf{u}_{I,1} \\ \mathbf{u}_{I,2} \\ \vdots \\ \mathbf{u}_{I,2K} \end{bmatrix}, \quad \mathbf{U}_Q(p, q) = \begin{bmatrix} \mathbf{u}_{Q,1} \\ \mathbf{u}_{Q,2} \\ \vdots \\ \mathbf{u}_{Q,2K} \end{bmatrix} \quad (22)$$

which can be combined to form a complex matrix \mathbf{U} as

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_{I,1} \\ \mathbf{u}_{I,2} \\ \vdots \\ \mathbf{u}_{I,2K} \end{bmatrix} + j \begin{bmatrix} \mathbf{u}_{Q,1} \\ \mathbf{u}_{Q,2} \\ \vdots \\ \mathbf{u}_{Q,2K} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_{2K} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{G1} \\ \mathbf{U}_{G2} \end{bmatrix} \quad (23)$$

where codes in group 1 (\mathbf{U}_{G1}) and group 2 (\mathbf{U}_{G2}) can be defined as

$$\mathbf{U}_{G1} = \begin{bmatrix} \mathbf{u}_{I,1} \\ \mathbf{u}_{I,2} \\ \vdots \\ \mathbf{u}_{I,K} \end{bmatrix} + j \begin{bmatrix} \mathbf{u}_{Q,1} \\ \mathbf{u}_{Q,2} \\ \vdots \\ \mathbf{u}_{Q,K} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_K \end{bmatrix} \quad (24)$$

$$\mathbf{U}_{G2} = \begin{bmatrix} \mathbf{u}_{I,K+1} \\ \mathbf{u}_{I,K+2} \\ \vdots \\ \mathbf{u}_{I,2K} \end{bmatrix} + j \begin{bmatrix} \mathbf{u}_{Q,K+1} \\ \mathbf{u}_{Q,K+2} \\ \vdots \\ \mathbf{u}_{Q,2K} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_{K+1} \\ \mathbf{u}_{K+2} \\ \vdots \\ \mathbf{u}_{2K} \end{bmatrix} \quad (25)$$

The code set given in (23) or (24) and (25) are the GPC code set we want. It can be seen from (23) that the obtained GPC

codes work in pairs as an in-phase and quadrature components, $\mathbf{u}_{I,k}$ and $\mathbf{u}_{Q,k}$, respectively, where $k = 1, \dots, 2K$. Therefore, they can be sent via orthogonal carriers in a QPSK modulator. At receiver side, the two components $\mathbf{u}_{I,k}$ and $\mathbf{u}_{Q,k}$ should undergo matched-filtering individually before being summed over to form a decision variable. The PG of the GPC codes is $4NK$ with its set size being $2K$.

V. CORRELATION PROPERTIES OF GPC CODES

In this section, we will show the following properties of the GPC codes.

- Their ACF side lobes are always zero, except at the time shifts equal to $2nP$, where $n = 1, \dots, \infty$ and $P = 2N$.
- Their intragroup CCFs (from either \mathbf{U}_{G1} or \mathbf{U}_{G2}) are always zero, except at the time shifts equal to $n2P$, where $n = 1, \dots, \infty$.
- Their intergroup CCFs are always zero for any time shifts.

A. ACF at Zero Shift

Assume $\mathbf{u}_{I,k}$ and $\mathbf{u}_{Q,k}$ form one GPC code pair, which are assigned to one user. From the definition of ACF given in (39), we can readily obtain the ACF at zero shift as

$$\begin{aligned} & \frac{1}{2} [\rho(\mathbf{u}_{I,k}; 0) + \rho(\mathbf{u}_{Q,k}; 0)] \\ &= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-0} \mathbf{u}_{I,k}(1, j+0) \mathbf{u}_{I,k}^*(1, j) \right. \\ & \quad \left. + \frac{1}{L} \sum_{j=1}^{L-0} \mathbf{u}_{Q,k}(1, j+0) \mathbf{u}_{Q,k}^*(1, j) \right] \\ &= 1 \end{aligned} \quad (26)$$

where $L = PK$.

