Lecture 18: Two Stage Miller OTA
Announcements

- HW4 Due Now
- HW5 assigned today and due Friday 10/29 (5:00PM)
- Exam 2 Wednesday Nov 3 9:10-10:10AM
  - Material through Lecture 17
  - Will post some reference exams this weekend
Agenda

• Two Stage Miller OTA
• OpAmp Characterization
Multi-Stage Amplifiers

- Single-stage amplifiers typically have to trade-off gain and swing range
- Multi-stage amplifiers allow for higher gain without sacrificing swing range
- The major challenge with multi-stage amplifiers is achieving adequate phase margin to insure stability in a feedback configuration
Two Stage Miller OTA

**DC Gain**

\[ A_{VDC} = A_{v1} A_{v2} = \left( -\frac{g_{m2}}{g_{o2} + g_{o4}} \right) \left( -\frac{g_{m8}}{g_{o8} + g_{o7}} \right) = \frac{g_{m2} g_{m8}}{(g_{o2} + g_{o4})(g_{o8} + g_{o7})} \]

\[ A_{VDC} = G_m R_{out} \]

\[ R_{out} = \frac{1}{g_{o8} + g_{o7}} \]

\[ G_m = -g_{m8} A_{v1} = \frac{g_{m8} g_{m2}}{g_{o2} + g_{o4}} \]
Frequency Response – No Compensation

\[
A(s) = \left( \frac{A_{VDC}}{1 + \frac{S}{\omega_p}} \right) \left( 1 + \frac{S}{\omega_p} \right)
\]

\begin{align*}
A_{VDC} &= -\frac{g_{m1} g_{m3}}{g_1 g_L} \\
\omega_p &= -\frac{g_1}{C_1} \quad \text{(LHP)} \\
\omega_p &= -\frac{g_L}{C_L} \quad \text{(LHP)} \\
GBW &= (A_{VDC}) \times \min(\omega_p, \omega_{p2}) \quad \text{(if dominant pole system, valid?)} \\
\text{Phase margin} &= 180 - \tan^{-1}\left(\frac{\omega_u}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p2}}\right)
\end{align*}
**Frequency Response – Miller Compensation (Ignoring $\omega_z$)**

\[ A_{VDC} = -\frac{g_{m1} g_{m3}}{g_1 g_L} \]

\[ \omega_{p1} = -\frac{g_1}{C_1 + \frac{g_{m3}}{g_L} C_M} \quad \text{(LHP)} \]

\[ \omega_{p2} = -\frac{g_{m3}}{C_1 + C_L} \quad \text{(LHP)} \]

\[ GBW' = (A_{VDC})^*|\omega_{p1}| \approx \frac{g_{m1}}{C_M} \]

\[ \text{Phase margin} = 180 - \tan^{-1}\left(\frac{GBW'}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{GBW'}{\omega_{p2}}\right) \]

**Phase compensation ➔ Pole splitting techniques!!**

After compensation

- Phase Margin > 45 degrees
- Bandwidth is reduced!!!
Frequency Response – Miller Compensation (Considering $\omega_z$)

\[ A(s) = A_{VDC} \frac{1 - \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)} \]

Parasitic (bad) RHP zero!!

\[ \omega_{ZERO} = \frac{g_{m3}}{C_M} \] (RHP)

Phase margin = $180 - \tan^{-1}\left(\frac{GBW'}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{GBW'}{\omega_{p2}}\right) - \tan^{-1}\left(\frac{GBW'}{\omega_{ZERO}}\right)$

After compensation
- Phase Margin > 45 degrees
- Bandwidth is reduced!!!
Parasitic (bad) RHP zero!!
Can be catastrophic if close or below \( \omega_u \)!

\[
\omega_{\text{ZERO}} = \frac{g_{m3}}{C_M}
\]

Phase margin = \( 180 - \tan^{-1}\left(\frac{\omega_u}{\omega_{p1}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p2}}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{\text{ZERO}}}\right) \)

After compensation

\( \Rightarrow \) Phase Margin \(<< 45 \text{ degrees} \)
\( \Rightarrow \) Phase is equivalent to having 3 poles below unity gain frequency \( \Rightarrow \) Unstable!

