Lecture 11: Frequency Response (cont.)
Announcements

• HW3 due Monday Oct 11 9:10AM
  • Posted on website with additional notes

• Reading
  • Johns/Martin 3.1-3.8, 3.11
Agenda

• Common-Source Amp Input Impedance
• Common-Drain Amp Frequency Response
• Differential Pairs
Common-Source Amp Frequency Response

\[ A_{dc} = -g_m r_o \]
\[ \omega_z = \frac{g_{m1}}{C_{gd1}} \]

\[ \omega_{p1} = -\frac{1}{R_{in}[C_{gs1} + C_{gd1}(1 + g_m r_o)]} \]
\[ \omega_{p2} \approx -\frac{g_{m1}C_{gd1}}{C_{gd1}C_{gs1} + C_{gs1}C_o + C_{gd1}C_o} \]

\[ \omega_{p1} = -\frac{1}{r_o(C_{gd1} + C_o)} \]
\[ \omega_{p2} \approx -\frac{1}{R_{in}(C_{gs1} + C_{gd1})} \]
Common-Source Amp Input Impedance

Neglecting Output Cap: \[ Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s} \]

Input impedance is purely capacitive \((C_{gs} + \text{Miller } C_{gd})\)

Considering Output Cap: \[ \frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GDS}(1 + g_m R_D + R_D C_{DBS})} \]

Low frequency is capacitive, but then impedance experiences a zero followed by a second pole
Small signal analysis: Common-drain (source follower) amplifier

- How this is done?
- Why?

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{g_m}{g_{m1} + g_{mb} + g_{01} + g_{02}}
\]
Common-Drain Amplifier: High Frequency Response

- Simplifying the schematic a bit for SSA
  - Ideal current source load and neglecting transistor $r_o$ and $g_{mb}$ (i.e. $\lambda=\gamma=0$)
  - Will result in an optimistic DC gain estimate
Common-Drain Amplifier: High Frequency Response

KCL @ Node $v_a$: $(v_a - v_i)G_S + v_a sC_{gd} + (v_a - v_o)sC_{gs} = 0$

KCL @ Node $v_o$: $(v_o - v_a)sC_{gs} - g_m(v_a - v_o) + v_o sC_L = 0$

After some algebra:

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GSS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$
Common-Drain Amplifier: High Frequency Response

\[
\frac{V_{out}(s)}{V_{in}} = \frac{g_m + C_{GS}s}{R_s(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_s C_{GD} + C_L + C_{GS})s + g_m}
\]

- From this simplified transfer function:

\[
A_{dc} = \frac{g_m}{g_m} = 1 \quad \text{(Optimistic)}
\]

**Exact**

\[
A_{dc} = \frac{g_m}{g_m + g_o + g_{mb}}
\]

\[
\omega_z = -\frac{g_m}{C_{gs}}
\]

2 poles, If we assume that they are spaced far apart:

\[
\omega_{p1} \approx \frac{g_m}{g_m R_s C_{GD} + C_L + C_{GS}} = \frac{1}{R_s C_{GD} + \frac{C_L + C_{GS}}{g_m}}
\]
Common-Drain Amp Input Impedance

\[ Z_{in} = \frac{1}{C_{GSS}} + \left(1 + \frac{g_m}{C_{GSS}}\right) \frac{1}{g_{mb} + C_{LS}} \]

Low Frequency: \[ Z_{in} \approx \frac{1}{C_{GSS}} \left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}} \]

Equivalent to a series capacitive term \( C_{gs} \left(\frac{g_{mb}}{g_m + g_{mb}}\right) \) and resistive term \( \frac{1}{g_{mb}} \)

High Frequency: \[ Z_{in} \approx \frac{1}{C_{GSS}} + \frac{1}{C_{LS}} + \frac{g_m}{C_{GSS}C_{LS}^2} \]

Series combination of \( C_{gs} \) and \( C_L \) and a negative resistance term \( -\left(\frac{g_m}{C_{gs}C_L\omega^2}\right) \)

The negative resistance term can be utilized in oscillator design
Differential Pair: Linear range is limited to $2\ V_{DSAT}$

If both transistors are saturated and $IB$ is ideal ($r_{IB}=\infty$)

\[
\begin{align*}
i_{d1} + i_{d2} &= IB \\
\frac{i_{d1}}{2} &= \frac{\mu_n C_{OX}}{2} \frac{W}{L} \left(V_{gs1} - V_T\right)^2 \\
\frac{i_{d2}}{2} &= \frac{\mu_n C_{OX}}{2} \frac{W}{L} \left(V_{gs2} - V_T\right)^2
\end{align*}
\]

