Physical Processes of Phase Noise in Differential LC Oscillators

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Introduction

There is an unprecedented interest among circuit designers today to obtain insight into mechanisms of phase noise in LC oscillators. For only with this insight is it possible to optimize oscillator circuits using low-quality integrated resonators to comply with the exacting phase noise specifications of modern wireless systems. Various numerical simulators are now available to assist the circuit designer [1], [2], [3], in some cases accompanied by qualitative interpretations [4]. At present, therefore, the situation of the oscillator designer is similar to the designer of amplifiers who is equipped only with SPICE, but who lacks physical insight and methods for simple yet accurate analysis with which to optimize a circuit.

Over the years, various attempts at phase noise analysis have produced results that are variations on Leeson's classic "heuristic derivation without formal proof" [5], [6]. These analyses are based on a linear model of an LC resonator in steady-state oscillation through application of either feedback or negative conductance. The results confirm Leeson by showing that phase noise is proportional to noise-to-carrier ratio and inversely to the square of resonator quality factor. However, without knowledge of the constant of proportionality, which Leeson leaves as an unspecified noise factor, the actual phase noise cannot be predicted.

It is now well understood that the large-signal periodic switching of a self-limited oscillator [7] underpins this noise factor [8]. At first sight, an accurate noise analysis of an oscillator subject to periodic bias currents appears intractable, however by using sensible approximations Huang has solved this problem for a Colpitts oscillator [9] and obtained good agreement between analysis and measurements of thermally induced phase noise. The mechanisms of flicker noise upconversion, which are important in CMOS oscillators, remain obscure.

In this paper we concentrate on an understanding of the popular differential LC oscillator. We introduce simple models to capture the nonlinear processes that convert voltage or current thermal noise in resistors or transistors into phase noise in the oscillator. The analysis does not require hypothetical elements, such as limiters or amplitude control loops, to fully explain phase noise. A simple expression at the end accurately specifies thermally induced phase noise, and lends substance to Leeson's original hypothesis.

Next, the upconversion of flicker noise into phase noise is traced to mechanisms first identified in the 1930's, but apparently since forgotten. Unlike thermally induced phase noise, which appears as phase modulation sidebands, flicker noise is shown to upconvert by bias-dependent frequency modulation.

The results are validated against SpectreRF simulations and measurements on two differential CMOS oscillators tuned by resonators with very different Q's.

Recognizing Phase Noise

For the purposes of analysis, a noise spectrum is considered as consisting of uncorrelated sinewaves in a 1 Hz bandwidth at any given frequency. Voltage or current noise produces amplitude and phase fluctuations when superimposed on a periodic signal (from now on, a large sinewave $V_0\sin(2\pi f_0 t)$). This is clearly seen [10] by isolating one sinewave v_n in the noise spectrum, say at a frequency offset $+f_m$ from the sinewave frequency f_0. Figure 1 shows this as a phasor v_n rotating relative to the sinewave phasor V_0, which is then decomposed into two equal collinear phasors at $+f_m$, and two anti-phase conjugate phasors which are assigned a negative relative frequency $-f_m$. Grouping the phasors pairwise as $\pm f_m$, it is seen that one pair modulates the amplitude of the sinewave with time (AM), while the other sweeps its phase (PM). Thus, half of any additive noise on a sinewave produces phase noise, the other half amplitude noise. When $\sin(\omega_0 t)$ is accompanied either by noise sinewave phasors $\pm\sin(\omega_0+\omega_m)t$, $\mp\sin(\omega_0-\omega_m)t$ or by $\pm\cos(\omega_0+\omega_m)t$, $\pm\cos(\omega_0-\omega_m)t$, then phase noise alone is present.

Simple Model of the Differential Oscillator

This paper treats the well-known tail-current biased differential LC oscillator (Figure 2). In steady state, the differential pair acts as a negative conductance that switches the tail current I_T into the LC resonator. Owing to filtering in the LC circuit, the square wave of current creates a sinusoidal voltage across the resonator of amplitude $(4/\pi)I_T R$. This voltage drives the differential pair into switching, thus sustaining oscillation. In a CMOS oscillator the amplitude may build up to several volts, eventually limited by the supply voltage.

