ECEN325: Electronics
Summer 2012

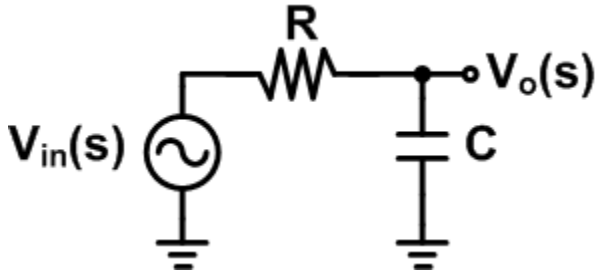
Bode Plot Example

Sam Palermo

Analog & Mixed-Signal Center

Texas A&M University

RC Transfer Function



$$V_o(s) = \frac{Z_C}{Z_R + Z_C} V_{in}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in}(s) = \frac{1}{1 + sRC} V_{in}(s)$$

AC Transfer Function, F(S)

$$F(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sRC}$$

where $s = j\omega = j2\pi f$

Bode Plots

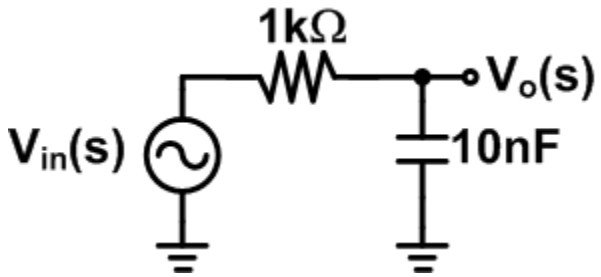
- Technique to plot the **Magnitude** (squared) and **Phase** response of a transfer function
  - Magnitude is plotted in Decibels (dB), which is a power ratio unit

$$\left|F(j\omega)\right|^2 \overset{dB}{\Rightarrow} 10\log_{10}\left(\left|F(j\omega)\right|^2\right)(\text{dB}) = 20\log_{10}\left(\left|F(j\omega)\right|\right)(\text{dB})$$

  - Phase is typically plotted in degrees

$$\text{Phase}\left(F(j\omega)\right) = \tan^{-1}\left(\frac{\text{Im}\left(F(j\omega)\right)}{\text{Re}\left(F(j\omega)\right)}\right)$$

RC Bode Plot Example



$$F(s)=\frac{V_o(s)}{V_{in}(s)}=\frac{1}{1+sRC}=\frac{1}{1+s10^{-5}}=\frac{1}{1+j\omega 10^{-5}}$$

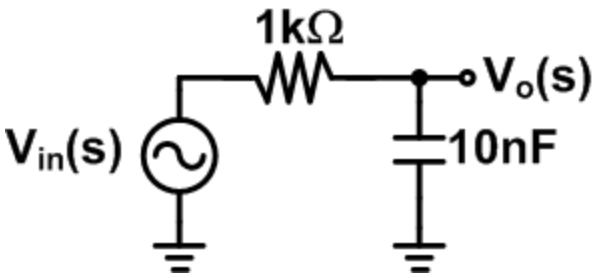
$$F(s)=\frac{1}{1+j\omega 10^{-5}}=\frac{1}{1-\frac{j\omega}{p_1}},\ \text{where } p_1=-10^5 \text{ rad/s}$$

Magnitude Squared (dB):

$$20\log_{10}\left|F(j\omega)\right|=20\log_{10}\left|\frac{1}{\sqrt{1+\left(\omega 10^{-5}\right)^2}}\right|=20\log_{10}(1)-20\log_{10}\left(\sqrt{1+\left(\omega 10^{-5}\right)^2}\right)$$

Phase: $\text{Phase}\left(F(j\omega)\right)=-\tan^{-1}\left(\omega 10^{-5}\right)$

RC Bode Plot Example



Magnitude:

$$20\log_{10}\left|F(j\omega)\right| = 20\log_{10}\left|\frac{1}{\sqrt{1+\left(\omega 10^{-5}\right)^2}}\right| = 20\log_{10}(1) - 20\log_{10}\left(\sqrt{1+\left(\omega 10^{-5}\right)^2}\right)$$

Phase: $\text{Phase}\left(F(j\omega)\right) = -\tan^{-1}\left(\omega 10^{-5}\right)$