B. ACF at Odd Time Shifts

Let τ_o be an odd integer or $\text{mod}(\tau_o, 1) = 1$. Thus, the ACF at an odd time shift becomes

$$\begin{aligned} & \frac{1}{2} [\rho(\mathbf{u}_{I,k}; \tau_o) + \rho(\mathbf{u}_{Q,k}; \tau_o)] \\ &= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-\tau_o} \mathbf{u}_{I,k}(1, j+\tau_o) \mathbf{u}_{I,k}^*(1, j) \right. \\ & \quad \left. + \frac{1}{L} \sum_{j=1}^{L-\tau_o} \mathbf{u}_{Q,k}(1, j+\tau_o) \mathbf{u}_{Q,k}^*(1, j) \right] \\ &= \frac{1}{2} \left\{ \frac{1}{L} \sum_{j=1}^{L-\tau_o} [\mathbf{H}_L(k, j+\tau_o) \mathbf{H}_L^*(k, j)] \right. \\ & \quad \left. \times [\mathbf{D}(1, \varsigma_1) \mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1) \mathbf{D}^*(2, \varsigma_2)] \right\} \\ &= 0 \end{aligned} \quad (27)$$

where it is to be noted that, if j is an odd integer, $j+\tau_o$ must be an even integer. On the other hand, if j is even, $j+\tau_o$ must be odd. Therefore, we always have $\varsigma_1 \neq \varsigma_2$ and $\mathbf{D}(1, \varsigma_1) \mathbf{D}^*(1, \varsigma_2) +$

$\mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2) = 0$. We have also taken $\varsigma_1 = -\text{mod}(j + \tau_o, 2) + 2$ and $\varsigma_2 = -\text{mod}(j, 2) + 2$, as defined in (21), into account in the above derivation.

C. ACF at Even Time Shifts

In the same way, if τ_e is an even time shift, we have

$$\begin{aligned} & \frac{1}{2} [\rho(\mathbf{u}_{I,k}; \tau_e) + \rho(\mathbf{u}_{Q,k}; \tau_e)] \\ &= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-\tau_e} \mathbf{u}_{I,k}(1, j + \tau_e) \mathbf{u}_{I,k}^*(1, j) \right. \\ & \quad \left. + \frac{1}{L} \sum_{j=1}^{L-\tau_e} \mathbf{u}_{Q,k}(1, j + \tau_e) \mathbf{u}_{Q,k}^*(1, j) \right] \\ &= \frac{1}{2} \left\{ \frac{1}{L} \sum_{j=1}^{L-\tau_e} [\mathbf{H}_L(k, j + \tau_e) \mathbf{H}_L^*(k, j)] \right. \\ & \quad \left. \times [\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2)] \right\} \\ &= 0 \end{aligned} \quad (28)$$

where $\tau_e \neq 2Pn$ ($n = 1, \dots, \infty$) and $\varsigma_1 = -\text{mod}(j + \tau_e, 2) + 2$ and $\varsigma_2 = -\text{mod}(j, 2) + 2$. If j is odd, $j + \tau_e$ is always odd. If j is even, $j + \tau_e$ will also be even. Thus, we have $\varsigma_1 = \varsigma_2$ and $\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2) = 1 + 1 = 2$. However, the ACF will become nonzero if $\tau_e = 2Pn$ ($n = 1, \dots, \infty$).

D. CCF at Zero Time Shift

Next, let us examine the CCF at zero shift. Assume $\mathbf{u}_{I,k}, \mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}, \mathbf{u}_{Q,k'}$ are two different GPC codes. Thus, the CCF at zero time shift between codes $\mathbf{u}_{I,k}, \mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}, \mathbf{u}_{Q,k'}$ can be written into

$$\begin{aligned} & \frac{1}{2} [\rho(\mathbf{u}_{I,k}, \mathbf{u}_{I,k'}; 0) + \rho(\mathbf{u}_{Q,k}, \mathbf{u}_{Q,k'}; 0)] \\ &= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-0} \mathbf{u}_{I,k}(1, j + 0) \mathbf{u}_{I,k'}^*(1, j) \right. \\ & \quad \left. + \frac{1}{L} \sum_{j=1}^{L-0} \mathbf{u}_{Q,k}(1, j + 0) \mathbf{u}_{Q,k'}^*(1, j) \right] \\ &= \frac{1}{2} \{0 + 0\} = 0 \end{aligned}$$

