Announcements

• Project Preliminary Report #2 now due Monday April 26 in class

• Exam 2 is April 30

• Project feedback meetings
  • Today 3:30-5
  • Wednesday 10:30-12
Agenda

- VCOs
Charge-Pump PLL Circuits

- Phase Detector
- Charge-Pump
- Loop Filter
- VCO
- Divider
Voltage-Controlled Oscillator

- **Time-domain phase relationship**

\[
\omega_{out}(t) = \omega_0 + \Delta\omega_{out}(t) = \omega_0 + K_{VCO}v_c(t)
\]

\[
\theta_{out}(t) = \int \Delta\omega_{out}(t)dt = K_{VCO} \int v_c(t)dt
\]
Voltage-Controlled Oscillators (VCO)

- Ring Oscillator
  - Easy to integrate
  - Wide tuning range (5x)
  - Higher phase noise

- LC Oscillator
  - Large area
  - Narrow tuning range (20-30%)
  - Lower phase noise
Barkhausen’s Oscillation Criteria

Closed-loop transfer function:

\[ \frac{H(j\omega)}{1 - H(j\omega)} \]

- Sustained oscillation occurs if \( H(j\omega) = 1 \)

- 2 conditions:
  - Gain = 1 at oscillation frequency \( \omega_0 \)
  - Total phase shift around loop is \( n360^\circ \) at oscillation frequency \( \omega_0 \)
Ring Oscillator Example

Three-stage ring oscillator

\[ H(s) = -\frac{A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3} \]

\[ \omega_{osc} = \sqrt{3}\omega_0 \]

\[ \tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = 60^\circ \]

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{-A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3} = \frac{-A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3 + A_0^3} \]

\[ \left[ \sqrt{1 + \left(\frac{\omega_{osc}}{\omega_0}\right)^2} \right]^3 = 1 \]

\[ A_0 = 2 \]
Ring Oscillator Example

- 4-stage oscillator
  - $A_0 = \sqrt{2}$
  - Phase shift = 45

- Easier to make a larger-stage oscillator oscillate, as it requires less gain and phase shift per stage

\[ H(s) = -\frac{A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3} \]
\[ \omega_{osc} = \sqrt{3}\omega_0 \]
\[ \tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = 60^\circ \]
\[ \frac{V_{out}(s)}{V_{in}(s)} = -\frac{A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3 + A_0^2} \]
\[ 1 + \left(\frac{\omega_{osc}}{\omega_0}\right)^2 = 1 \]
\[ A_0 = 2 \]
LC Oscillator Example

LC tank impedance

\[ |Z_{eq}(s = j\omega)|^2 = \frac{R_s^2 + L_1^2\omega^2}{(1 - L_1C_1\omega^2)^2 + R_s^2C_1^2\omega^2} \]

\[ L_P = L_1\left(1 + \frac{R_s^2}{L_1^2\omega^2}\right), \quad C_P = C_1, \quad R_P \approx \frac{L_1^2\omega^2}{R_s} \]

[\text{Razavi}]
LC Oscillator Example

- Phase condition satisfied at:
  \[ \omega_1 = \frac{1}{\sqrt{L_p C_p}} \]

- Gain condition satisfied when:
  \[ (g_m R_p)^2 \geq 1 \]
Supply-Tuned Ring Oscillator

\[ T_{VCO} = 2nT_D \approx \frac{2nC_{\text{stage}}}{\beta(V_c - V_{th})} \]

\[ K_{VCO} = \frac{\partial f_{VCO}}{\partial V_c} = \frac{\beta}{2nC_{\text{stage}}} \]

[Sidiropoulos VLSI 2000]
Current-Starved Ring Oscillator

Current-starved VCO.
Capacitive-Tuned Ring Oscillator

\[ C_{eff} = \frac{C}{1 + sCR} \]
Symmetric Load Ring Oscillator

- Symmetric load provides frequency tuning at excellent supply noise rejection
- See Maneatis papers for self-biased techniques to obtain constant damping factor and loop bandwidth (% of ref clk)
LC Oscillator

- A variable capacitor (varactor) is often used to adjust oscillation frequency.

- Total capacitance includes both tuning capacitance and fixed capacitances which reduce the tuning range.

\[ \omega_{osc} = \frac{1}{\sqrt{L_P C_P}} = \frac{1}{\sqrt{L_P \left( C_{tune} + C_{fixed} \right)}} \]
Varactors

- pn junction varactor
  - Avoid forward bias region to prevent oscillator nonlinearity

- MOS varactor
  - Accumulation-mode devices have better Q

[Perrott]

[Razavi]
Oscillator Noise

Jitter

PHASE NOISE

[McNeill]
Oscillator Phase Noise Model

\[ L(\Delta f) = 10 \log \left( \frac{\text{Noise Spectral Density}}{\text{Carrier Power}} \right) \text{ (dBc/Hz)} \]

Leeson’s Model:

\[ L(\Delta f) = 10 \log \left( \frac{2FkT}{P_{\text{sig}}} \left( 1 + \left( \frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right) \left( 1 + \frac{\Delta f_{1/f^3}}{|\Delta f|} \right) \right) \]

- For improved model see Hajimiri papers

[Perrott]
Open-Loop VCO Jitter

- Measure distribution of clock threshold crossings
- Plot $\sigma$ as a function of delay $\Delta T$

D.U.T. [McNeill]

CSA803A

Average delay = $NT_0$

$\Delta T$

$\sigma_{\Delta T}$
Open-Loop VCO Jitter

- Jitter $\sigma$ is proportional to $\sqrt{\Delta T}$
- $\kappa$ is VCO time domain figure of merit
VCO in Closed-Loop PLL Jitter

- PLL limits $\sigma$ for delays longer than loop bandwidth $\tau_L$

$$\tau_L = \frac{1}{2\pi f_L}$$
Converting Phase Noise to Jitter

- RMS Phase Jitter

\[
J_{\text{phase}} = \frac{1}{2\pi f_{\text{vco}}} \sqrt{\int S_{\phi}(f) df}
\]

- Integration range depends on application
  - \( f_{\text{min}} \) set by standard
    - Ex. Assumed CDR tracking bandwidth
  - Usually stop integration at \( f_o/2 \) to avoid capturing carrier and harmonics
Next Time

- PLL wrap-up
- CDRs