# ECEN689: Special Topics in High-Speed Links Circuits and Systems
# Spring 2010

## Lecture 22: ISI and Random Noise


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# Announcements

- HW6 due Wednesday April 7 (in class)
- Exam 2 will be either April 28 or 30
- Reading
    - Dally 6.1-6.3

# Agenda

- Common noise sources
  - ISI
  - Random noise

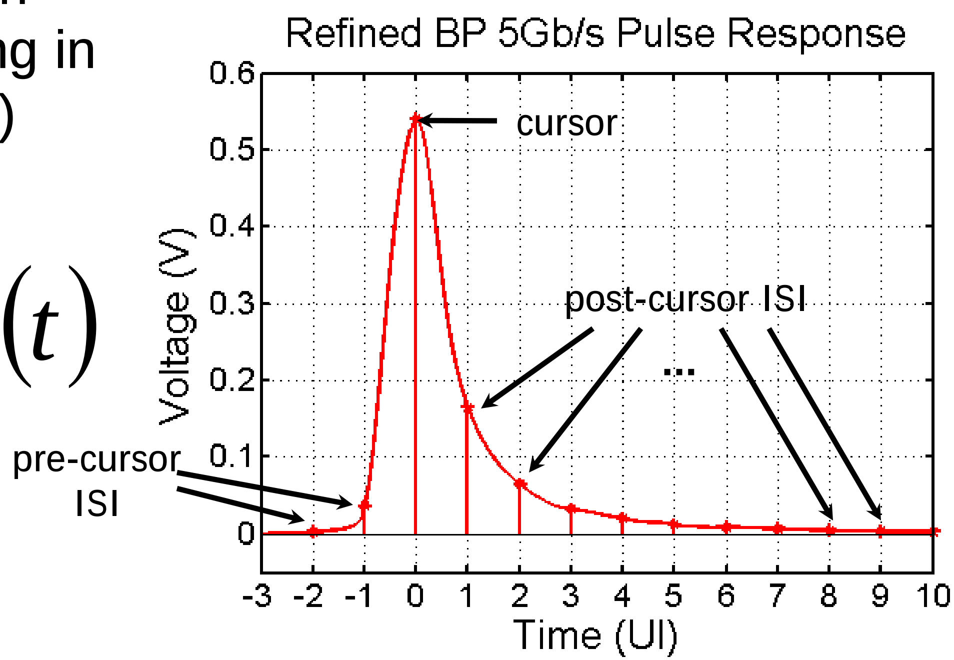
# Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

# Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)

$$y^{(d_k)}(t) = c^{(d_k)}(t) * h(t)$$



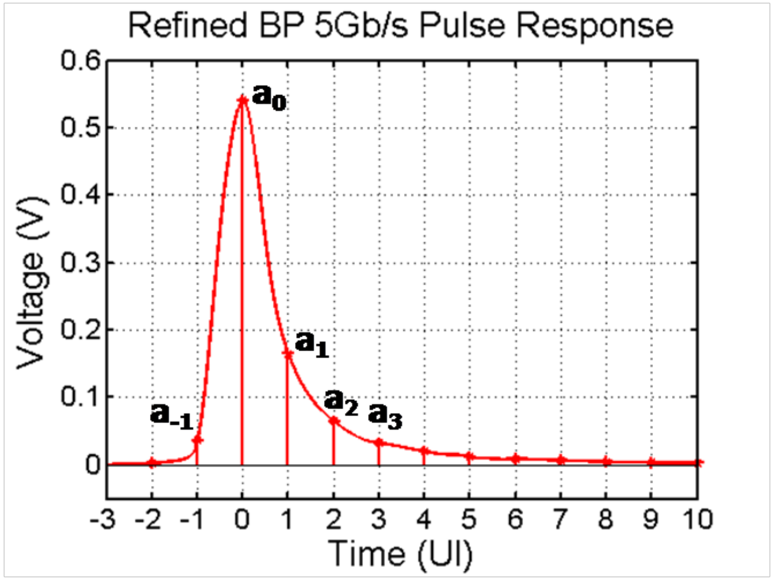

$y^{(1)}(t)$ sampled relative to pulse peak:

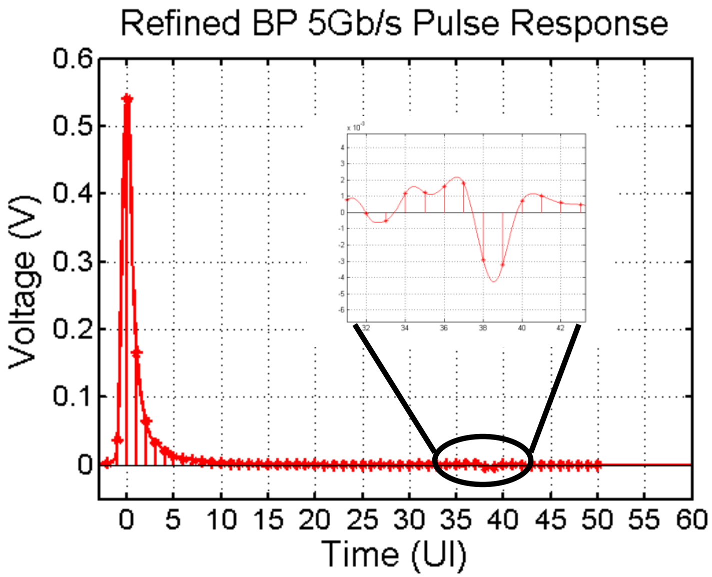
[... 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 ...]

k =[ ... -2 1 0 1 2 3 4 5 6 ...]