where $\varsigma = -\text{mod}(j, 2) + 2$, and for $x \in \{1, 2\}$ we have

$$\begin{aligned} & \sum_{j=1}^L [\mathbf{H}_K(k, j) \mathbf{H}_K^*(k', j)] [\mathbf{D}(x, \varsigma) \mathbf{D}^*(x, \varsigma)] \\ &= \sum_{j=1}^L [\mathbf{H}_K(k, j) \mathbf{H}_K^*(k', j)] \sum_{j=1}^L [\mathbf{D}(x, \varsigma) \mathbf{D}^*(x, \varsigma)] \\ &= 0 \times \sum_{j=1}^L [\mathbf{D}(1, \varsigma) \mathbf{D}^*(1, \varsigma)] = 0 \end{aligned} \quad (29)$$

E. CCF at Odd Time Shifts

To calculate CCF at odd time shifts between $\mathbf{u}_{I,k}, \mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}, \mathbf{u}_{Q,k'}$, we can proceed as follows:

$$\begin{aligned} & \frac{1}{2} [\rho(\mathbf{u}_{I,k}, \mathbf{u}_{I,k'}; \tau_o) + \rho(\mathbf{u}_{Q,k}, \mathbf{u}_{Q,k'}; (\mathbf{u}_{I,k}, \mathbf{u}_{I,k'}; \tau_o))] \\ &= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-\tau_o} \mathbf{u}_{I,k}(1, j + \tau_o) \mathbf{u}_{I,k'}^*(1, j) \right. \\ & \quad \left. + \frac{1}{L} \sum_{j=1}^{L-\tau_o} \mathbf{u}_{Q,k}(1, j + \tau_o) \mathbf{u}_{Q,k'}^*(1, j) \right] \\ &= \frac{1}{2} \left\{ \frac{1}{L} \sum_{j=1}^{L-\tau_o} [\mathbf{H}_K(k, j + \tau_o) \mathbf{H}_K^*(k', j)] \right. \\ & \quad \left. \times [\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2)] \right\} \\ &= 0 \end{aligned} \quad (30)$$

where $\varsigma_1 = -\text{mod}(j + \tau_o, 2) + 2$ and $\varsigma_2 = -\text{mod}(j, 2) + 2$. We have also taken into account $\varsigma_1 \neq \varsigma_2$ and $\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2) = 0$ in the proof.

F. CCF at Even Time Shifts

To calculate CCFs at even time shifts between $\mathbf{u}_{I,k}, \mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}, \mathbf{u}_{Q,k'}$, we have to deal with two different cases: intragroup CCFs (two pairs are from either group 1 or group 2) and intergroup CCFs (two pairs are from different groups). The two GPC code groups have been defined in (23), or (24) and (25).

Assume that $\mathbf{u}_{I,k}, \mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}, \mathbf{u}_{Q,k'}$ are from the same group. For $\tau_e \neq 2Pn$, where $n = 1, \dots, \infty$, we have

$$\begin{aligned} & \frac{1}{2} [\rho(\mathbf{u}_{I,k}, \mathbf{u}_{I,k'}; \tau_e) + \rho(\mathbf{u}_{Q,k}, \mathbf{u}_{Q,k'}; \tau_e)] \\ &= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-\tau_e} \mathbf{u}_{I,k}(1, j + \tau_e) \mathbf{u}_{I,k'}^*(1, j) \right. \\ & \quad \left. + \frac{1}{L} \sum_{j=1}^{L-\tau_e} \mathbf{u}_{Q,k}(1, j + \tau_e) \mathbf{u}_{Q,k'}^*(1, j) \right] \\ &= \frac{1}{2} \left\{ \frac{1}{L} \sum_{j=1}^{L-\tau_e} [\mathbf{H}_K(k, j + \tau_e) \mathbf{H}_K^*(k', j)] \right. \\ & \quad \left. \times [\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2)] \right\} \\ &= \frac{1}{2} \left\{ \frac{1}{L} \times 0 \times [\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2)] \right\} \\ &= 0 \end{aligned} \quad (31)$$

where again we have considered $\varsigma_1 = -\text{mod}(j + \tau_e, 2) + 2$ and $\varsigma_2 = -\text{mod}(j, 2) + 2$ and, thus, $\varsigma_1 = \varsigma_2$. Therefore, we always have $\mathbf{D}(1, \varsigma_1)\mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1)\mathbf{D}^*(2, \varsigma_2) = 1 + 1 = 2$. However, if $\tau_e = 2Pn$, the CCF at even shift will not be zero, where $n = 1, \dots, \infty$.

