Nonlinear Control Techniques for Low-Voltage, Low-Power Applications:

Class D Audio Amplifiers

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1. Introduction to audio amplifiers

In a sound system, the power amplifier supplies power to the loudspeaker.

The typical speaker input impedance is low, usually in the 4 Ω to 8 Ω range. Thus, the power amplifier must be able to supply the high peak currents required to drive the low impedance.

Standard audible frequency band is from 20 Hz to 20 KHz

For high-fidelity sound system, THD must be < 0.1 %

Least detectable amount of harmonic distortion by humans is ~ 0.3 % in average

Telephone: THD ~ 10 % BW ~ 4 KHz
Main power amplifier classes

Linear
- Class A, B, AB
  - Linearity (happy)
  - Efficiency (sad)

Nonlinear
- Class D
  - Linearity (sad)
  - Efficiency (happy)
Power amplifier classes

Class A amplifier
• Current flows continuously
• High sound quality
• Poor efficiency (25 %)

Class B amplifier
• Current flows half of the period
• Linearity compromised by crossover
• Higher efficiency (78.5 %)

Class AB amplifier
• Hybrid between classes A and B
• Good sound quality
• Higher efficiency (78.5 %)

Class D amplifier
• Switching amplifier
• Ideal THD 0%
• Highest efficiency (100 %)
2. Conventional class D audio amplifier

Continuously switch the output from one rail to another at supersonic frequency (Pulse Width Modulation -PWM-)

There are two main areas of application for class D amplifiers:

1. Low Power Outputs
   - From few milliwatts to around 5 W
   - Hearing aids, mobile phones, personal stereos, laptop computer audio, etc.
   - Portable products, battery driven
   - High efficiency required

2. High Power Outputs
   - From 80 W to 1400 W
   - Home theatre systems, car audio systems, etc.
   - Keeps dissipation and heat sink size minimum
2. Conventional class D audio amplifier

History
• First proposed around 1950’s
• 5 % THD in 1976 state of the art amplifier

Basic Principles
• No output devices operating in the linear mode
• Output stage switches at supersonic frequency
• There is no inherent supply rejection
• Switching frequencies from 50KHz to 1MHz
• PWM is generated comparing the audio signal with a triangle wave carrier signal
• Triangle wave needs to be linear to prevent distortion

Efficiency
• 100 % at all output levels ideally
• Between 80% and 90% due to parasitic losses in practice
2. Conventional class D audio amplifier

Pulse Width Modulation (PWM)

Varies the duty cycle of the converter switches at a high switching frequency (supersonic) to achieve a target average low frequency output voltage.

![Class D Audio Amplifier (THD = 0.00%)](image)

- Class D amplifier THD is calculated considering all harmonics below 20KHz (max. audible frequency)
- Ideal class D amplifier THD is 0.0 %
- Performance is degraded due to system nonlinearities

Nonlinearity sources

- Passive components nonlinearities
- MOS on-resistance
- Non-ideal triangle carrier signal
2. Conventional class D audio amplifier

Passive components nonlinearities (†)

The low saturation current of the load inductor causes frequency dependent non-linearity.

Saturation current of inductor is defined as the $I_{DC}$ current level where the effective inductance value is decreased to 90% of its value at zero DC current.

An approximate expression of nonlinear inductance is obtained by series representation

$THD(\%) = 100 \frac{V_{third harmonic}}{V_{fundamental}}$

$\begin{align*}
THD(\%) &= 100 \frac{3 \cdot L \cdot V^2 \cdot 0.1 \cdot 2\pi \cdot f_{in}}{4 \cdot R^3 \cdot I_{sat}^2} \\
&= \frac{3 \cdot L \cdot V^2 \cdot 0.1 \cdot 2\pi \cdot f_{in}}{4 \cdot R^3 \cdot I_{sat}^2}
\end{align*}$

where $L$, $V$, $f_{in}$, $R$ and $I_{sat}$ are the inductance at zero DC current, the output voltage, the input signal frequency, the speaker resistance and the inductor saturation current respectively

