

**Texas A&M University**  
**Department of Electrical and Computer Engineering**

**ECEN 620 – Network Theory (Broadband Circuit Design)**

**Fall 2020**

**Exam #1**

**Instructor: Sam Palermo**

- Please write your name in the space provided below
- Please verify that there are 6 pages in your exam
- You may use one double-sided page of notes and equations for the exam
- Good Luck!

Problem	Score	Max Score
1		40
2		60
<b>Total</b>		<b>100</b>

Name: SAM PALERMO

UIN: \_\_\_\_\_

## Problem 1 (40 points)

Select the proper PLL architecture to track a frequency ramp of  $10^{12} \text{ rad/s}^2$  with a steady-state phase error of 0.01 rad. Assume that  $K_{VCO} = 2\pi \cdot (1\text{GHz/V})$  and that the loop filter utilizes a 1nF capacitor. Additional circuitry should be added to the loop filter to achieve a  $\zeta=2$ . Give the value of  $K_{PD}$  (include correct units), draw the loop filter and label the filter component values.

A 2nd-Order Type 2 PLL is required to track a frequency ramp

$$E(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Frequency Ramp:  $\phi_{in}(s) = \frac{\Lambda}{s^3}$

$$\phi_{e,ss} = \lim_{s \rightarrow 0} \frac{\Lambda}{s^3} \frac{s \cdot s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\Lambda}{\omega_n^2} = 0.01 \text{ rad}$$

$$\Rightarrow \omega_n = \sqrt{\frac{\Lambda}{\phi_{e,ss}}} = \sqrt{\frac{10^{12} \text{ rad/s}^2}{0.01 \text{ rad}}} = 10^7 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{K_{PD} K_{VCO}}{C}}$$

$$K_{PD} = \frac{\omega_n^2 C}{K_{VCO}} = \frac{(10^7 \text{ rad/s})^2 (1\text{nF})}{2\pi (1\text{GHz/V})} = \frac{100 \mu\text{A}}{2\pi}$$

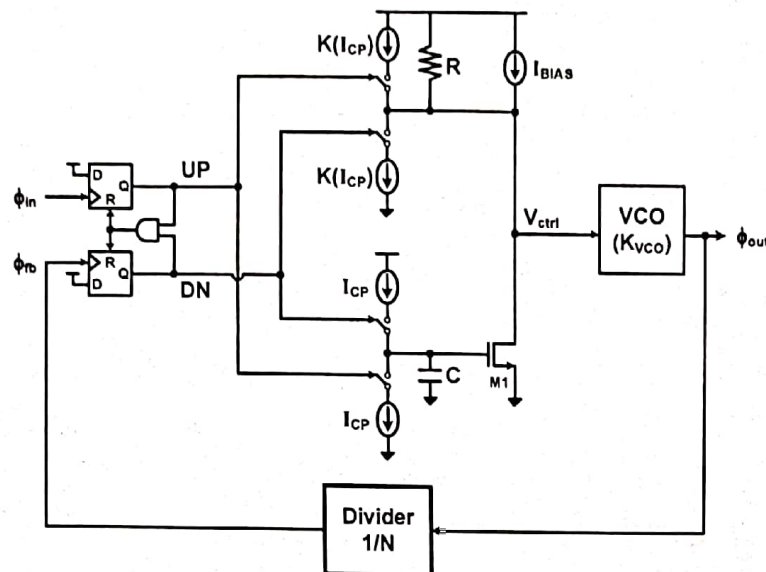
$$\boxed{K_{PD} = \frac{100 \mu\text{A}}{2\pi}}$$

$$R = \frac{2\zeta}{\omega_n C} = \frac{2(2)}{(10^7 \text{ rad/s})(1\text{nF})} = 400 \Omega$$

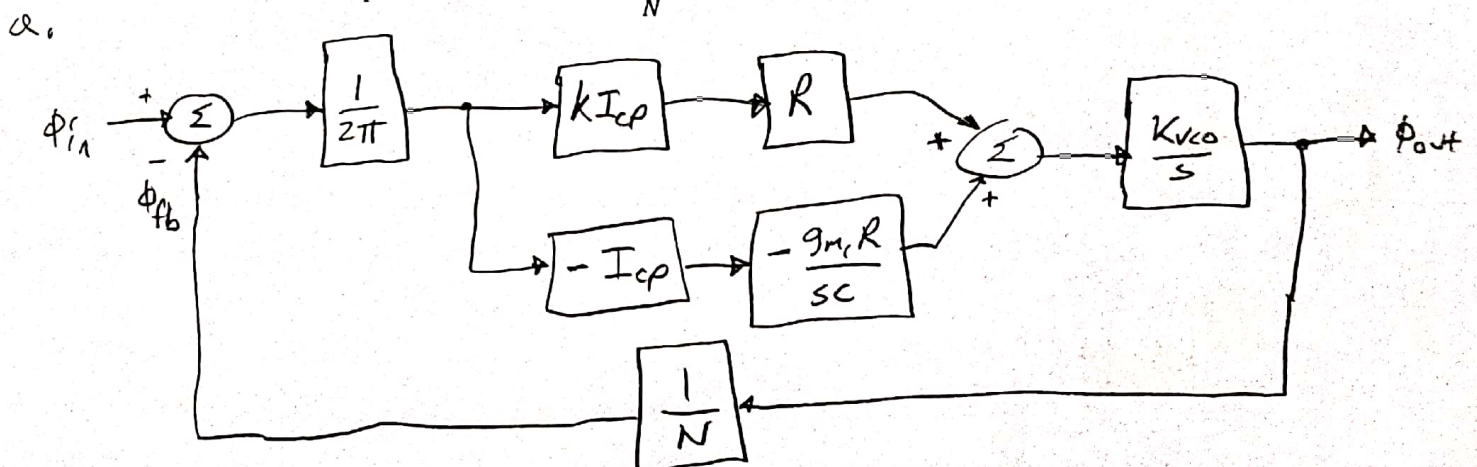


## Problem 2 (60 points)

For the PLL shown below, assume that the VCO gain is  $K_{VCO}$  is positive, all transistors are operating in saturation with  $r_o = \infty$  and you can ignore any transistor device capacitors.