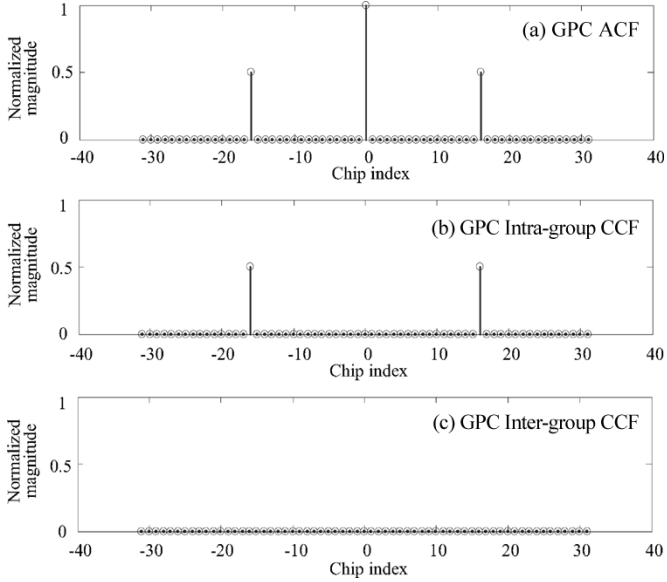


Fig. 2. (a) Normalized ACF, (b) normalized intragroup CCFs, and (c) normalized intergroup CCFs for GPC codes. The GPC codes offer an IFW with its width being $8N$ (where $N = 4$ in this case) in both their ACF and intragroup CCF, where N is the length of the element code of the CC codes used to generate the GPC codes. The PG of the code set is 64. The code length is 32 and the set size is 4.

On the other hand, if $\mathbf{u}_{I,k}$, $\mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}$, $\mathbf{u}_{Q,k'}$ are from different groups, we have

$$\begin{aligned}
& \frac{1}{2} [\rho(\mathbf{u}_{I,k}, \mathbf{u}_{I,k'}; \tau_e) + \rho(\mathbf{u}_{Q,k}, \mathbf{u}_{Q,k'}; \tau_e)] \\
&= \frac{1}{2} \left[\frac{1}{L} \sum_{j=1}^{L-\tau_e} \mathbf{u}_{I,k}(1, j + \tau_e) \mathbf{u}_{I,k'}^*(1, j) \right. \\
&\quad \left. + \frac{1}{L} \sum_{j=1}^{L-\tau_e} \mathbf{u}_{Q,k}(1, j + \tau_e) \mathbf{u}_{Q,k'}^*(1, j) \right] \\
&= \frac{1}{2} \left\{ \frac{1}{L} \sum_{j=1}^{L-\tau_e} [\mathbf{H}_K(k, j + \tau_e) \mathbf{H}_K^*(k', j)] \right. \\
&\quad \left. \times [\mathbf{D}(1, \varsigma_1) \mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1) \mathbf{D}^*(2, \varsigma_2)] \right\} \\
&= \frac{1}{2} \left\{ \frac{1}{L} \times 0 \times [\mathbf{D}(1, \varsigma_1) \mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1) \mathbf{D}^*(2, \varsigma_2)] \right\} \\
&= 0 \tag{32}
\end{aligned}$$

where if the index j is odd, then $j + \tau_e$ must be odd; otherwise, if j is even, $j + \tau_e$ must be even. This makes $\varsigma_1 = \varsigma_2$ and $\mathbf{D}(1, \varsigma_1) \mathbf{D}^*(1, \varsigma_2) + \mathbf{D}(2, \varsigma_1) \mathbf{D}^*(2, \varsigma_2) = 1 + 1 = 2$, where $\varsigma_1 = -\text{mod}(j + \tau_e, 2) + 2$ and $\varsigma_2 = -\text{mod}(j, 2) + 2$. Therefore, if the two GPC codes $\mathbf{u}_{I,k}$, $\mathbf{u}_{Q,k}$ and $\mathbf{u}_{I,k'}$, $\mathbf{u}_{Q,k'}$ are from different groups, their CCFs at any even shifts are always zero.

Fig. 2 shows the ACF and CCF for a sample GPC code set generated using the approach given in this paper.