In previous work on noise in mixers [11], we have shown how a simple model of the switching differential pair is sufficient to explain all frequency translations of noise. This model is used here. Suppose that some noise (v_n) accompanies the resonator sinewave. Assuming that a small fraction of the resonator voltage around the zero crossing is enough to fully switch the differential pair, then the noise simply advances or retards the instant of zero crossing (Figure 3(a)). The randomly pulse-width modulated current at the switch output may be decomposed into the original periodic square wave in the absence of noise, superimposed with pulses of constant height but random width (Figure 3(b)). In turn, these pulses may be approximated by a train of impulses at twice the oscillation frequency multiplying the original noise waveform $v_n(t)$ (Figure 3(c)).

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Thermally Induced Phase Noise

Resonator Noise

Now consider a current source $i_n \sin((\omega_0+\omega_m)t+\phi)$ representing noise in the loss conductance of the resonator, where $i_n^2=4kT/R$. According to the model above, this modulates the zero crossing instants of the differential pair, producing a current which, in addition to the usual square wave, also consists of current pulses sampling this noise at $2\omega_0$. After sampling, frequency components appear at $\omega_0 \pm \omega_m$, $3\omega_0 \pm \omega_m$, ... However, usually the resonator will filter the 3rd and higher harmonics, leaving $\omega_0 \pm \omega_m$ as the only important terms. These will induce a symmetric voltage response in the resonator, and through feedback arrive at steady state. The steady-state oscillation, in general, is of the form:

$$v_{out} = V_0 \sin \omega_0 t + A \sin(\omega_0 - \omega_m)t + B \cos(\omega_0 - \omega_m)t$$
$$+C \sin(\omega_0 + \omega_m)t + D \cos(\omega_0 + \omega_m)t$$

and here $A=-C=i_n \times (L\omega_0^2/4\omega_m)$, while $B=D\simeq 0$. The relative signs of A and C prove that the steady-state response to current noise in the resonator's resistor is phase noise in the oscillator. The single-sideband phase noise density is found by the ratio of the sideband power at a given frequency to the power in the fundamental oscillation frequency. Thus, the thermally induced phase noise density due to resonator loss is:

$$\mathcal{L}(\omega_m) = N_1 N_2 \frac{kTR}{V_0^2}\left(\frac{\omega_0}{2Q\omega_m}\right)^2$$

where $N_1=2$, the number of loss sources (in the left and right resonators) and $N_2=4$ because uncorrelated quadrature noise originating at $\omega_0 \pm \omega_m$ contributes to SSB phase noise at offset ω_m.

Tail Current Noise

The switching action of the differential pair commutates noise in the tail currents like a single-balanced mixer. The noise is translated up and down in frequency, and enters the resonator. The resulting voltage drives the differential pair, the noise components modulating the zero crossing instants. The resulting impulses of current feed back into the resonator. The steady-state solution is found by solving simultaneous equations of a form that anticipates the end result, much like in any feedback circuit.

The single-balanced mixer shows the largest conversion gain around the fundamental switching frequency, 1/3rd the current conversion gain around the 3rd harmonic, and so on. Therefore, only mixing by the fundamental at ω_0 is important. Noise originating in the tail current at ω_m upconverts to $\omega_0 \pm \omega_m$. Similarly, noise at $2\omega_0 \pm \omega_m$ downconverts to $\omega_0 \pm \omega_m$.

Analysis shows that the upconversion produces coefficients A=C, B=−D, both of which indicate AM only. It should be noted that AM noise superimposed on the resonator fundamental frequency does not modulate the zero crossings of the switching differential pair, and therefore does not propagate in the feedback loop back into the resonator. However, the downconversion results in phase noise only, with A=−C, and $B=D\simeq 0$. The phase noise caused by thermal noise originally at $2\omega_0$ is:

$$\mathcal{L}(\omega_m) = \frac{32}{9}\gamma g_m R \frac{kTR}{V_0^2}\left(\frac{\omega_0}{2Q\omega_m}\right)^2$$

where γ is the noise factor of a single FET, classically 2/3. It is important to note that the AM noise resulting from upconversion, if impressed across a varactor at the resonator, will modulate the varactor, thus the oscillation frequency by AM-to-FM conversion [12]. Although the process is different, the resulting sidebands are indistinguishable from PM noise sidebands. Unlike the other mechanisms of phase noise, this effect depends on the varactor characteristics and VCO tuning range and it may be significant only in certain situations.