By Linearity: $y^{(0)}(t) = -1 * y^{(1)}(t)$

# Peak Distortion Analysis Example



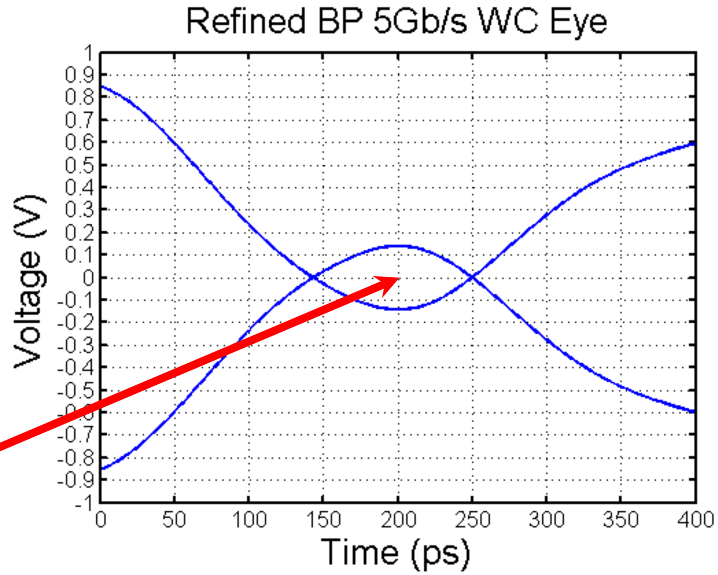




$$y_0^{(1)}(t) = 0.540$$

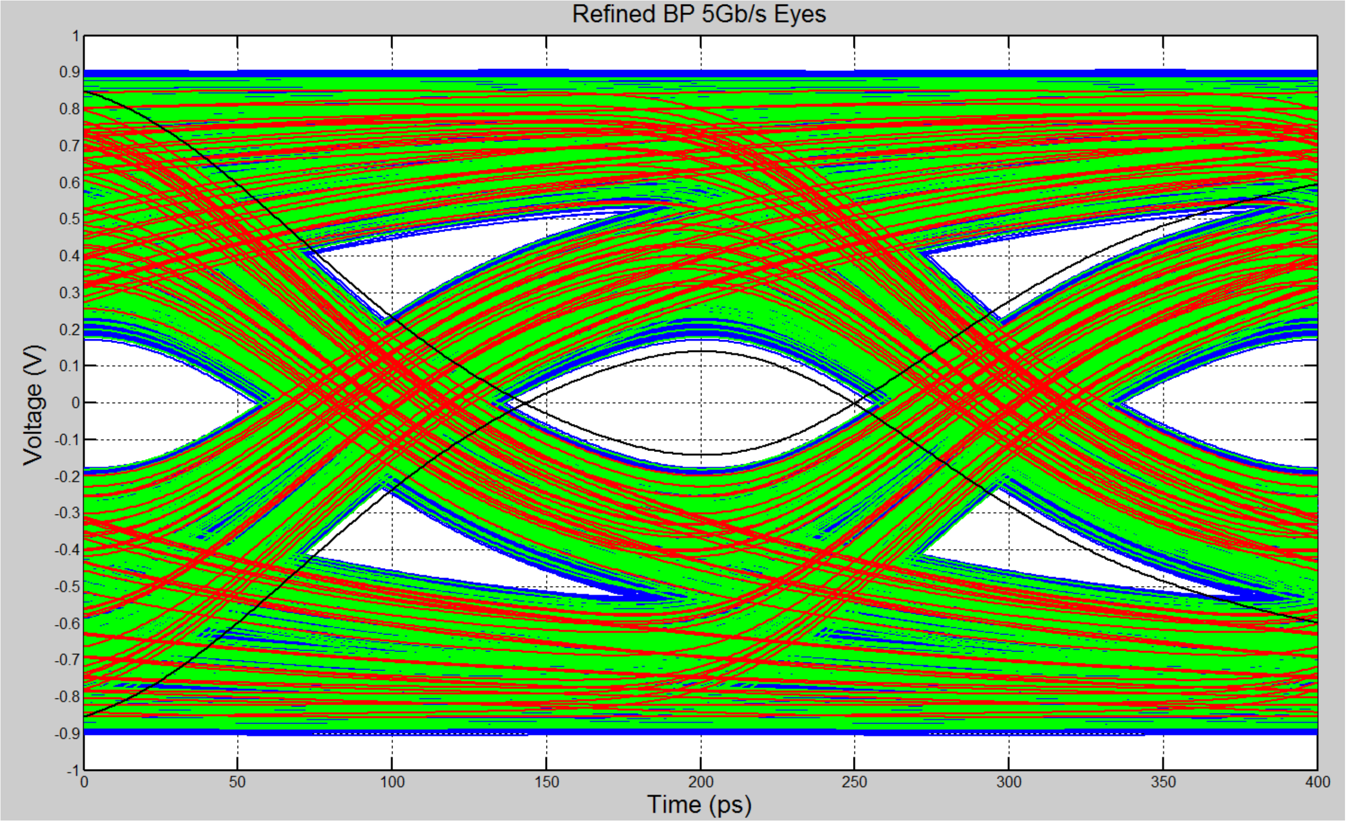
$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.007$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.389$$