VI. CONCLUSION

In this paper, a new approach to generate GPC codes has been proposed. Each GPC code set consists of two subgroups. Their intergroup CCFs are always zero at any shifts. Their ACF side-

lobes and intragroup CCFs are also zero, except for the time shifts of $2Pn$, where $n = 1, \dots, \infty$, $P = 2N$, and N is the element code length of the CC codes used to generate the GPC codes. Every GPC code set offers an IFW, which has a width of $8N$ and is uniform across entire code set.

APPENDIX A

PROOF OF FLOCKWISE ORTHOGONALITY OF CC CODES

To generate a CC code set, first we need two $N \times N$ orthogonal matrices, \mathbf{A} and \mathbf{B} , whose elements satisfy $|a_{ij}| = 1$ and $|b_{ij}| = 1$, as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_N \end{bmatrix} \tag{33}$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N} \\ b_{21} & b_{22} & \cdots & b_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NN} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_N \end{bmatrix} \tag{34}$$

based on which we can obtain the third matrix \mathbf{C} as

$$\mathbf{C} = \begin{bmatrix} b_{11}\mathbf{a}_1 & b_{12}\mathbf{a}_2 & \cdots & b_{1N}\mathbf{a}_N \\ b_{21}\mathbf{a}_1 & b_{22}\mathbf{a}_2 & \cdots & b_{2N}\mathbf{a}_N \\ \vdots & \vdots & \vdots & \vdots \\ b_{N1}\mathbf{a}_1 & b_{N2}\mathbf{a}_2 & \cdots & b_{NN}\mathbf{a}_N \end{bmatrix}. \tag{35}$$

Now, another $N \times N$ orthogonal matrix \mathbf{D} is introduced with all its elements being $|d_{ij}| = 1$ as follows:

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1N} \\ d_{21} & d_{22} & \cdots & d_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ d_{N1} & d_{N2} & \cdots & d_{NN} \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_N \end{bmatrix}. \tag{36}$$

Based on \mathbf{A} and \mathbf{D} , we can obtain a vector \mathbf{Z}_{xy} or

$$\mathbf{Z}_{xy} = [a_{x1}d_{y1} \ a_{x2}d_{y2} \ \cdots \ a_{xN}d_{yN}] \quad (x, y = 1, \dots, N) \tag{37}$$

such that the k th code in a CC code set can be generated as

$$\begin{aligned}
\mathbf{E}_k &= \begin{bmatrix} b_{k1}\mathbf{Z}_{11} & b_{k2}\mathbf{Z}_{21} & \cdots & b_{kN}\mathbf{Z}_{N1} \\ b_{k1}\mathbf{Z}_{12} & b_{k2}\mathbf{Z}_{22} & \cdots & b_{kN}\mathbf{Z}_{N2} \\ \vdots & \vdots & \vdots & \vdots \\ b_{k1}\mathbf{Z}_{1N} & b_{k2}\mathbf{Z}_{2N} & \cdots & b_{kN}\mathbf{Z}_{NN} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{e}_{k1} \\ \mathbf{e}_{k2} \\ \vdots \\ \mathbf{e}_{kN} \end{bmatrix} \quad (k = 1, \dots, N). \tag{38}
\end{aligned}$$

In general, the partial ACF of $\mathbf{c}_{k,m}$ and partial CCF between $\mathbf{c}_{k,m}$ and $\mathbf{c}_{k',m}$ can be defined, as shown in (39) and (40) at the bottom of the next page, respectively. It is noted that zero partial ACF and zero partial CCF will always ensure zero periodic or zero aperiodic ACFs and CCFs. Thus, the proofs followed will be focused only on partial ACF and partial CCF. Let us define

$$\begin{cases} g_1(x) = \text{mod}(x, N) + N\delta(\text{mod}(x, N)) \\ g_2(x) = \frac{(x - \text{mod}(x, N))}{N} + (1 - \delta(\text{mod}(x, N))) \end{cases} \tag{41}$$

where $\text{mod}(x, N)$ stands for the residual of x after being divided by N and $\delta(x)$ will be unit at $x = 0$ and zero elsewhere. Also, for notation simplicity in the proof, we define

$$\begin{cases} p = g_1(x) \\ p' = g_1(x') \\ q = g_2(x) \\ q' = g_2(x') \end{cases}. \quad (42)$$

