Announcements

• HW3 due now
• HW4 assigned soon and due Friday 10/22
• Reading
  • Johns/Martin Chapter 4
Agenda

- MOSFET Noise
- Filtered Noise
- OTA Noise Example
Resistor Noise Model

- An equivalent voltage or current generator can model the resistor thermal noise

\[ V_{Rn}^2 = P_n R = 4kTR\Delta f \]

\[ I_{Rn}^2 = \frac{P_n}{R} = \frac{4kT}{R}\Delta f \]

- Recall the PSD is white (uniform w/ frequency)
Diode Noise Model

- Shot noise in diodes is caused by pulses of current from individual carriers in semiconductor junctions.
- White spectral density

\[ r_d' = \frac{kT}{qI_D} \] (noiseless)

\[ V_d^2(f) = 2kTr_d \]

\[ I_d^2(f) = 2qI_D \]

- Where \( q = 1.6 \times 10^{-19} \text{C} \) and \( I_D \) is the diode DC current.
Thermal Noise

=> Spectral Density of the thermal noise drain current (CMOS transistor biased @ linear region)

\[ DS = \frac{2}{d R kT} i_{DS}^2 \]

Resistor

\[ i_{n1}^2 = \frac{4kT}{R_1} \]

Transistor

\[ i_d^2 = \frac{4kT}{R_{DS}} \]

\[ R_{DS} \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS})} \]
White Noise

@ Triode region

\[
\frac{i_d^2}{W/L} = \left[4kT\mu C_{\text{ox}}\right]\left[V_{GS} - V_T - V_{DS}\right]
\]

Low current noise => W/L ↓ => \(g_m\) or \(g_o\) ↓

@ Saturation

\[
g_o = \frac{1}{R_{DS}} \rightarrow \frac{2}{3}g_m
\]

\[
i_d^2 = \frac{8}{3}kTg_m
\]

\[
\Rightarrow i_d^2 = \left(\frac{8kT}{3}\right)\left(\mu C_{\text{ox}}\right)\left(\frac{W}{L}\right)\left(V_{GS} - V_T\right)
\]
MOSFET 1/f (Flicker) Noise

- Caused by traps near Si/SiO₂ interface that randomly capture and release carriers

\[ i_d^2(f) = \frac{K_F g_m^2}{WLC_{ox} f} \]

- \( K_F \) is strongly dependent on the technology
1/f Noise Corner Frequency

- This is the frequency at which the flicker noise density equals the thermal noise density:

\[ \frac{K_F g_m^2}{W L C_{ox} f_{co}} = 4kT \gamma g_m \]

\[ f_{co} = \frac{K_F}{4kT \gamma C_{ox} W L} g_m = \frac{K_F}{4kT \gamma C_{ox} W L} \frac{1}{L} \left( \frac{g_m}{I_D} \right) \left( \frac{I_D}{W} \right) \]

- For a given \( g_m/I_D \) (which sets \( I_D/W \)), the only way to reduce \( f_{co} \) is to use longer channel devices.
Output and input referred noise

Current noise is the real one

Thermal Noise
\[ i_d = g_m V_{gs} \]
\[ i_d^2 = g_m^2 V_{gs}^2 \]

Voltage noise representation is an artifact to facilitate system analysis

\[ V_{gseq}^2 = \frac{8}{3} \frac{kT}{g_m} \]

Flicker Noise
\[ i_d^2 = \frac{K_f g_m^2}{W L C_{ox} f} \]

Referred to the input
\[ v_{eq}^2 = \frac{K_f g_m^2}{W L C_{ox} f} \frac{1}{g_m^2} \]
\[ v_{eq}^2 = \frac{K_f}{C_{ox} \left( \frac{1}{W L} \right) \left( \frac{1}{f} \right)} \]
Equivalent input referred voltage noise

\[ V_{eq}^2 = \frac{i_{dth}^2 + i_{df}^2}{g_m^2} \]

Equivalent input referred noise voltage means that all current noise sources are accounted as drain current and represented by an “equivalent” noise voltage at transistor gate.

\[ V_{eq}^2 = \frac{8kT}{3g_m} + \frac{K_F}{C_{ox}} \frac{1}{WL} \frac{1}{f} \]

\[ V_{eq\text{total}}(\text{RMS}) = \sqrt{\int_{BW}^{} V_{eq}^2(f)df} \]
NOISE COMPONENTS (values provided are for a 0.8 µm technology)

Noise density (V²/Hz)

$$v_{eq}^2 = v_{th}^2 + v_{1/f}^2$$

$$v_{eq}^2 = \frac{8 \ kT}{3 \ g_m} \ df + \frac{K_F}{WLC_{OX} f} \ df$$

\[ K_F \]
\[ C_{OX} \]

= 9.8x10⁻⁹ V²/µm – Hz (NMOS)

= 0.5x10⁻⁹ V²/µm – Hz (PMOS)