2. Conventional class D audio amplifier

MOS on-resistance (†)

The class D amplifier is more power efficient (ideally 100% efficiency) than linear amplifiers because its output PWM is switch mode. The on-resistance is not negligible (~200mΩ) and it will result in a slightly “amplitude modulated” signal at the PWM output. This modulation becomes more pronounced at higher modulation indexes.

where \( f_c \), \( f_s \), \( M \) and \( W_p \) are the carrier signal frequency, the input signal frequency, the modulation index and the output transistor size respectively.

2. Conventional class D audio amplifier

Carrier signal nonlinearity

Carrier signal non-idealities will affect directly the linearity performance in the class D audio amplifier.

There are three main carrier signal modulation schemes:

- Triangle wave modulation
- Sawtooth wave modulation
- Exponential wave modulation

- Harmonic frequency calculation of PWM is complex and is typically done using an FFT analysis of a simulated waveform but it usually leads to errors and miscalculations.

- Analytical calculations of harmonic components is usually done by using a Double Fourier Integral Analysis (DFIA). Mathematical expression is quite complex but accurate.
2. Conventional class D audio amplifier

Carrier signal nonlinearity

• A novel mathematical analysis method to model the carrier waveform has been proposed in (†)
  
  • Assume a exponential carrier signal instead of a triangle wave which may be generated by charging/discharging an RC integrator circuit with square pulses.
  
  • Shift the nonlinearity of the exponential carrier to the input modulating signal and then apply the Double Fourier Integral Analysis
  
    1. Remove the nonlinearity of the trailing-edge exponential carrier by transform it to a linearized exponential carrier (linear sawtooth carrier)
    2. Transform the initially-linear modulating signal (audio signal) to a transformed (nonlinear) modulating signal
    3. Repeat (1) and (2) for the leading-edge exponential carrier.
    4. Derive the Double Fourier coefficients of the double-sided PWM output by summing the Fourier coefficients of the trailing-edge and leading-edge PWM outputs.

This mathematical analysis is accurate but its complexity and procedure are extensive

2. Conventional class D audio amplifier

Carrier signal nonlinearity

• We propose to use a simpler method to analyze carrier signal nonlinearity based on the PWM Analysis by Duty Cycle Variation for any kind of periodic carrier signal in the following presentation

Overview of Double Fourier Integral Analysis of a PWM Waveform

• Fourier decomposition is based on the principle that any regular time-varying waveform \( f(t) \) can be expressed as an infinite series of sinusoidal harmonics:

\[
f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)
\]

where

\[
a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos m\omega t \, d(\omega t)
\]

\[
b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin m\omega t \, d(\omega t)
\]
2. Conventional class D audio amplifier

The analytical solution for the harmonic components of a PWM waveform assumes the existence of two time variables

\[ x(t) = \omega_c t \]
\[ y(t) = \omega_o t \]

where \( \omega_c \) and \( \omega_o \) are the carrier angular frequency and the input signal (audio wave) angular frequency respectively.

The objective is to find a function \( f(t) \) which describes the PWM signal as a periodic function of \( x \) and \( y \) by using Double Fourier Integral Analysis (†)

The purpose of the Double Fourier Integral Analysis is to express the PWM waveform as a function of a double variable controlled waveform.