Thus, from (37) and (38), we have

$$\begin{aligned} \mathbf{E}_k(m, x)\mathbf{E}_{k'}(m, x') &= (b_{kq}\mathbf{Z}_{qm}(1, p))(b_{k'q'}\mathbf{Z}_{q'm}(1, p')) \\ &= (b_{kq}a_{qp}d_{mp})(b_{k'q'}a_{q'p'}d_{mp'}). \end{aligned} \quad (43)$$

If relative time shift i is not equal to multiples of N , the ACF of a CC code \mathbf{E}_k becomes

$$\begin{aligned} \rho(\mathbf{E}_k; i) &= \frac{1}{N} \sum_{m=1}^N \frac{1}{N^2} \sum_{j=1}^{N^2-i} \mathbf{E}_k(m, j+i)\mathbf{E}_k(m, j) \\ &= \frac{1}{N^3} \sum_{j=1}^{N^2-i} (b_{k(g_2(j+i))}a_{(g_2(j+i))(g_1(j+i))}) \\ &\quad \times (b_{k(g_2(j))}a_{(g_2(j))(g_1(j))}) \mathbf{d}_{(g_1(j+i))}\mathbf{d}_{(g_1(j))}^* \\ &= \begin{cases} 1, & \text{if } i = 0 \\ 0, & \text{if } (i \neq 0) \cap (\text{mod}(i, N) \neq 0) \\ & \cap (\text{mod}(i, N^2) \neq 0) \end{cases} \end{aligned} \quad (44)$$

where N is the dimension of matrix \mathbf{D} defined in (36), whose orthogonality has been used to yield above result. Similarly, the CCF of two CC codes, \mathbf{E}_k and $\mathbf{E}_{k'}$, with their relative time shift i being not equal to multiples of N can be written as

$$\begin{aligned} \rho(\mathbf{E}_k, \mathbf{E}_{k'}; i) &= \frac{1}{N} \sum_{m=1}^N \frac{1}{N^2} \sum_{j=1}^{N^2-i} \mathbf{E}_k(m, j+i)\mathbf{E}_{k'}(m, j) \\ &= \frac{1}{N^3} \sum_{j=1}^{N^2-i} (b_{k(g_2(j+i))}a_{(g_2(j+i))(g_1(j+i))}) \\ &\quad \times (b_{k'(g_2(j))}a_{(g_2(j))(g_1(j))}) \mathbf{d}_{(g_1(j+i))}\mathbf{d}_{(g_1(j))}^* \\ &= \begin{cases} 1, & \text{if } (i = 0) \cap (k = k') \\ 0, & \text{if } (i \neq 0) \cap (\text{mod}(i, N) \neq 0) \\ & \cap (\text{mod}(i, N^2) \neq 0) \end{cases}. \end{aligned} \quad (45)$$

If the relative time shift i is equal to the multiples of N but not equal to the multiples of N^2 , we have $g_1(j+i) = g_1(j)$ for

any j different i . In this case, the ACF of a code \mathbf{E}_k and CCF between codes \mathbf{E}_k and $\mathbf{E}_{k'}$ will become

$$\begin{aligned} \rho(\mathbf{E}_k; i) &= \frac{1}{N} \sum_{m=1}^N \frac{1}{N^2} \sum_{j=1}^{N^2-i} \mathbf{E}_k(m, j+i)\mathbf{E}_k(m, j) \\ &= \frac{1}{N^2} \sum_{j'=1}^{N-\frac{i}{N}} (b_{k(g_2(j'+i))}b_{k(g_2(j'))}) \mathbf{a}_{(g_2(j'+i))}\mathbf{a}_{(g_2(j'))}^* \\ &= \begin{cases} 1, & \text{if } (i = 0) \cap (\text{mod}(i, N) = 0) \\ & \cap (\text{mod}(i, N^2) \neq 0) \\ 0, & \text{if } (i \neq 0) \cap (\text{mod}(i, N) = 0) \\ & \cap (\text{mod}(i, N^2) \neq 0) \end{cases} \end{aligned} \quad (46)$$