$$s(t) = 2(0.540 - 0.007 - 0.389) = 0.288$$



# Worst-Case Eye vs Random Data Eye

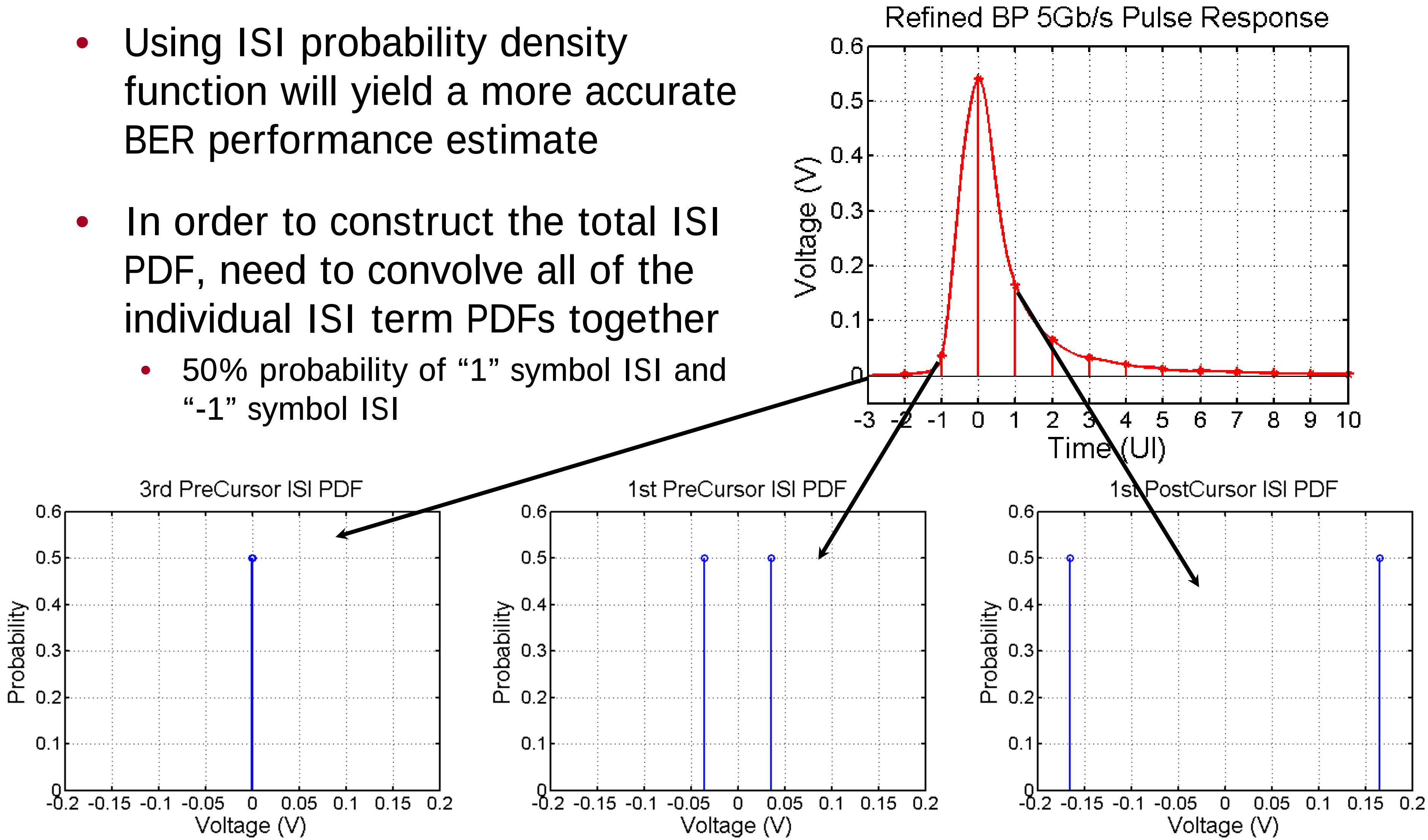



**Worst-Case Eye**
**100 Random Bits**
**1000 Random Bits**
**1e4 Random Bits**

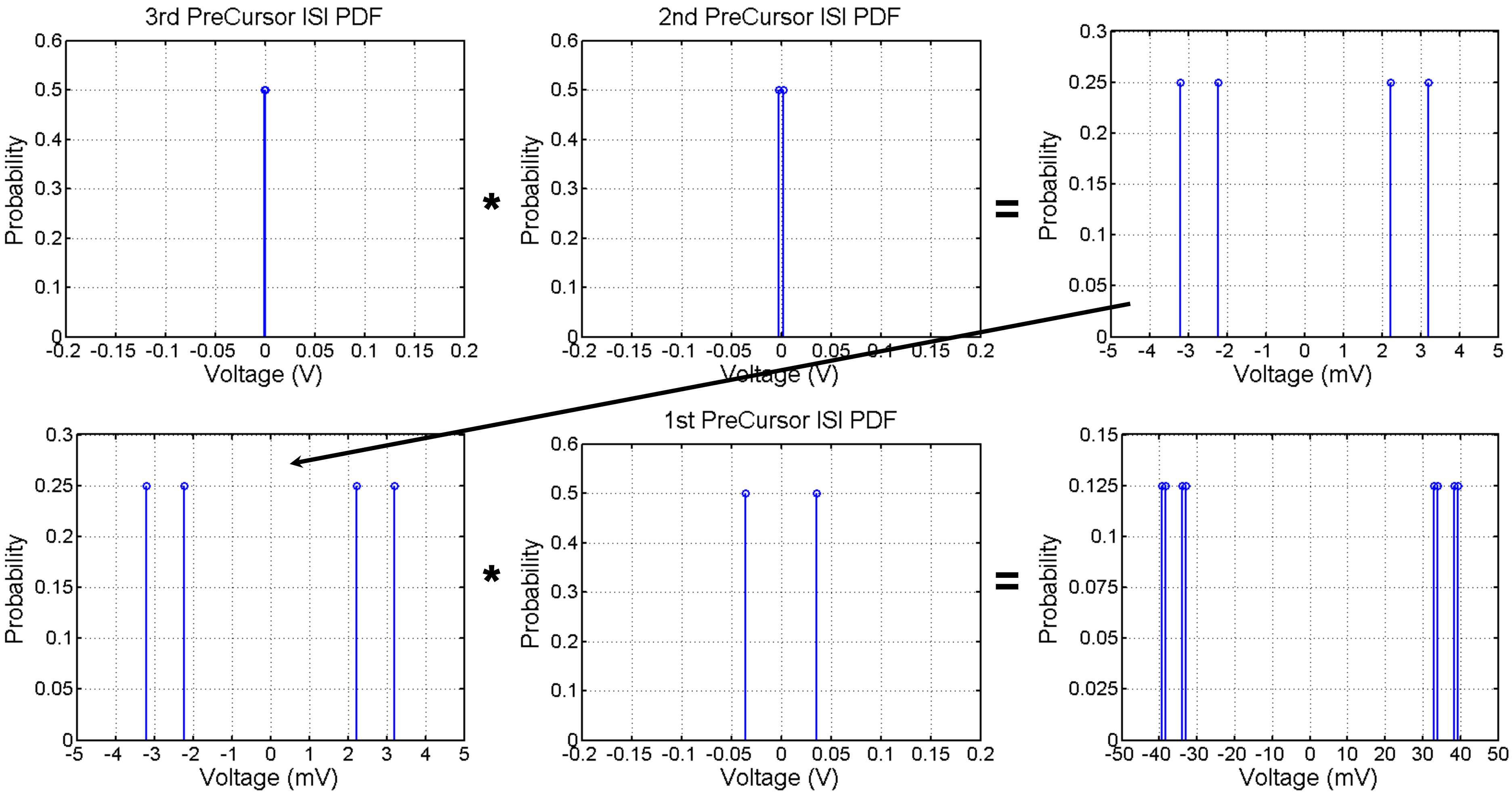
- Worst-case data pattern can occur at very low probability!
- Considering worst-case is too pessimistic