2. Conventional class D audio amplifier

In general, any double-variable time-varying function \( f(t) \) can be expressed, by using the Double Fourier Integral Analysis, in the following form (†)

\[
f(t) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} \left( A_{0n} \cos(n\omega_o t) + B_{0n} \sin(n\omega_o t) \right) + \sum_{m=1}^{\infty} \left( A_{m0} \cos(m\omega_c t) + B_{m0} \sin(m\omega_o t) \right)
\]

\[
+ \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left( A_{mn} \cos(m\omega_c t + n\omega_o t) + B_{mn} \sin(m\omega_c t + n\omega_o t) \right)
\]

where \( m \) is the carrier index variable and \( n \) is the baseband index variable

• The variables \( m \) and \( n \) define the angular frequency of each harmonic component

• The magnitudes of the harmonics components are the \( A_{mn} \) and \( B_{mn} \) coefficients

This function \( f(t) \) will provide an exact solution to determine the harmonic components of a PWM opposed to the traditional method of computing an FFT of the waveform, which will always be sensitive to the time resolution of the simulation and the periodicity of the overall waveform.

(†) D. G. Holmes and T. A. Lipo, *PWM For Power Converters*, Wiley Inter-science, USA, 2003
2. Conventional class D audio amplifier

Analyzing the function $f(t)$ we have that

$$\frac{A_{00}}{2}$$

where $m = n = 0$, corresponds to the DC offset component of the PWM (if any)

$$\sum_{n=1}^{\infty} (A_{0n} \cos(n\omega_ot) + B_{0n} \sin(n\omega_ot))$$

where $m = 0$, represents the fundamental component and baseband harmonics (if any)

$$\sum_{m=1}^{\infty} (A_{m0} \cos(m\omega_c t) + B_{m0} \sin(m\omega_o t))$$

where $n = 0$, defines the carrier wave harmonics (high-frequency components)

$$\sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} (A_{mn} \cos(m\omega_c t + n\omega_o t) + B_{mn} \sin(m\omega_c t + n\omega_o t))$$

where $m, n \neq 0$, represents the sideband harmonics

- The different summation terms in the PWM function $f(t)$ will depend on the type of carrier wave
- In some cases, it will be easier to express the summations using the Bessel function of the first kind ($J$)
2. Conventional class D audio amplifier

Example 1: Sine-Sawtooth Modulation

The PWM function \( v_{an}(t) \) for a sawtooth carrier signal expressed in terms of its harmonics components is

\[
v_{an}(t) = V_{DC} + V_{DC} M \cos(\omega_o t) + 2 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \left[ \cos m \pi - J_0(m \pi M) \right] \sin m \omega_c t
\]

\[+ 2 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_n(m \pi M) \left[ \sin n \frac{\pi}{2} \cos (m \omega_c t + n \omega_o t) - \cos n \frac{\pi}{2} \sin (m \omega_c t + n \omega_o t) \right]\]

As we expected, the THD of a class D audio amplifier will be 0.0 % since there are no baseband harmonics generated as we only have the fundamental tone.
Example 2: Sine-Triangle Modulation

The PWM function \( v_{an}(t) \) for a triangle carrier signal expressed in terms of its harmonics components is

\[
\begin{align*}
    v_{an}(t) &= V_{DC} + V_{DC} \cdot M \cos(\omega_c t) + 4 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} J_0 \left( m \frac{\pi}{2} M \right) \sin m \frac{\pi}{2} \cos m \omega_c t \\
    &+ 4 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_n \left( m \frac{\pi}{2} M \right) \sin \left( [m + n] \frac{\pi}{2} \right) \cos(m \omega_c t + n \omega_o t)
\end{align*}
\]

As in the previous case, the THD of a class D audio amplifier will be 0.0 % since there are no baseband harmonics generated as we only have the fundamental tone.
2. Conventional class D audio amplifier

Carrier signal nonlinearity

- Previous examples demonstrated that class D audio amplifier provides 0.0% THD ideally.
- In reality, THD > 0.0% because the carrier signals are not ideal.
- In fact, for an ideal triangle wave carrier signal \( f(x) \) we would have

\[
 f(x) = \frac{8}{\pi^2} \sum_{n=1,3,5,...}^{\infty} \frac{(-1)^{(n-1)/2}}{n^2} \sin\left(\frac{2n \pi x}{T}\right)
\]

- And, for an ideal sawtooth wave carrier \( f(y) \) we would have

\[
 f(y) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2n \pi y}{T}\right)
\]

We would need an infinite bandwidth system to generate a perfect carrier signal!