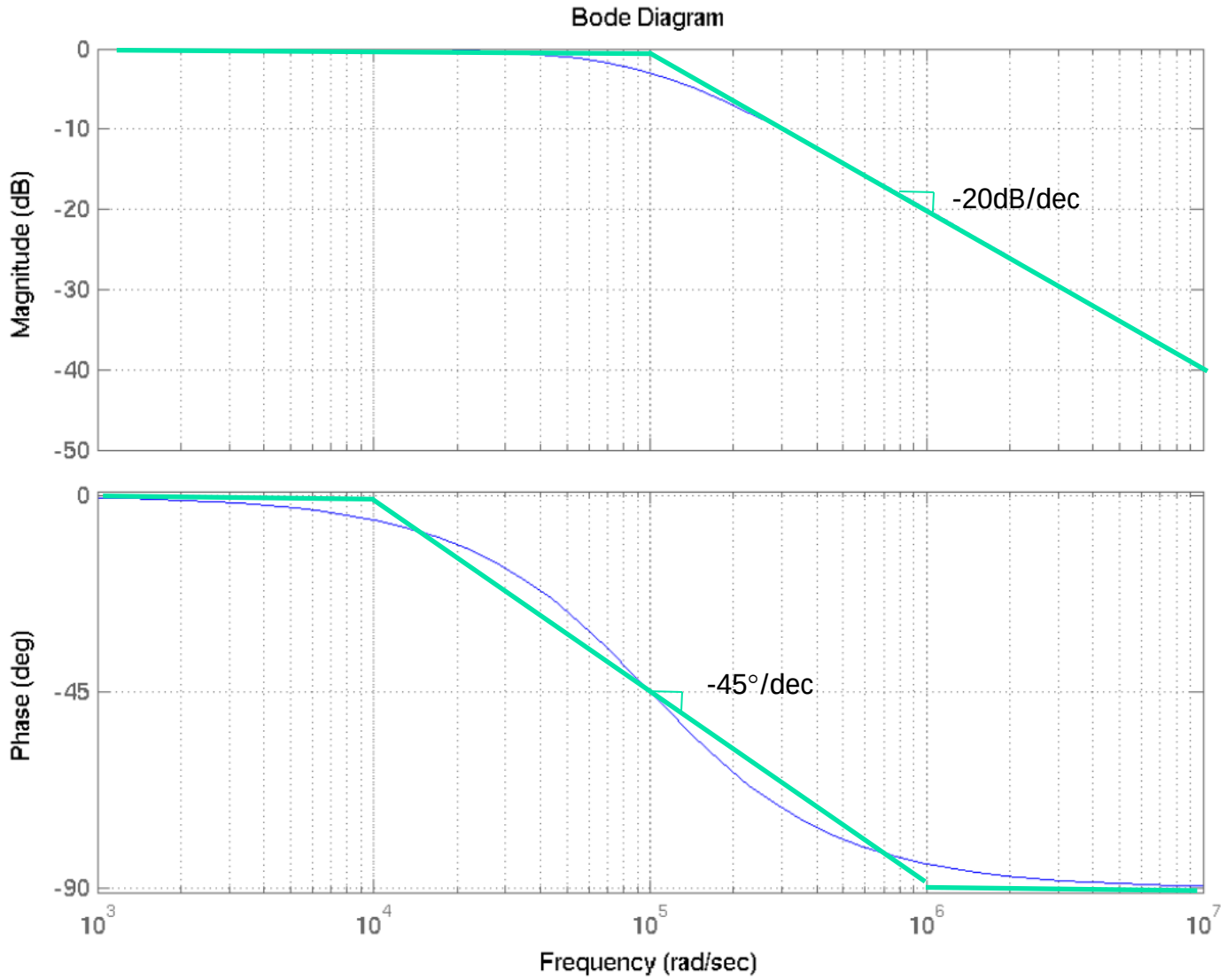
ω (rad/s)	|F(jω)|	|F(jω)|²	$20\log_{10}\|F(j\omega)\|$ (dB)	Phase (F(jω)) (°)
10^3	0.9999	0.9999	~0	~0
10^4	0.995	0.990	-0.043	-5.71
5×10^4	0.894	0.800	-0.969	-26.6
10^5	0.707	0.500	-3.01	-45.0
5×10^5	0.196	0.039	-14.2	-78.7
10^6	0.100	0.010	-20.0	-84.3
10^7	10^{-2}	10^{-4}	-40.0	-89.4
10^8	10^{-3}	10^{-6}	-60.0	-89.9

~$20\log_{10}$ (1) = 0dB

~$-20\log_{10}$ ($\omega 10^{-5}$) = -20dB/dec

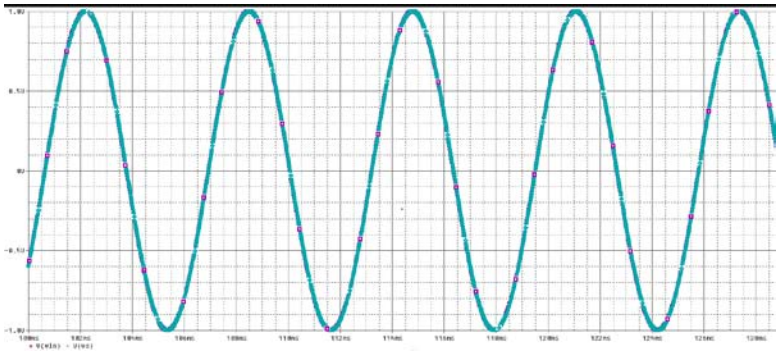
-45°/dec

RC Bode Plot Example

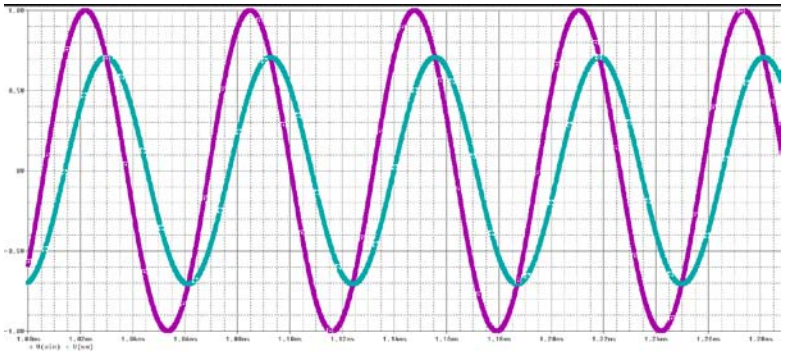


Transient Response

$\omega = 10^3$ rad/s



$\omega = 10^5$ rad/s



$\omega = 10^6$ rad/s