Differential Pair Noise

Noise originating in the differential pair is unlike the previous two cases. There, only certain parts of the noise spectrum contributed significantly to the total phase noise. White noise in the resonator is filtered at harmonics of the resonant frequency. White noise in the tail current only experiences a significant conversion gain around the second harmonic of the oscillation frequency. However, the simple model says that an impulse train samples white noise in the differential pair, which if true, will cause it to accumulate without bound at any specified offset frequency ω_m.

In reality, any practical differential pair requires a non-zero input voltage excursion to switch, and this is provided by the oscillation waveform across the resonator. Therefore, noise in the differential pair is actually not sampled by impulses, but by time windows of finite width. The window height is proportional to transconductance, and width is set by tail current, and slope of the oscillation waveform at zero crossing. The input-referred noise spectral density of the differential pair is inversely proportional to transconductance. Thus, the narrower the sampling window, that is, the larger the sampling bandwidth, the lower the noise spectral density [11]. Analysis shows that the noise bandwidth product is constant, and produces pure phase noise. After taking into account the accumulation of frequency translations throughout the sampling bandwidth, the following compact yet exact expression is reached:

$$\mathcal{L}(\omega_m) = \frac{32 I_T R \gamma}{\pi V_0}\frac{kTR}{V_0^2}\left(\frac{\omega_0}{2Q\omega_m}\right)^2$$

We note that [8] has arrived at a similar analysis for the first two sources of noise, but was unable to obtain a closed-form expression for this last term.

Proving Leeson's Hypothesis

Leeson originally postulated that thermally induced phase noise in any oscillator takes the form:

$$\mathcal{L}(\omega_m) = \frac{4FkTR}{V_0^2}\left(\frac{\omega_0}{2Q\omega_m}\right)^2$$

where F is an unspecified noise factor. By summing the expressions obtained above for thermally induced phase noise arising from the resonator, differential pair and tail bias current, respectively, for the differential oscillator Leeson's noise factor is:

$$F = 2 + \frac{8\gamma R I_T}{\pi V_0} + \gamma \tfrac{8}{9} g_{mbias} R$$

We emphasize that this simple expression captures all nonlinear effects and frequency translations. At low bias currents while the

amplitude of oscillation is smaller than the power supply, the differential pair acts as a pure current switch driving the resonator and $V_0=(4/\pi)RI_T$ [13]. Then the second term comprising F simplifies to 2γ. This means that as tail current increases and assuming $g_{mbias}R$ is held constant, the noise factor remains constant and phase noise improves as V_0^2, that is, as I_T^2. This has been observed by others [13]. However, beyond a critical tail current the amplitude V_0 is pegged constant, limited by supply voltage. Further increases in I_T will cause the differential pair's contribution to noise factor to rise, degrading phase noise proportionally to I_T (Figure 4). Therefore, for least phase noise the tail current should be just enough to drive the amplitude to its maximum possible value.

Flicker Noise Upconversion

Close-in to the oscillation frequency, the slope of the phase noise spectrum in all CMOS VCO's turns from –20 to –30 dB/decade. This is ascribed to the upconversion of flicker noise in FETs. To understand this, let us first see if the analysis above explains this upconversion.

Flicker noise in the tail current source at frequency ω_m indeed upconverts to $\omega_0 \pm \omega_m$ and enters the resonator, but as AM, not PM noise. Therefore, in the absence of a high gain varactor to convert AM to FM, flicker noise in the tail current will not appear as phase noise. Next consider flicker noise in the differential pair. The preceding analysis says that this modulates zero crossings, and injects a noise current into the resonator consisting of flicker noise sampled by an impulse train with frequency $2\omega_0$. Thus noise originating at frequency ω_m produces currents at ω_m and at $2\omega_0 \pm \omega_m$. Both frequencies are strongly attenuated in the resonator, and neither explains flicker-induced phase noise at $\omega_0 \pm \omega_m$. One can only conclude that the mechanisms of flicker noise upconversion are quite different than for thermally induced phase noise.