Unfortunately, band-limited systems degrade the performance of the overall class D amplifier generating undesired baseband components.
2. Conventional class D audio amplifier

Carrier signal nonlinearity

- Double Fourier Integral Analysis is a complex and tedious mathematical derivation
- Instead, we can use the PWM Analysis by Duty Cycle variation if the input signal (audio wave) is assumed to be constant within each carrier cycle, i.e. $\omega_c \gg \omega_o$, which is usually the case.

Example 1a: Sine-Sawtooth Modulation

- Normalizing the period of the sawtooth to $2\pi$ and its amplitude to 1, we have

\[
\nu_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)
\]

\[
a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} \nu_{an}(t) \cos mx \, dx
\]

\[
b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} \nu_{an}(t) \sin mx \, dx
\]

- We need to calculate the interval where $\nu_{an}(t)$ switches from 0 to 1
2. Conventional class D audio amplifier

Example 1a: Sine-Sawtooth Modulation (cont.)

- For $a_m$ coefficients, when $m \neq 0$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx = 2 \frac{V_{DC}}{\pi} \int_{-\pi}^{\pi} \cos mx \, dx = 2 \frac{V_{DC}}{\pi} \left[ \sin(m\pi M \cos y) + \sin m\pi \right]$$

- For $b_m$ coefficients, when $m \neq 0$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx = 2 \frac{V_{DC}}{\pi} \int_{-\pi}^{\pi} \sin mx \, dx = 2 \frac{V_{DC}}{\pi} \left[ \cos m\pi - \cos(m\pi M \cos y) \right]$$

- When $m = 0$ we have

$$a_0 = 2V_{DC} \left(1 + M \cos y\right) \quad b_0 = 0$$

- After some mathematical manipulation and applying the Jacobi-Anger expansions, we get the same expression obtained by using the Double Fourier Integral Analysis

$$v_{an}(t) = V_{DC} + V_{DC} M \cos(\omega_o t) + 2 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \left[ \cos m\pi - J_0(m\pi M) \right] \sin m\omega_o t$$

$$+ 2 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_n(m\pi M) \left[ \sin n \frac{\pi}{2} \cos(m\omega_o t + n\omega_o t) - \cos n \frac{\pi}{2} \sin(m\omega_o t + n\omega_o t) \right]$$
2. Conventional class D audio amplifier

Example 2a: Sine-Triangle Modulation

- Normalizing the period of the sawtooth to $2\pi$ and its amplitude to 1, we have

$$v_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)$$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx$$

- We need to calculate the interval where $v_{an}(t)$ switches from 0 to 1

- For $a_m$ coefficients, when $m \neq 0$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx = \frac{V_{DC}}{\pi} \int_{-\frac{\pi}{2}(1+M \cos y)}^{\frac{\pi}{2}(1+M \cos y)} \cos mx \, dx = 4 \frac{V_{DC}}{\pi} \left[ \sin \left( m \frac{\pi}{2} (1 + M \cos y) \right) \right]$$
2. Conventional class D audio amplifier

Example 2a: Sine-Triangle Modulation

- For $b_m$ coefficients, since the triangle wave is an even function

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx = 2 \frac{V_{DC}}{\pi} \int_{-\pi}^{\pi} \sin mx \, dx = 0$$

- When $m=0$ we have

$$a_0 = 2V_{DC} (1 + M \cos y)$$

- After some mathematical manipulation, as well as in the previous example, and applying the Jacobi-Anger expansions, we get the same expression obtained by using the Double Fourier Integral Analysis

$$v_{an}(t) = V_{DC} + V_{DC} M \cos(\omega_o t) + 4 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} J_0 \left( \frac{m \pi}{2} M \right) \sin \left( \frac{m \pi}{2} \cos m \omega_c t \right)$$

$$+ 4 \frac{V_{DC}}{\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_n \left( \frac{m \pi}{2} M \right) \sin \left( \frac{[m+n] \pi}{2} \right) \cos (m \omega_c t + n \omega_o t)$$
2. Conventional class D audio amplifier