FOR LOW-FREQUENCY APPLICATIONS,
WHEREIN 1/F NOISE IS DOMINANT,
PMOS DEVICES MUST BE USED.
Filtered Noise

- Noise output spectral density is a function only of the magnitude of the transfer function, and not its phase.
- With multiple uncorrelated noise sources, combined output is also uncorrelated.

\[
v_{n_i}^2(f) \rightarrow A(s) \rightarrow v_{no}^2(f) = |A(j2\pi f)|^2 v_{n_i}^2(f)
\]

\[
v_{no}^2(f) = \sum_{i=1,2,3} |A_i(j2\pi f)|^2 v_{ni}^2(f)
\]
First-Order RC Circuit Example

What is the total output noise power?
First-Order RC Circuit Example

\[ A(s) = \frac{v_{out}(s)}{v_R} = \frac{1}{1 + sRC} \]

\[ v_{out}^2(f) = |A(j2\pi f)|^2 v_R^2(f) = \frac{1}{1 + 4\pi^2 f^2 R^2 C^2} 4kTR \]

To calculate Total Noise Power integrate over all frequencies

\[ v_{out}^2 = \int_0^\infty \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2} \]

Using

\[ \int \frac{dx}{x^2 + 1} = \tan^{-1} x \]

\[ v_{out}^2 = \frac{2kT}{\pi C} \tan^{-1}(2\pi fRC) \bigg|_0^{f=\infty} = \frac{2kT}{\pi C} \left[ \frac{\pi}{2} - 0 \right] = \frac{kT}{C} \]
Noise is generated by R but integrated noise is function of C (??)

\[ v_{total}^2 = \int_0^\infty \left( \frac{1}{1 + (\omega RC)^2} \right) (4kTR) \, df = \frac{kT}{C} \]

To get more insight, let's have a closer look on the operations!

Notice that:
When R increases thermal noise increases too but the corner frequency decreases, leading to a constant area under the curves!
Noise Bandwidth

- The noise bandwidth is equal to the frequency span of a brickwall filter having the same output noise rms value

\[ v_0^2 B_n = \int_{0}^{\infty} v_{no}^2 df \]

For a first-order filter \( B_n = \frac{\pi}{2} \omega_p \)

Validating with previous slides derivation:

Total Noise Output = \( v_0^2 B_n = (4kTR) \left( \frac{\pi}{2} \right) \left( \frac{1}{2\pi RC} \right) = \frac{kT}{C} \)
Output referred noise: Take advantage of SYMMETRIES!

Output referred current noise density

Superposition: Every transistor contributes; consider one at the time.

Analysis: You can use standard circuit analysis techniques but at the end of the day you have to consider POWER.

Output noise density: Each noise component represent the RMS value of random uncorrelated noise! Then add the power noise components

\[ i_{out}^2 = \frac{8}{3} kTg_{m1} \]
Output referred noise: Take advantage of SYMMETRIES!

Output referred current noise density due to the P-type devices:

Left hand side transistor:

\[ i_{out2}^2 \approx i_{d2}^2 = \frac{8}{3} kTg_{m2} \]

Right hand side transistor

\[ i_{out2}^2 = \frac{8}{3} kTg_{m2} \]
Output and input referred noise

Output referred current noise density

\[ i_{\text{out}}^2 = 2 \left( \frac{8}{3} kT g_{m1} \right) + 2 \left( \frac{8}{3} kT g_{m2} \right) \]

Input referred noise density \( (V^2/Hz) \)

\[ v_{\text{in,eq}}^2 = 2 \left( \frac{8 kT}{3 g_{m1}} \right) + 2 \left( \frac{8 kT g_{m2}}{3 g_{m1} g_{m1}} \right) \]

In this case, noise due to the current source is mainly common-mode noise

Be careful because this is not always the case!
Integrated Input referred noise

Input referred thermal noise density \((V^2/\text{Hz})\)

\[
v_{\text{in,eq}}^2 = 2 \left( \frac{8}{3} \frac{kT}{g_{m1}} \right) + 2 \left( \frac{8}{3} \frac{kT}{g_{m1}} \frac{g_{m2}}{g_{m1}} \right)
\]

Input referred noise level (volts)

\[
\text{Noise}(V_{\text{RMS}}) = \sqrt{\int_{BW} V_{\text{in,eq}}^2 \, df}
\]

Example: for thermal noise, the noise level becomes (assuming a single-pole system)

\[
\text{Noise}(V_{\text{RMS}}) \approx \sqrt{\frac{16kT}{3}} \sqrt{\frac{1}{g_{m1}}} \sqrt{1 + \frac{g_{m2}}{g_{m1}}} \left( \sqrt{\frac{\pi}{2}} BW \right)
\]

I should advise you to use:

\[
\text{Noise}(V_{\text{RMS}}) \approx \sqrt{\frac{8kT}{g_{m1}}} \sqrt{1 + \frac{g_{m2}}{g_{m1}}} \left( \sqrt{BW} \right) \quad 4kT \approx 16 \times 10^{-21} \text{ coul.V}
\]
Next Time

• OTAs