and

$$\begin{aligned} \rho(\mathbf{E}_k, \mathbf{E}_{k'}; i) &= \frac{1}{N} \sum_{m=1}^N \frac{1}{N^2} \sum_{j=1}^{N^2-i} \mathbf{E}_k(m, j+i)\mathbf{E}_{k'}(m, j) \\ &= \frac{1}{N^2} \sum_{j'=1}^{N-\frac{i}{N}} (b_{k(g_2(j'+i))}b_{k'(g_2(j'))}) \\ &\quad \times \mathbf{a}_{(g_2(j'+i))}\mathbf{a}_{(g_2(j'))}^* \\ &= \begin{cases} 1, & \text{if } (i = 0) \cap (\text{mod}(i, N^2) = 0) \\ & \cap (k = k') \\ 0, & \text{if } (i \neq 0) \cap (\text{mod}(i, N) \neq 0) \\ & \cap (\text{mod}(i, N^2) = 0) \end{cases} \end{aligned} \quad (47)$$

respectively. In the proof shown above, we have used the orthogonality of matrix \mathbf{A} defined in (33). Finally, we can calculate $\rho(\mathbf{E}_k, \mathbf{E}_{k'}; i)$ with i being equal to the multiples of N^2 as

$$\begin{aligned} \rho(\mathbf{E}_k, \mathbf{E}_{k'}; i) &= \frac{1}{N} \sum_{m=1}^N \frac{1}{N^2} \sum_{j=1}^{N^2-i} \mathbf{E}_k(m, j+i)\mathbf{E}_{k'}(m, j) \\ &= \frac{1}{N^3} \sum_{j=1}^{N^2-i} (b_{kg_2(j+i)}a_{g_2(j+i)g_1(j)}) \\ &\quad \times (b_{k'g_2(j)}a_{g_2(j)g_1(j)}) \\ &\quad \times \sum_{m=1}^N (d_{m(g_1(j))}) (d_{m(g_1(j))}) \\ &= \frac{1}{N} \sum_{j'=1}^N (b_{k(g_2(j'))}b_{k'(g_2(j'))}) \\ &= \frac{1}{N} \mathbf{b}_k \mathbf{b}_{k'}^* = \begin{cases} 1, & \text{if } k = k' \\ 0, & \text{if } k \neq k' \end{cases} \end{aligned} \quad (48)$$

where the orthogonality of matrix \mathbf{B} , defined in (34), has been used here.

$$\rho(\mathbf{c}_{k,m}; i) = \begin{cases} \frac{1}{N} \sum_{j=1}^{N-i} \mathbf{c}_{k,m}(j+i)\mathbf{c}_{k,m}^*(j), & \text{if } i = 0, 1, \dots, N-1 \\ \frac{1}{N} \sum_{j=1-i}^N \mathbf{c}_{k,m}(j+i)\mathbf{c}_{k,m}^*(j), & \text{if } i = -1, -2, \dots, -N+1 \end{cases} \quad (39)$$

$$\rho(\mathbf{c}_{k,m}, \mathbf{c}_{k',m}; i) = \begin{cases} \frac{1}{N} \sum_{j=1}^{N-i} \mathbf{c}_{k,m}(j+i)\mathbf{c}_{k',m}^*(j), & \text{if } i = 0, 1, \dots, N-1 \\ \frac{1}{N} \sum_{j=1-i}^N \mathbf{c}_{k,m}(j+i)\mathbf{c}_{k',m}^*(j), & \text{if } i = -1, -2, \dots, -N+1 \end{cases} \quad (40)$$

To summarize, we have proved that the CC codes do possess ideal ACF and CCF for whatever relative time shifts i , or

$$\begin{aligned} \rho(\mathbf{E}_k, \mathbf{E}_{k'}; i) &= \frac{1}{N} \sum_{m=1}^N \frac{1}{N^2} \sum_{j=1}^{N^2-i} \mathbf{E}_k(m, j+i) \mathbf{E}_{k'}^*(m, j) \\ &= \begin{cases} 1, & \text{if } (k = k') \cap (i = 0) \\ 0, & \text{if } (k \neq k') \cap (i \neq 0) \end{cases} \end{aligned} \quad (49)$$