Carrier signal nonlinearity

- We propose to generalize the PWM Analysis by Duty Cycle variation and apply it to a any carrier waveform to calculate analytically the THD in a class D audio amplifier

Example 3: Sine-Non-Ideal Triangle Modulation

- Recall that a triangle wave carrier signal is constructed by an infinite sum of sinusoidal functions and since there are not unlimited bandwidth systems, the number of harmonics (n) in a triangle wave carrier signal is finite
2. Conventional class D audio amplifier

Sine-Non-Ideal Triangle Modulation

• There are two trivial cases in a triangle wave shaped carrier signal

  1. When the number of harmonics is infinite we have an ideal triangle wave carrier signal and the THD of the class D amplifier is 0.0%

  2. When the number of harmonics is 1 then we have a sine-cosine modulation PWM and the THD of the class D amplifier will depend on the modulation index (M)

• Lets examine the case where the non-ideal triangle wave carrier signal contains one single harmonic component

**Example 3: Sine-Cosine Modulation**

\[
v_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)
\]

\[
a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx
\]

\[
b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx
\]
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

- For $a_m$ coefficients, when $m \neq 0$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx = 2 \frac{V_{DC}}{\pi} \int \cos mx \, dx = 4 \frac{V_{DC}}{\pi} \left[ \sin \left( m \arccos \left( -\frac{\pi^2}{8} M \cos y \right) \right) \right]$$

- For $b_m$ coefficients, since the cosine is an even function

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx = 2 \frac{V_{DC}}{\pi} \int \sin mx \, dx = 0$$

- When $m = 0$ we have

$$a_0 = 4 \frac{V_{DC}}{\pi} \arccos \left( -\frac{\pi^2}{8} M \cos y \right)$$
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

• Finally, the function $v_{an}(t)$ will be given by

$$v_{an}(t) = 2 \frac{V_{DC}}{\pi} \arccos \left( \arcsin \left[ 2 \sum_{k=1}^{\infty} \sin \left( k \frac{\pi}{2} \right) J_k \left( -\frac{\pi^2}{8} M \right) \cos(ky) \right] \right)$$

$$+ \sum_{m=1}^{\infty} 4 \frac{V_{DC}}{\pi} \left[ \sin \left( m \arccos \left( -\frac{\pi^2}{8} M \cos y \right) \right) \right] \cos mx$$

• We can see that the fundamental component is not alone in the expression but comes with baseband harmonics product of the cosine shaped-carrier waveform. Such baseband harmonics will produce the harmonic distortion in the class D audio amplifier.

• It can be appreciated that as we increment the modulation index $M$, the distortion increments exponentially.

• Same procedure can be applied for a given number of harmonics components present in the triangle wave carrier, however, the only closed-form solution exists when $n = 1$ and $n = \infty$. The solution when $1 < n < \infty$ must be calculated numerically.
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

• In order to verify the mathematical derivation and its results, we have created a simple SIMULINK model to simulate a class D audio amplifier and compare the traditional FFT method and the analytical solution to find the THD in the amplifier.

Non-ideal triangle wave carrier signal with $n = 1$.

Non-ideal triangle wave carrier signal with $n = 5$. 
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

Class D Amplifier THD (n = 15)

Class D Amplifier THD (n = 25)

Non-ideal triangle wave carrier signal with n = 15

Class D Amplifier THD (n = 99)

Non-ideal triangle wave carrier signal with n = 25

Non-ideal triangle wave carrier signal with n = 99
2. Conventional class D audio amplifier

Example 3: Sine-Cosine Modulation (cont.)