After compensation

\( \Rightarrow \) Phase Margin > 45 degrees
\( \Rightarrow \) Bandwidth is reduced!!!
Adding a series resistance

![Circuit Diagram]

$$A(s) = \frac{A_{VDC} \left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right) \left(1 + \frac{s}{\omega_{p3}}\right)}$$

$$\omega_{p3} \approx -\frac{1}{R_Z C_1}$$ (Generally high frequency & can be ignored)

$$\omega_z = \frac{1}{\left(\frac{1}{g_{m3}} - R_Z\right) C_M}$$

Can design $R_Z$ to improve phase margin

Non-zero $R_Z$ will push RHP to a higher frequency (initially)

$$R_Z = \frac{1}{g_{m3}}$$ pushes the RHP zero to infinity

$$R_Z > \frac{1}{g_{m3}}$$ pushes zero from RHP to LFP

$$R_Z = \frac{C_L + C_M + C_1}{g_{m3} C_M}$$ can cancel $\omega_{p2}$
Two Stage Miller OTA Noise

\[ i_{eq\_out}^2 = \frac{8kT}{3} \left[ \left( 2g_{m1} + 2g_{m2} \right) \left( \frac{g_{m3}}{g_1} \right)^2 + g_{m3} + g_{m4} \right] \]

\[ v_{eq\_in}^2 = \frac{i_{eq\_out}^2}{3g_{m1}} = \frac{8kT}{3g_{m1}} \left[ 2 + 2 \frac{g_{m2}}{g_{m1}} + \frac{g_{m3} + g_{m4}}{g_{m1}} \right] \left( \frac{g_{m3}}{g_1} \right)^2 \]

- Output referred current noise density
- Input referred voltage noise density

\[ \text{Spectral noise density at low frequencies (capacitors are neglected)} \]
OPAMP Characterization

Main parameters to be measured:

• DC gain (10^4-10^6) V/V
• Frequency Limitations
  • Bandwidth (Few Hertz~1kHz)
  • Gain-Bandwidth product (1~100 Mhz)
• Output resistance
• Input Impedance
• Signal Swing
  • Common-mode input range
  • Output swing
• Stability
• DC Offset
• Slew-rate
• CMRR
• PSRR

For this section, see:
CMOS Analog design, Allen & Holberg
OPAMP Characterization

DC gain ($10^4$-$10^6$ V/V):

• Very difficult to measure in open-loop due to DC offsets.

How to measure/characterize it?

• Stabilize for DC

For DC, the OPAMP operates in closed loop!!

• For frequencies higher than $1/R_CC_C$, the OPAMP operates in open-loop with a grounded load given by $R_C$. 

\[ V_0 \neq 0 \]
OPAMP: DC Characterization

How to measure/characterize it?

• At DC
\[
\frac{v_o}{v_i} = \frac{A(s)}{1 + A(s)} \approx 1
\]

• If \(A(s)B(s) << 1\) then the measured gain is dominated by the OPAMP transfer function!

\[
B(s) = \frac{1}{sC_C R_C + \frac{1}{sC_C}} = \frac{1}{1 + sR_C C_C}
\]

\[
\omega_{P1} > \left[ \frac{A_{DC}}{R_C C_C} \right] \frac{1}{1 + sR_C C_C}
\]

OPAMP (open-loop)
OPAMP Characterization

DC Offset

\[ V_0 \sim V_{\text{offset}} \]
OPAMP Characterization: GBW and stability

\[ A(s) \]

\[ V_{in} \]

\[ V_0 \]

\[ A_{VDC} \]

\[ V_i \]

\[ t \]

\[ \omega_{p1} \]

GBW

Enough phase margin
OPAMP Characterization: GBW and stability

\[ A_{\text{VDC}}(\omega) \quad \text{OPAMP (open-loop)} \]

Not enough phase margin
OPAMP Characterization: Slew-Rate (max speed)

\[ V_{in} \quad + \quad A(s) \quad - \quad V_{0} \]

Slew-Rate:
\[ \text{max } \frac{d}{dt} V_{o(t)} \]

Max output level: \( V_{DD} \)
Min output level: \( V_{SS} \)
Use a slow triangular input signal such that the raising and falling edges are not determined by slew rate limitations.
OPAMP Characterization: Input and Output impedance

At Low frequencies:
\[ Z_{\text{measured}} \neq Z_o \] (Why????)

At medium frequencies:
\[ Z_{\text{measured}} = Z_o || R_C \]

Be sure that the OPAMP (all internal transistors) is properly biased during characterization!!

RC and CC as large as possible!!
Next Time

• OpAmp Feedback & Stability
• Common-Mode Feedback Techniques