APPENDIX B

PROOF OF n -CHIP ORTHOGONALITY OF CC CODES

A. Chipwise ($n = 1$) Orthogonality of A CC Code Set

Let us first consider the case with $n = 1$. Thus, only one chip will be involved in the orthogonality proof here. Based on the definitions given in (41) and (42), the chipwise CCF between codes k and k' can be written as

$$\begin{aligned} &\sum_{m=1}^N \mathbf{E}_k(m, x) \mathbf{E}_{k'}(m, x') \\ &= \sum_{m=1}^N (b_{kq} \mathbf{Z}_{qm}(1, p)) (b_{k'q'} \mathbf{Z}_{q'm}(1, p')) \\ &= (b_{kq} b_{k'q'} a_{qp} a_{q'p'}) \sum_{m=1}^N (d_{mp} d_{mp'}) \\ &= \begin{cases} \pm N, & \text{if } p' = p \\ 0, & \text{if } p' \neq p \end{cases} \end{aligned} \quad (50)$$

which has taken into account the orthogonality property of matrix \mathbf{D} , defined in (36).

It is to be noted that the same result will be obtained for either $k = k'$ or $k \neq k'$, which means that the same result will apply for either ACF or CCF of a CC code set. This result tells us that, as long as the relative time shifts, x and x' , between two CC codes, $\mathbf{E}_k(m, x)$ and $\mathbf{E}_{k'}(m, x')$, satisfy the following condition:

$$\begin{aligned} \text{mod}(x, N) + N\delta(\text{mod}(x, N)) \\ = \text{mod}(x', N) + N\delta(\text{mod}(x', N)) \end{aligned} \quad (51)$$

both chipwise ACF and chipwise CCF of a CC code set is always zero.

B. N -chip ($n = N$) Orthogonality of A CC Code Set

Now, we should consider the orthogonality of a CC code set that involves N consecutive chip shifts. It is to be noted that each element code in a CC code set contains N^2 chips and, thus, N chips span only $1/N$ of the whole element code length. Also, it is assumed that the relative time shift between any two codes of concern takes the multiples of N chips. For two CC codes

$$\begin{aligned} \mathbf{E}_k &= \begin{bmatrix} b_{k1} \mathbf{Z}_{11} & b_{k2} \mathbf{Z}_{21} & \cdots & b_{kN} \mathbf{Z}_{N1} \\ b_{k1} \mathbf{Z}_{12} & b_{k2} \mathbf{Z}_{22} & \cdots & b_{kN} \mathbf{Z}_{N2} \\ \vdots & \vdots & \vdots & \vdots \\ b_{k1} \mathbf{Z}_{1N} & b_{k2} \mathbf{Z}_{2N} & \cdots & b_{kN} \mathbf{Z}_{NN} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{e}_{k1} \\ \mathbf{e}_{k2} \\ \vdots \\ \mathbf{e}_{kN} \end{bmatrix} \end{aligned} \quad (52)$$

$$\begin{aligned} \mathbf{E}_{k'} &= \begin{bmatrix} b_{k'1} \mathbf{Z}_{11} & b_{k'2} \mathbf{Z}_{21} & \cdots & b_{k'N} \mathbf{Z}_{N1} \\ b_{k'1} \mathbf{Z}_{12} & b_{k'2} \mathbf{Z}_{22} & \cdots & b_{k'N} \mathbf{Z}_{N2} \\ \vdots & \vdots & \vdots & \vdots \\ b_{k'1} \mathbf{Z}_{1N} & b_{k'2} \mathbf{Z}_{2N} & \cdots & b_{k'N} \mathbf{Z}_{NN} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{e}_{k'1} \\ \mathbf{e}_{k'2} \\ \vdots \\ \mathbf{e}_{k'N} \end{bmatrix} \end{aligned} \quad (53)$$

we can readily write down their N -chip CCF as

$$\begin{aligned} \sum_{y=1}^N b_{kx} \mathbf{Z}_{xy} (b_{k'x'} \mathbf{Z}_{x'y})^H &= \sum_{y=1}^N b_{kx} b_{k'x'} \sum_{z=1}^N (a_{xz} a_{x'z}) (d_{yz})^2 \\ &= \sum_{y=1}^N b_{kx} b_{k'x'} \sum_{z=1}^N (a_{xz} a_{x'z}) \times 1 \\ &= \sum_{y=1}^N b_{kx} b_{k'x'} \times 0 = 0 \end{aligned} \quad (54)$$

where \mathbf{x}^H stands for the Hermitian transform of \mathbf{x} . The above result shows that N -chip shifted ACF and CCF of a CC code set are always zero.

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