Solving these equations => Valid for $|v_1 - v_2| < 2^{1/2}V_{DSAT1}$

\[
\begin{align*}
i_{d1} &= \frac{IB}{2} + \frac{g_m (v_1 - v_2)}{2} \sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2} \\
i_{d2} &= \frac{IB}{2} - \frac{g_m (v_1 - v_2)}{2} \sqrt{1 - \left(\frac{v_1 - v_2}{2V_{DSAT1}}\right)^2}
\end{align*}
\]
The diff pair is a nonlinear circuit

\[ i_{d1} - i_{d2} = g_{ml} (v_1 - v_2) \left( 1 - \frac{(v_1 - v_2)^2}{2V_{DSAT1}^2} \right) \]

Non-linear term

If \( v_d = v_1 - v_2 < V_{DSAT1} \implies \)

\[ i_{d1} - i_{d2} = \sqrt{\mu_n C_{OX} \frac{W}{L}} I_B (v_1 - v_2) \]

Note:
Linear range increases for large \( V_{DSAT1} \)
\( V_{GS} \) is also increased (limited by VSS)
Basic Operational Transconductance Amplifier

DESIGN CONSIDERATIONS:

- $V_d = v_1 - v_2 < V_{DSAT}$
- $V_{1,2} - VSS > V_{GS1} + V_{DSATB}$

For small signals, ignoring the capacitors:

$$i_{out} = \sqrt{\mu_n C_{OX}} \frac{W}{L} I_B (v_1 - v_2)$$

or

$$i_{out} = g_m (v_1 - v_2)$$  Sensitive to differential signals

$$v_{out} = g_m r_{out} (v_1 - v_2)$$

$$r_{out} = r_{o1} IIr_{op}$$

For an ideal current source and ignoring the effects of $g_m b$, and transistor mismatches, then $i_{out} = 0$ for $v_1 = v_2$ ==> rejection to common-mode (noise) signals present at the input!
Consider node $V_X$

$$V_X = -\frac{0.5g_m R_X}{1 + SR_X C_X} V_d$$

$$R_X = r_{01} \parallel r_{0P} \parallel \left(\frac{1}{g_{mp}}\right)$$

$$C_X \approx 2C_{gsp} + C_{dbp} + \left(1 + \text{miller factor}\right)C_{dgp} + C_{dbn}$$
Low Frequency Response

\[ R_O = R_{ON} \parallel R_{OP} \]

\[ v_o = -g_m \left( \frac{v_d}{2} \right) R_o - g_m \left( -\frac{v_d}{2} \right) R_x \left( -g_{mp} R_o \right) \]

\[ R_x \approx \frac{1}{g_{mp}} \]

\[ v_o = -g_m v_d R_o \]
Frequency Response

\[ v_o = -g_m \left( \frac{v_d}{2} \right) \frac{R_o}{1 + sR_oC_o} - g_m \left( \frac{v_d}{2} \right) \left( \frac{1}{g_{mp}} \right) \frac{g_{mp}R_o}{1 + sR_oC_o} \]

\[ v_o = -g_m R_o \left( \frac{v_d}{2} \right) \frac{1}{1 + sR_oC_o} \left[ 1 + \frac{1}{1 + s \frac{C_x}{g_{mp}}} \right] \]

\[ v_o = -g_m R_o \left( \frac{v_d}{2} \right) \frac{1}{1 + sR_oC_o} \left[ 2 + s \frac{C_x}{g_{mp}} \right] \]
**Frequency Response**

Poles are associated with both resistors and capacitors.

\[ V_x = \frac{g_m R_x}{1 + SR_x C_x} V_d \]

**IMPORTANT REMARKS !!!!**

- DC-GAIN IS PROPORTIONAL TO \( R_x \)
- POLE FREQUENCY IS PROPORTIONAL TO \( 1/C_x R_x \)
- GAIN-BANDWIDTH PRODUCT \((=g_m/C_x)\) IS CONSTANT

TRADEOFF BETWEEN GAIN AND BANDWIDTH
Next Time

- Single-Stage Amplifiers (cont.)
  - Common-Gate
  - Cascode Stage

- Noise