The mathematical model predicts with high accuracy the result in the SIMULINK simulation!

Exercise: Can you provide an analytical expression for the THD output spectrum when a sawtooth carrier signal with only one harmonic is used to PWM an audio signal?
2. Conventional class D audio amplifier

Carrier signal nonlinearity

• Lets now analyze the case when an exponential waveform is used as a carrier signal.

• An exponential waveform is usually employed as a carrier signal due to its simple implementation.

Example 4: Sine-Exponential Modulation

• The exponential waveform is generated by charging/discharging a simple RC integrator with square pulses.

Defining a set of normalized exponential waves

\[
f(x) = \begin{cases} 
V_{dc} \left( \frac{e^{-\frac{x}{T_0}}}{1 - error} - error \right), & -\frac{T}{2} < x < 0 \\
2 - \frac{1}{1 - error} & 0 < x < \frac{T}{2}
\end{cases}
\]
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

Normalizing the exponential carrier to a period of $2\pi$ we can calculate the Fourier coefficients based on the PWM Analysis by Duty Cycle Variation

$$v_{an}(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos mx + b_m \sin mx)$$

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \cos mx \, dx$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} v_{an}(t) \sin mx \, dx$$
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

- For $a_m$ coefficients, when $m \neq 0$

\[
a_m = 2V_{DC} \frac{\pi}{\pi} \left[ \cos mx \, dx \right] = -t_0 \ln \left( 1 - \frac{1}{2} \left( 1 - \text{error} \left( \frac{M}{V_{DC}} \cos y + 1 \right) \right) \right) - \pi - t_0 \ln \left( \frac{M}{2V_{DC}} \cos y (1 - \text{error}) + \frac{1 + \text{error}}{2} \right)
\]

\[
a_m = 2V_{DC} \frac{\pi}{m \pi} \left[ \sin \left( -mt_0 \ln \left( 1 - \frac{1}{2} \left( 1 - \text{error} \left( \frac{M}{V_{DC}} \cos y + 1 \right) \right) \right) \right) - \sin \left( m \left( -\pi - t_0 \ln \left( \frac{M}{2V_{DC}} \cos y (1 - \text{error}) + \frac{1 + \text{error}}{2} \right) \right) \right) \right]
\]

- For $b_m$ coefficients

\[
b_m = 2V_{DC} \frac{\pi}{\pi} \left[ \sin mx \, dx \right] = -t_0 \ln \left( 1 - \frac{1}{2} \left( 1 - \text{error} \left( \frac{M}{V_{DC}} \cos y + 1 \right) \right) \right) - \pi - t_0 \ln \left( \frac{M}{2V_{DC}} \cos y (1 - \text{error}) + \frac{1 + \text{error}}{2} \right)
\]

\[
b_m = -2V_{DC} \frac{\pi}{m \pi} \left[ \cos \left( -mt_0 \ln \left( 1 - \frac{1}{2} \left( 1 - \text{error} \left( \frac{M}{V_{DC}} \cos y + 1 \right) \right) \right) \right) - \cos \left( m \left( -\pi - t_0 \ln \left( \frac{M}{2V_{DC}} \cos y (1 - \text{error}) + \frac{1 + \text{error}}{2} \right) \right) \right) \right]
\]
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

• For \( a_m \) coefficients, when \( m \neq 0 \)

\[
a_0 = 2 \frac{V_{DC}}{\pi} \left[ -t_0 \ln \left( 1 - \frac{1}{2} (1-error) \left( \frac{M}{V_{DC}} \cos y + 1 \right) \right) \right. \\
\left. -\pi - t_0 \ln \left( \frac{M}{2V_{DC}} \cos y (1-error) + \frac{1+error}{2} \right) \right]
\]

\[
a_0 = 2 \frac{V_{DC}}{\pi} \left[ -t_0 \ln \left( 1 - \frac{1}{2} (1-error) \left( \frac{M}{V_{DC}} \cos y + 1 \right) \right) \right. \\
\left. + \left( \pi + t_0 \ln \left( \frac{M}{2V_{DC}} \cos y (1-error) + \frac{1+error}{2} \right) \right) \right]
\]

• As in the previous example, the coefficient \( a_0 \) will have the fundamental tone as well as baseband harmonics that will degrade the class D audio amplifier THD.

