ECEN689: Special Topics in High-Speed Links Circuits and Systems
Spring 2010

Lecture 22: ISI and Random Noise

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Announcements

• HW6 due Wednesday April 7 (in class)
• Exam 2 will be either April 28 or 30
• Reading
  • Dally 6.1-6.3
Agenda

- Common noise sources
  - ISI
  - Random noise
Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)

\[ y^{(d_k)}(t) = c^{(d_k)}(t) \ast h(t) \]

\[ y^{(1)}(t) \] sampled relative to pulse peak:

\[ \ldots 0.003 \ 0.036 \ 0.540 \ 0.165 \ 0.065 \ 0.033 \ 0.020 \ 0.012 \ 0.009 \ \ldots \]

\[ k = [ \ldots -2 \ 1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ \ldots ] \]

By Linearity: \( y^{(0)}(t) = -1 \ast y^{(1)}(t) \)
Peak Distortion Analysis Example

\[ y_0^{(1)}(t) = 0.540 \]

\[
\sum_{k=-\infty}^{\infty} y^{(1)}(t-kT)_{y(t-kT)<0} = -0.007
\]

\[
\sum_{k=-\infty}^{\infty} y^{(1)}(t-kT)_{y(t-kT)>0} = 0.389
\]

\[ s(t) = 2(0.540 - 0.007 - 0.389) = 0.288 \]
Worst-Case Eye vs Random Data Eye

- Worst-case data pattern can occur at very low probability!
- Considering worst-case is too pessimistic
Constructing ISI Probability Density Function (PDF)

- Using ISI probability density function will yield a more accurate BER performance estimate.

- In order to construct the total ISI PDF, need to convolve all of the individual ISI term PDFs together:
  - 50% probability of “1” symbol ISI and “-1” symbol ISI.
Convolving Individual ISI PDFs Together

- Keep going until all individual PDFs convolved together
Complete ISI PDF

ISI PDF

Probability

Voltage (V)

-0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
Data 1 PDF is centered about the cursor value and varies from a maximum positive value to the worst-case value predicted by PDA.
- This worst-case value occurs at a low probability!
Cursor Cumulative Distribution Function (CDF)

- Data 1 error probability for a given offset is equal to the Data 1 CDF

\[
BER(X) = \int_{-\infty}^{X} (PDF) \, dx
\]
Combining Cursor CDFs

Data 1 CDF

Data CDF

Data -1 CDF

Refined BS 4Gbits Eyes
Bit-Error-Rate (BER) Distribution Eye

- Statistical BER analysis tools use this technique to account for ISI distribution and also other noise sources
  - Example from Stateye
    - Note: Different channel & data rate from previous slides
Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise
Random Noise

• Random noise is unbounded and modeled statistically
  • Example: Circuit thermal and shot noise

• Modeled as a continuous random variable described by
  • Probability density function (PDF)
  • Mean, $\mu$
  • Standard deviation, $\sigma$

$$PDF = P_n(x), \ \mu_n = \int_{-\infty}^{\infty} xP_n(x)dx, \ \sigma_n^2 = \int_{-\infty}^{\infty} (x - \mu_n)^2 P_n(x)dx$$
Gaussian Distribution

- Gaussian distribution is normally assumed for random noise
- Larger sigma value results in increased distribution spread

\[ P_n(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu_n)^2}{2\sigma^2}} \]
Signal with Added Gaussian Noise

- Finite probability of noise pushing signal past threshold to yield an error
Cumulative Distribution Function (CDF)

- The CDF tells what is the probability that the noise signal **exceeds** a certain value.

\[
\Phi_n(x) = \int_{u=-\infty}^{x} P_n(u) \, du = \int_{u=-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(u-\mu_n)^2}{2\sigma^2}} \, du
\]
Error and Complimentary Error Functions

- **Error Function:**
  \[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{u=0}^{x} \exp(-u^2)du \]

- **Relationship between normal CDF and Error Function:**
  \[ \Phi(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right] \]

- **The complementary error function gives the probability that the noise will exceed a given value**
  \[ Q(x) = 1 - \Phi(x) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right] \]
  \[ = \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right) \]
  \[ Q_{\mu\sigma}(x) = \frac{1}{2} \text{erfc} \left( \frac{x - \mu}{\sigma \sqrt{2}} \right) \]
Bit Error Rate (BER)

- Using erfc to predict BER:

- Need a symbol of about $7\sigma$ for BER=$10^{-12}$
  - Peak-to-peak value will be 2x this
Next Time

• Noise Sources
• Timing Noise
• BER Analysis Techniques