# Constructing ISI Probability Density Function (PDF)

- Using ISI probability density function will yield a more accurate BER performance estimate
- In order to construct the total ISI PDF, need to convolve all of the individual ISI term PDFs together
  - 50% probability of “1” symbol ISI and “-1” symbol ISI

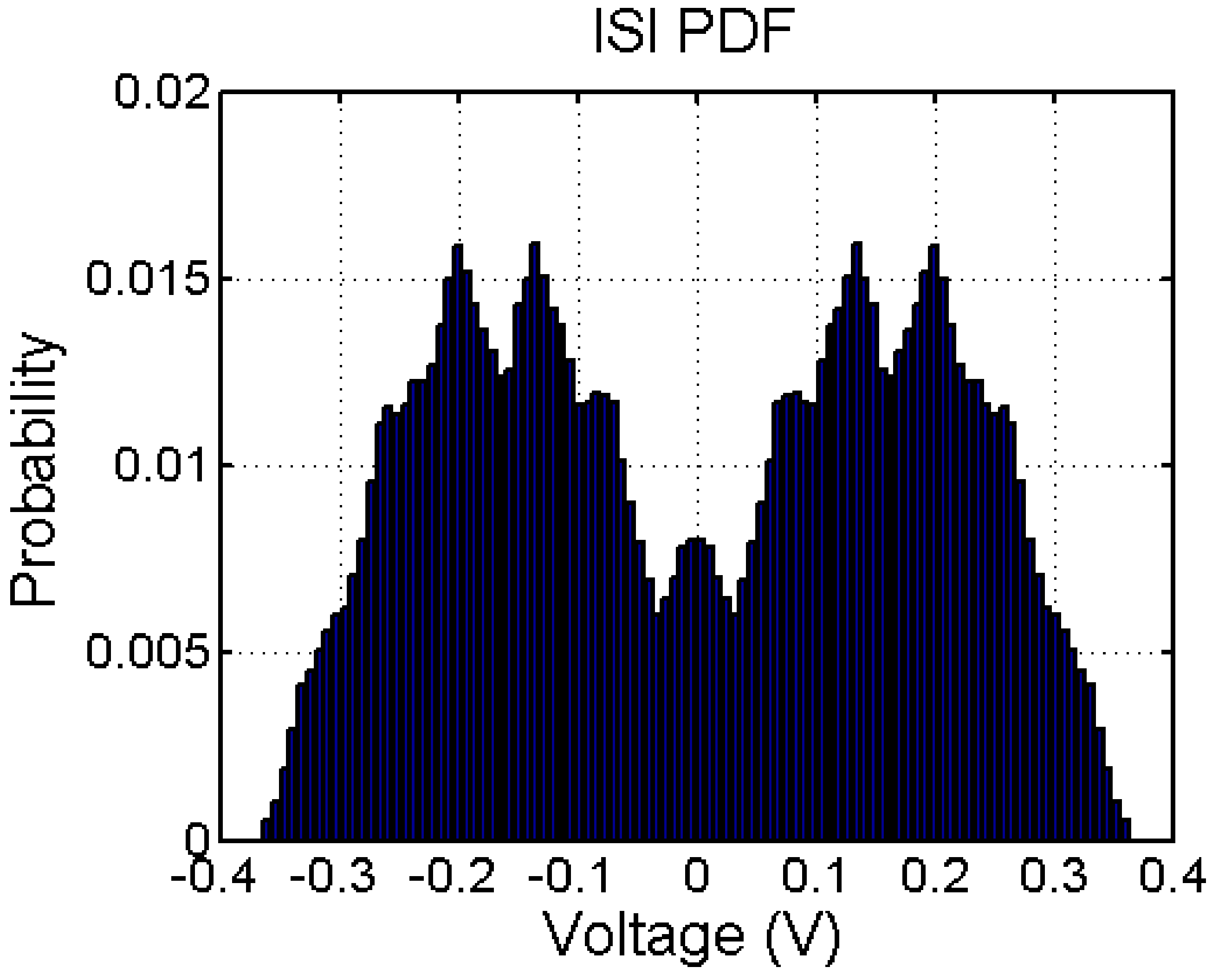


# Convolving Individual ISI PDFs Together

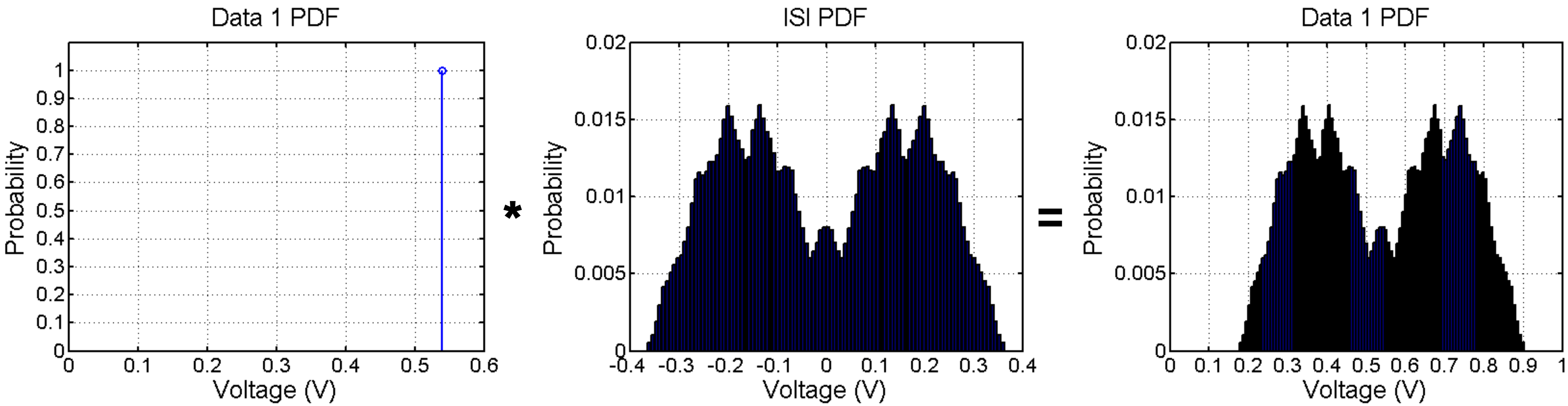



- Keep going until all individual PDFs convolved together