• As the ‘error’ parameter increases, the exponential wave behaves in a quasi-triangular way and the baseband harmonics magnitude decrease.

• Like in the previous example, a SIMULINK model was created and simulated in order to compare the results of both procedures.
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)
2. Conventional class D audio amplifier

Example 4: Sine-Exponential Modulation (cont.)

The mathematical model predicts with high accuracy the result in the SIMULINK simulation!

This analysis method can be further applied to any carrier wave or even multiphase systems where multilevel PWM is generated.
3. Proposed class D audio amplifier (†)
(single ended architecture)

Class D amplifier with sliding surface

3. Proposed class D audio amplifier: PWM generation

Traditional architecture

- Ideally fixed frequency
- Jitter (degrades linearity)
- Non-linear circuit
- Carrier generator adds complexity
- Non-ideal triangle wave

Proposed architecture

- No dedicated triangle wave
- Linearity no compromised
- Good transient response
- Variable frequency (450KHz-600KHz)
3. Proposed class D audio amplifier

Error function

\[ V_1 = e_1 = (V_A - V_{OUT}) \]

Switching function

\[ V_3 = s(e_1, e_2) = e_1 + \alpha e_2 = e_1 + \alpha \dot{e}_1 = (1 + \alpha s)e_1(s) \]

Phase portraits in a class D audio amplifier

Class D Amplifier with sliding surface
3. Proposed class D audio amplifier

Error function

\[ V_1 = e_1 = (V_A - V_{OUT}) \]

Switching function

\[ V_3 = s(e_1) = e_1 + \alpha \dot{e}_1 = (1 + \alpha s)e_1(s) \]
3. Proposed class D audio amplifier

Efficiency vs. input signal

Sliding mode phases
A – Initial condition
B – Reaching mode
C – Sliding surface
D – Sliding equilibrium point

Output spectrum (300 mVpp)
3. Proposed class D audio amplifier

- THD versus audio frequency input
- THD versus audio voltage input
- SNR versus audio frequency input
- PSRR versus audio frequency input
3. Proposed class D audio amplifier

<table>
<thead>
<tr>
<th>Design</th>
<th>THD</th>
<th>η</th>
<th>Supply</th>
<th>Load</th>
<th>I₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>0.28%</td>
<td>92%</td>
<td>2.5 V</td>
<td>8 Ω</td>
<td>25.2 μA</td>
</tr>
<tr>
<td>[2]</td>
<td>0.11%</td>
<td>70%</td>
<td>5.0 V</td>
<td>8 Ω</td>
<td>-</td>
</tr>
<tr>
<td>[3]</td>
<td>0.03%</td>
<td>76%</td>
<td>4.2 V</td>
<td>8 Ω</td>
<td>4.7 mA</td>
</tr>
<tr>
<td>[4]</td>
<td>0.20%</td>
<td>90%</td>
<td>5.0 V</td>
<td>4 Ω</td>
<td>-</td>
</tr>
<tr>
<td>[5]*</td>
<td>0.08%</td>
<td>85%</td>
<td>5.0 V</td>
<td>4 Ω</td>
<td>8.0 mA</td>
</tr>
<tr>
<td>[6]*</td>
<td>0.40%</td>
<td>87%</td>
<td>2.7 V</td>
<td>4 Ω</td>
<td>2.8 mA</td>
</tr>
<tr>
<td>[7]</td>
<td>0.04%</td>
<td>79%</td>
<td>3.6 V</td>
<td>8 Ω</td>
<td>2.5 mA</td>
</tr>
<tr>
<td>[8]</td>
<td>0.10%</td>
<td>92%</td>
<td>12 V</td>
<td>8 Ω</td>
<td>-</td>
</tr>
<tr>
<td>This work</td>
<td>0.08%</td>
<td>91%</td>
<td>2.7 V</td>
<td>8 Ω</td>
<td>2.0 mA</td>
</tr>
</tbody>
</table>