- a) Draw the phase domain small signal model of the loop.
- b) Find the expressions for the product of the forward path gain and feedback factor,  $\frac{G(s)}{N}$ , and determine the pole-zero locations of  $\frac{G(s)}{N}$ .



$$b. \frac{G(s)}{N} = \frac{1}{2\pi} \left[ K I_{CP} R + I_{CP} \frac{g_{m1} R}{sC} \right] \left( \frac{K_{VCO}}{s} \right) \left( \frac{1}{N} \right)$$

$$\frac{G(s)}{N} = \frac{K I_{CP} R K_{VCO}}{2\pi} \left( s + \frac{g_{m1}}{KC} \right) \frac{1}{s^2 N}$$

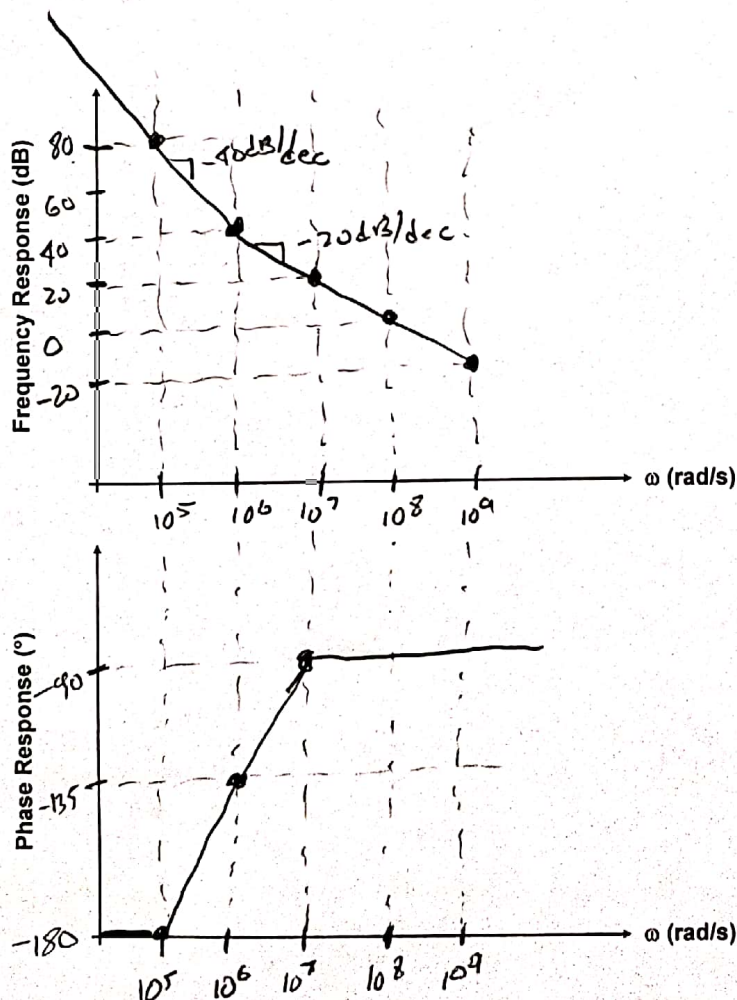
1 zero at  $-\frac{g_{m1}}{KC}$   
2 poles at  $\phi$

- c) Assume that  $I_{CP}=10\mu A$ ,  $K=10$ ,  $R=50k\Omega$ ,  $K_{VCO}=2\pi \cdot (1GHz/V)$ ,  $g_{m1}=20\mu A/V$  and  $N=50$ . What is the  $C$  value required for the  $\frac{G(s)}{N}$  transfer function zero,  $\omega_z$ , to equal  $1Mrad/s$ ? Sketch the  $\frac{G(s)}{N}$  Bode Plot (magnitude and phase). What is the phase margin?

$$\omega_z = \frac{g_{m1}}{KC} \Rightarrow C = \frac{g_{m1}}{K\omega_z} = \frac{20\mu A/V}{10(1Mrad/s)} = 2pF$$

$$\frac{G(s)}{N} = \frac{10(10\mu A)(50k\Omega)(2\pi \cdot 1GHz/V)}{2\pi} \left( s + \frac{20\mu A/V}{10(2pF)} \right) = \frac{5 \times 10^9 (s + 10^6)}{s^2 50}$$

$$A + \omega = 10^5 \quad \left| \frac{G(j10^5)}{N} \right| \approx 10^4$$



$$C = 2pF$$

$$\text{Phase Margin} = 90^\circ$$

d) What is the phase relationship between  $\phi_{in}$  and  $\phi_{fb}$  when is PLL locked?

Due to the PFD, the loop will lock with a  $0^\circ$  phase difference.

$$\phi_{in} - \phi_{fb} = 0^\circ$$

## Scratch Paper