# Complete ISI PDF



# Cursor PDF – Data 1

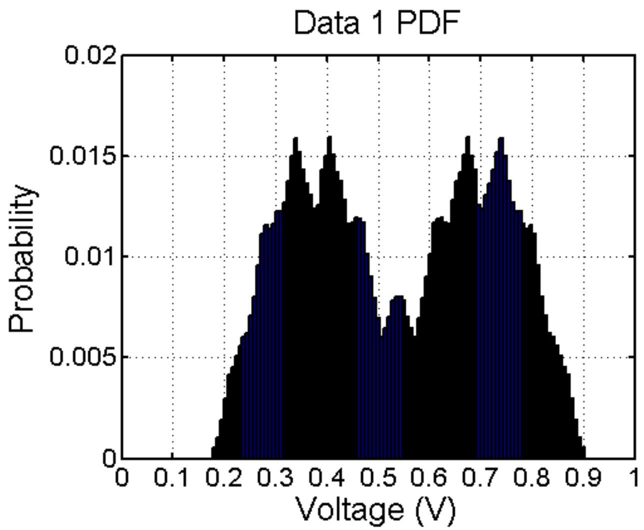


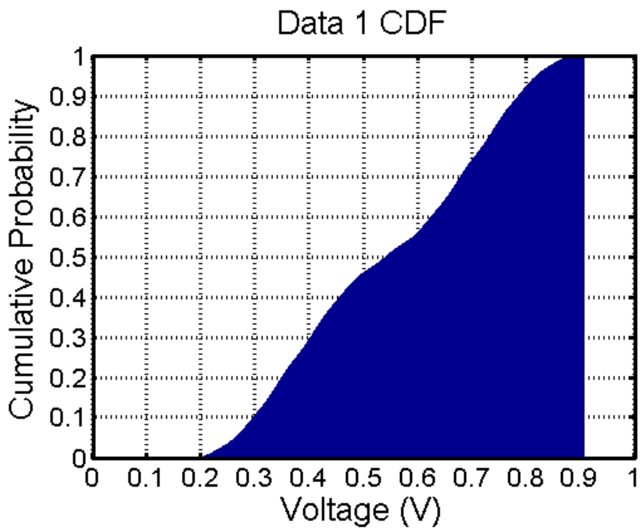

- Data 1 PDF is centered about the cursor value and varies from a maximum positive value to the worst-case value predicted by PDA
    - This worst-case value occurs at a low probability!

# Cursor Cumulative Distribution Function (CDF)

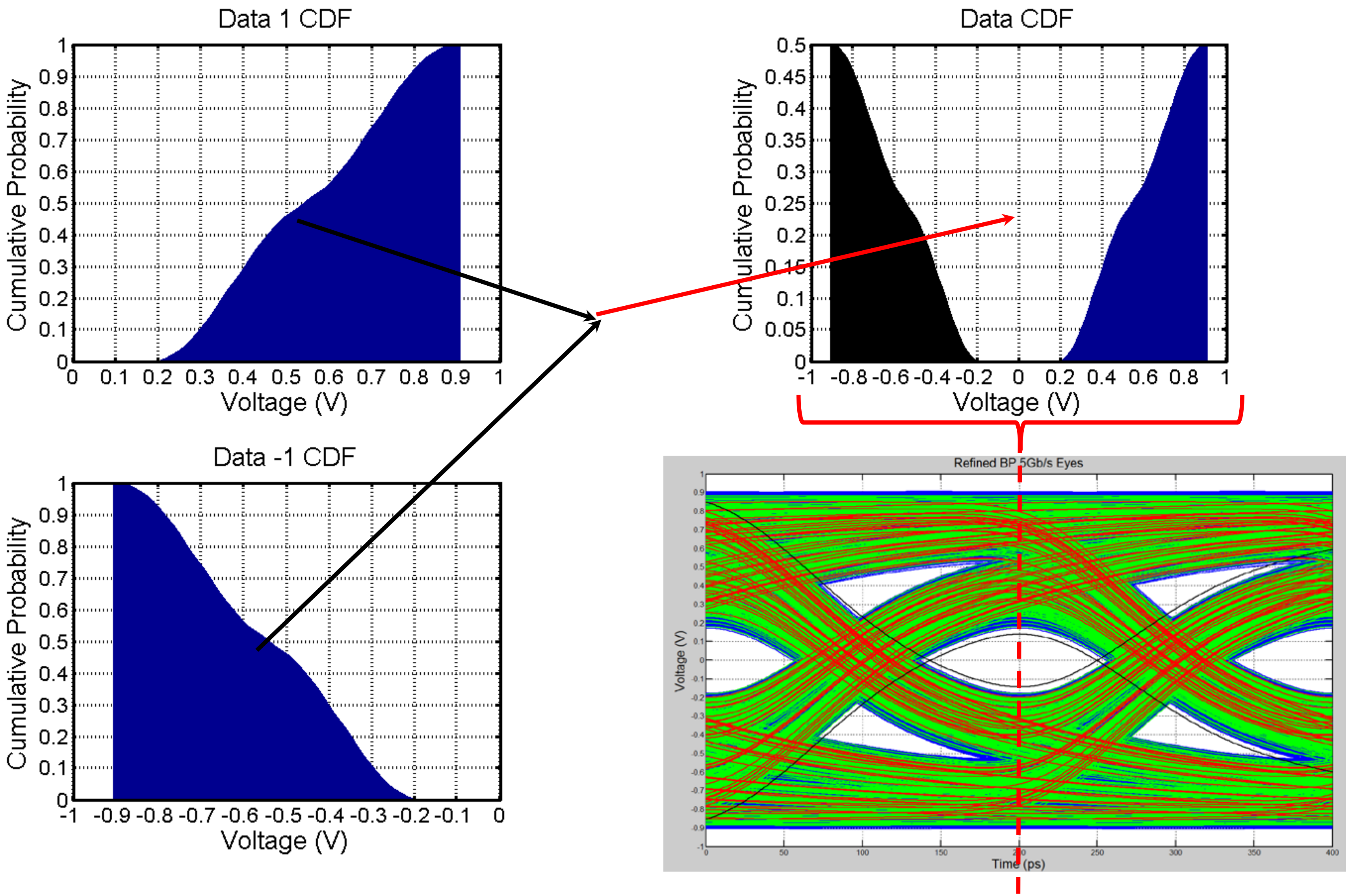
- Data 1 error probability for a given offset is equal to the Data 1 CDF