Fundamental Sources of FM in Oscillators

In 1934, Groszkowski [15] while studying electronic oscillators realized that the steady-state oscillation frequency seldom coincides with the natural frequency of the resonator which tunes the oscillator. He found that the discrepancy arises because the active device in the oscillator, such as the differential pair current switch in the circuit considered here, drives the resonator with a harmonic-rich waveform. The harmonics will flow into the lower impedance capacitor (Figure 5) and upset the exact reactive power balance between the L and the C required for steady state. Now the frequency of oscillation must shift down until the reactive power in the inductor increases to equal the reactive power in the capacitor due to the fundamental and all harmonics. The shift, $\Delta\omega$, is:

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{2Q^2}\sum_{n=2}^{\infty}\frac{n^2(1-n^2)}{(1-n^2)^2+n^2/Q^2}\cdot m_n^2$$

where m_n is the normalized level of the nth harmonic. $\Delta\omega$ is the sum of all negative terms, which means that oscillation frequency slows down with more harmonic content. Now the harmonic content at the output of a periodically switching differential pair is a function of the tail current. In the autonomous oscillator, the drive to the differential pair is also a function of tail current. The sensitivity $\partial\omega/\partial I_T$ is responsible for an "indirect" FM [7] due to flicker noise in I_T.

However, this is not the only mechanism of indirect FM. At RF, active device capacitance is also significant, and it no longer appears as a pure negative resistance to the resonator. For example, the differential pair commutates current flowing in the capacitor C_T at the tail, which presents a negative capacitor (or, equivalently, an inductor in a narrowband sense) at the differential output (Figure 6). This speeds up the oscillation frequency. Flicker noise in the differential pair FETs modulates the duty cycle of commutation, and therefore the effective negative capacitance. Here, too, Groszkowski gives a method of systematic analysis [16], which captures the reactive components in the active devices by measuring the area κ enclosed by hysteresis in the dynamic negative resistance curve.

$$\frac{\Delta\omega}{\omega_0} = -\frac{\kappa}{2Q^2\omega_0 L} + \frac{1}{2Q^2}\sum_{n=2}^{\infty}\frac{n^2(1-n^2)}{(1-n^2)^2+n^2/Q^2}\cdot m_n^2$$

Thus the sensitivity of the reactance to bias current or offset voltage in the differential pair is estimated, which is another means whereby flicker noise modulates the frequency of oscillation.

Validation of Analysis

The phase noise model was validated on two CMOS differential LC oscillators. One oscillator uses a low Q, on-chip inductor, while the other uses off-chip inductors with large Q. Flicker noise is modelled as a bias-independent, gate-referred voltage source [14]. The measured data and SpectreRF simulations are plotted with predictions based on this paper. Excellent agreement (Figure 7) is found across the entire spectrum, which encompasses thermally induced phase noise and upconverted flicker noise.

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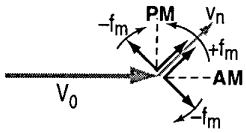

Figure 1. Noise phasor added to a sinewave decomposes into PM and AM sidebands.

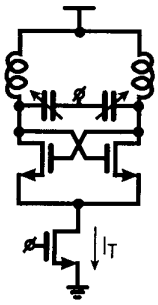

Figure 2. Differential LC oscillator biased by tail current.