$$BER(X) = \int_{-\infty}^{X} (PDF)\,dx$$



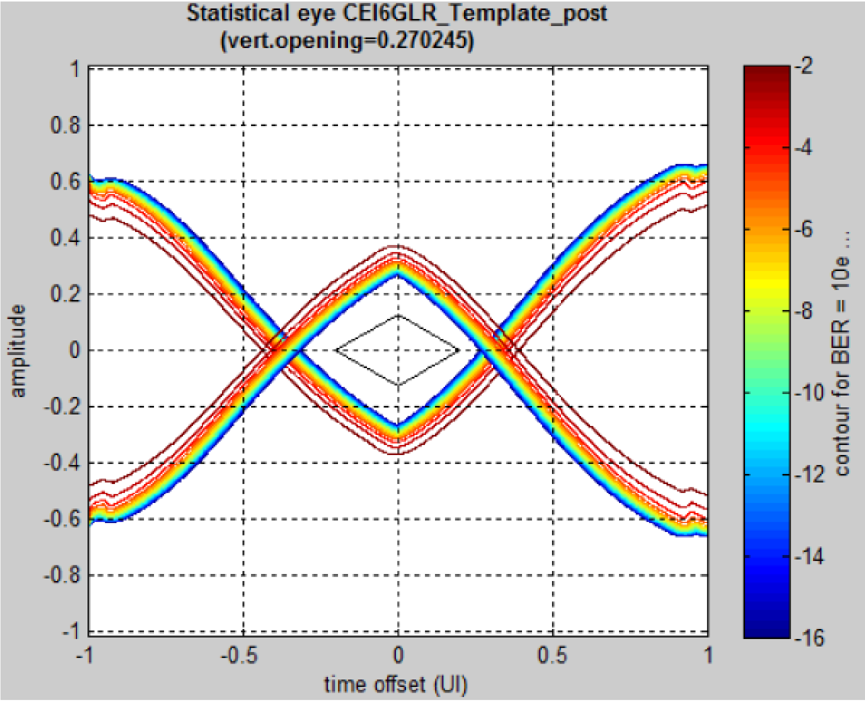



# Combining Cursor CDFs



# Bit-Error-Rate (BER) Distribution Eye

- Statistical BER analysis tools use this technique to account for ISI distribution and also other noise sources
  - Example from Stateye
    - Note: Different channel & data rate from previous slides



# Common Noise Sources

- Power supply noise
- Receiver offset
- Crosstalk
- Inter-symbol interference
- Random noise

# Random Noise

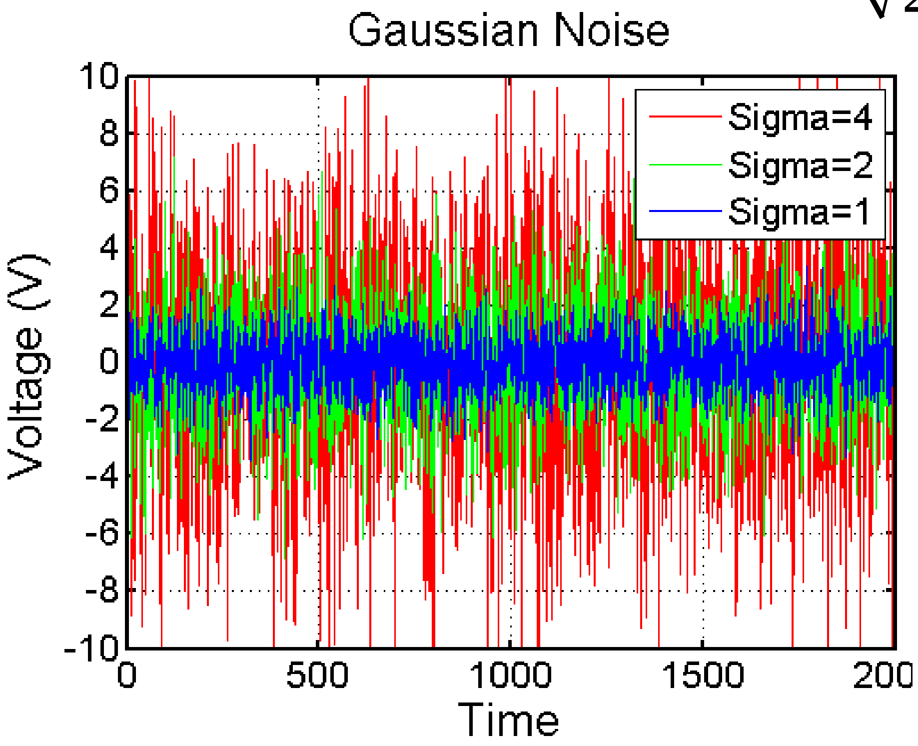
- Random noise is unbounded and modeled statistically
  - Example: Circuit thermal and shot noise
- Modeled as a continuous random variable described by
  - Probability density function (PDF)
  - Mean, $\mu$
  - Standard deviation, $\sigma$

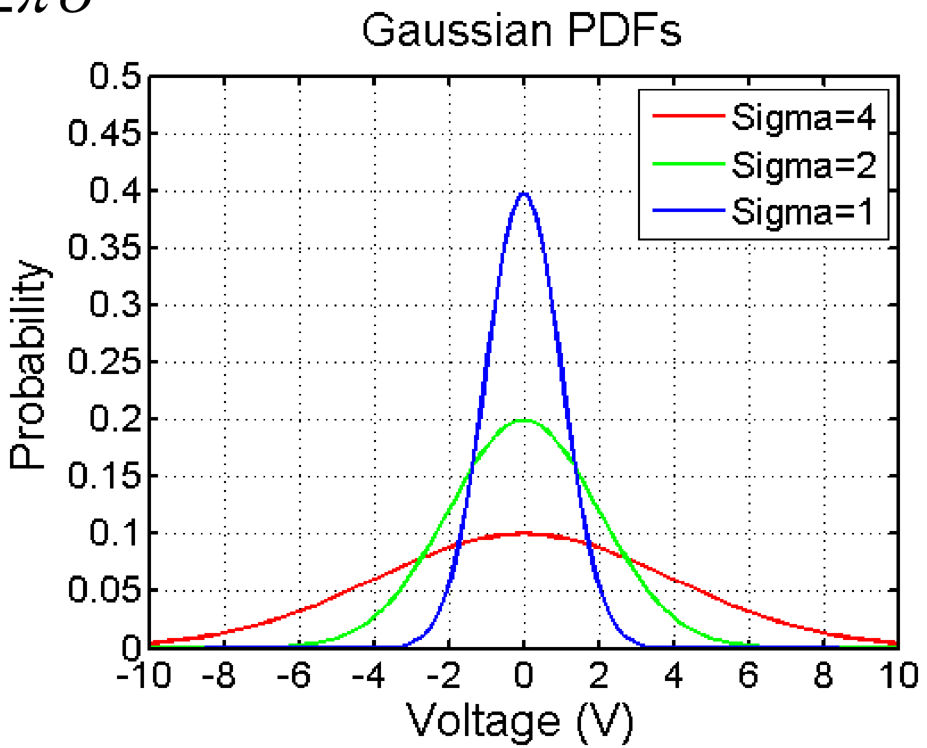
$$PDF = P_n(x), \quad \mu_n = \int_{-\infty}^{\infty} x P_n(x) dx, \quad \sigma_n^2 = \int_{-\infty}^{\infty} (x - \mu_n)^2 P_n(x) dx$$

# Gaussian Distribution

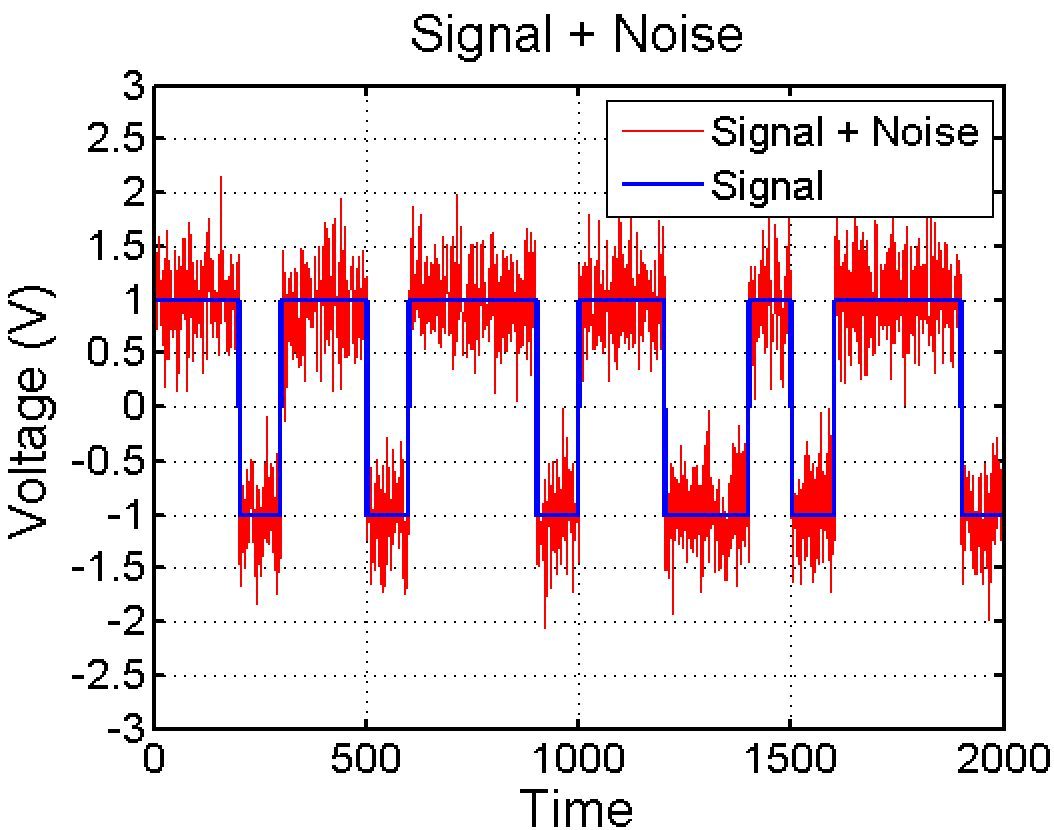
- Gaussian distribution is normally assumed for random noise
  - Larger sigma value results in increased distribution spread

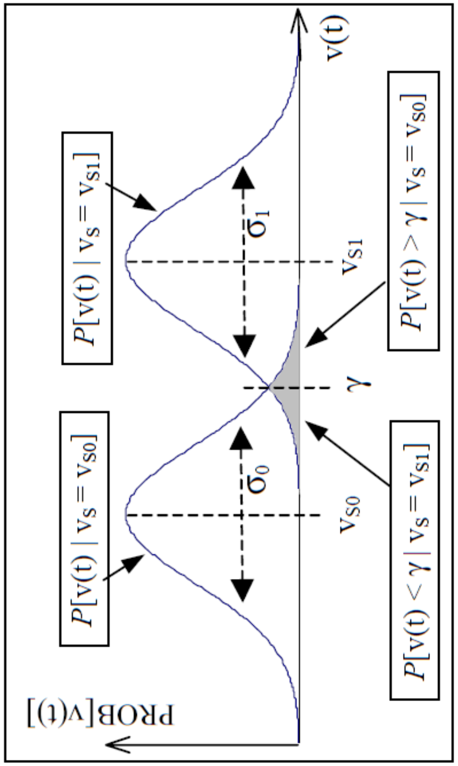
$$P_n(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu_n)^2}{2\sigma^2}}$$