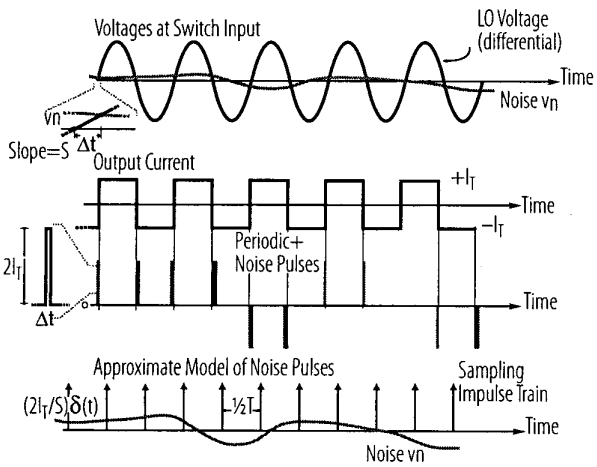

Figure 3. (a) Noise at input of differential pair modulates instants of zero crossing. (b) Output current consists of square wave, plus random noise pulses. (c) Noise pulses modelled as a train of impulses sampling noise waveform.

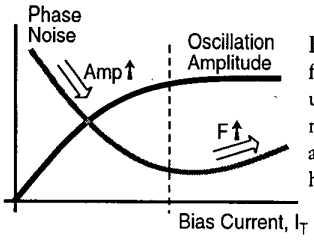

Figure 4. Increasing tail current first causes amplitude to rise, until limited by supply. Phase noise diminishes with rising amplitude, then worsens due to higher noise factor.

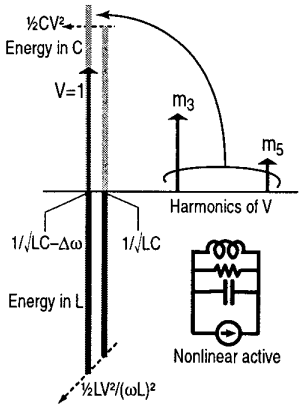

Figure 5. Harmonics of oscillating current flow into capacitor, increasing its reactive energy. Steady state frequency shifts down until inductor energy balances.

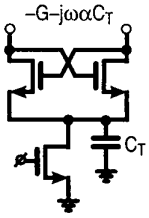

Figure 6. Capacitors associated with active device appear as reactances across the resonator, shifting frequency.

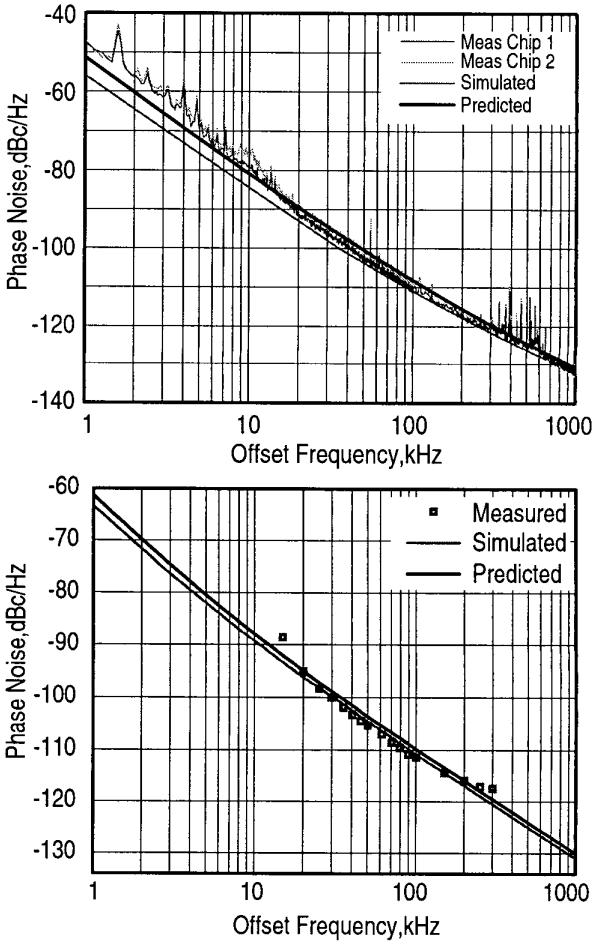

Figure 7. Validation of the analysis presented in this paper. Measured phase noise is compared with predictions from analysis, and with SpectreRF simulations. (a) 0.35-μm CMOS 1.1 GHz oscillator using resonator with loaded Q of 6. (b) 0.25-μm CMOS 830 MHz oscillator using discrete inductor with loaded of Q of 25.