# Signal with Added Gaussian Noise

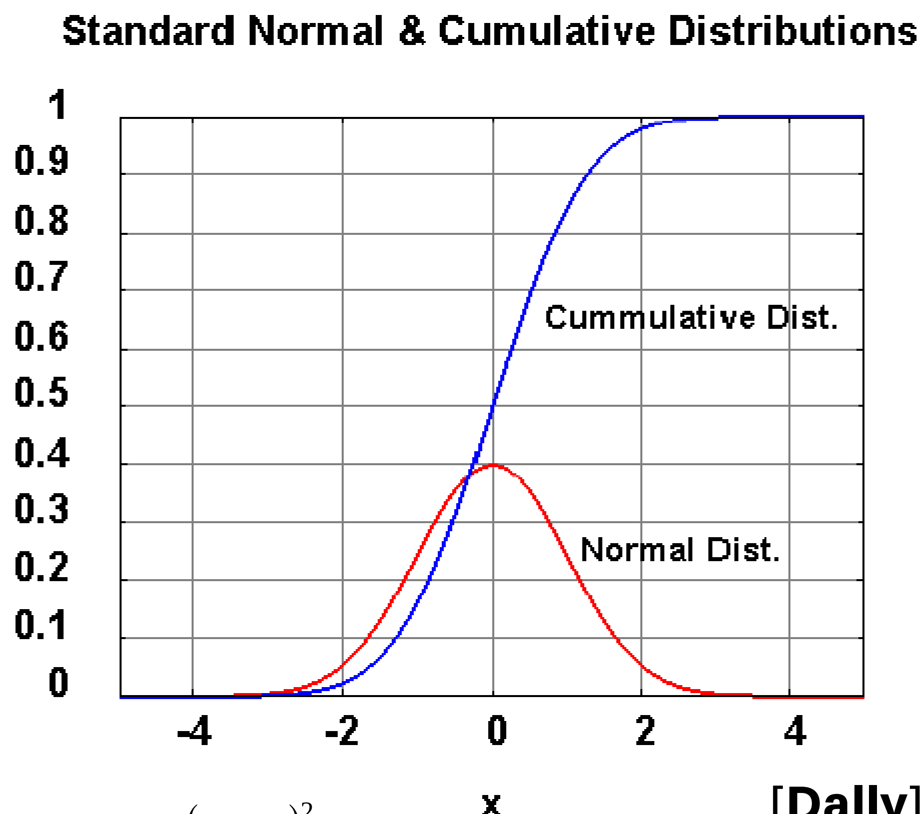



- Finite probability of noise pushing signal past threshold to yield an error

# Cumulative Distribution Function (CDF)

- The CDF tells what is the probability that the noise signal **exceeds** a certain value




[Dally]

$$\Phi_n(x) = \int_{u=-\infty}^{x} P_n(u)\,du = \int_{u=-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(u-\mu_n)^2}{2\sigma^2}}\,du$$

# Error and Complimentary Error Functions

- Error Function:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{u=0}^{x} \exp(-u^2) du$$

- Relationship between normal CDF and Error Function:

$$\Phi(x) = \frac{1}{2}\left[1 + erf\left(\frac{x}{\sqrt{2}}\right)\right]$$

- The complementary error function gives the probability that the noise will exceed a given value

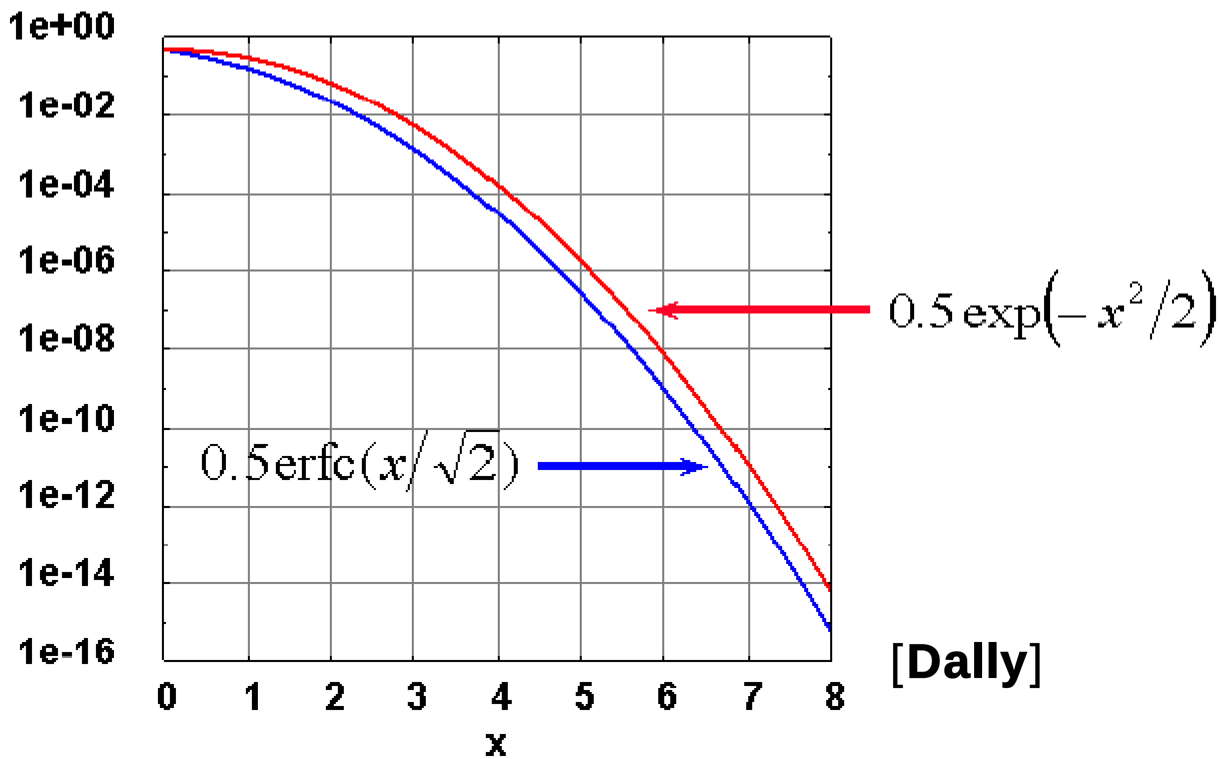
$$Q(x) = 1 - \Phi(x) = \frac{1}{2}\left[1 - erf\left(\frac{x}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$$

$$Q_{\mu\sigma}(x) == \frac{1}{2} erfc\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)$$

# Bit Error Rate (BER)

- Using erfc to predict BER:




- Need a symbol of about $7\sigma$ for BER=$10^{-12}$
  - Peak-to-peak value will be 2x this

# Next Time

- Noise Sources
- Timing Noise
- BER Analysis Techniques