4. Class D audio amplifiers: two design approaches

**Motivation**

- Single ended architecture generates even-order distortion tones which degrades linearity
- Single ended version generates “quasi” differential output by adding an extra inverter (causes delay and distortion)

**Advantages of fully-differential version**

- Even-order cancellation enhances linearity
- No delay in signal paths

**Improvements from single-ended version**

- Reduction of building blocks (operations are done in a single OPAMP)
- Comparator design is done with internal positive feedback instead of poly-resistors
4. Class D audio amplifiers: two design approaches

Output waveforms

Output spectrum
4. Class D audio amplifiers: two design approaches

Motivation

- Multilevel converters present better linearity as number of level increases
- High frequency components are pushed to higher frequencies (for three level modulation, carrier $f_s$ is pushed to $2xfs$)
- Possibility of adding an extra-level by using H-bridge already present in class D output stage

Advantages of three-level architecture

- Multi-level modulation = linearity improvement
- No additional hardware cost

Characteristics

- Two identical switching surfaces are created (Two binary comparators are used)
- Each switching surface is fed by the audio signal shifted 180 degrees from each other
- Each output stage operates at two different levels
- Differential output becomes multi-level!!!
4. Class D audio amplifiers: two design approaches

Three-level PWM

Output waveform

Output spectrum
4. Class D audio amplifiers: two design approaches

Class D fully-differential and multilevel have similar efficiencies and linearity but multilevel modulation gives better SNR and PSRR due to the extra level of quantization.
## Table of comparison

$$FM = \frac{\eta}{I_0 \times THD \times 100e3}$$

<table>
<thead>
<tr>
<th>Design</th>
<th>THD</th>
<th>$\eta$</th>
<th>Supply</th>
<th>Load</th>
<th>$I_0$</th>
<th>SNR</th>
<th>PSRR</th>
<th>$f_s$</th>
<th>$P_{O, \text{max}}$</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>[3]</td>
<td>0.20%</td>
<td>-</td>
<td>3.0V</td>
<td>8Ω</td>
<td>-</td>
<td>81dB</td>
<td>-</td>
<td>1.5MHz</td>
<td>381 mW</td>
<td>1.20 mm²</td>
</tr>
<tr>
<td>[4]</td>
<td>0.07%</td>
<td>92%</td>
<td>2.5V</td>
<td>8Ω</td>
<td>25.2uA</td>
<td>80dB</td>
<td>85dB</td>
<td>200KHz</td>
<td>330mW</td>
<td>0.60 mm²</td>
</tr>
<tr>
<td>[5]</td>
<td>0.50%</td>
<td>85%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>85dB</td>
<td>40dB</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>[6]</td>
<td>0.11%</td>
<td>70%</td>
<td>5.0V</td>
<td>8Ω</td>
<td>-</td>
<td>-</td>
<td>90dB</td>
<td>1.0MHz</td>
<td>250mW</td>
<td>12.5 mm²</td>
</tr>
<tr>
<td>[7]</td>
<td>0.03%</td>
<td>76%</td>
<td>4.2V</td>
<td>8Ω</td>
<td>4.7mA</td>
<td>98dB</td>
<td>70dB</td>
<td>410KHz</td>
<td>700mW</td>
<td>0.44 mm²</td>
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<td>90%</td>
<td>5.0V</td>
<td>4Ω</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>450KHz</td>
<td>1250mW</td>
<td>12.3 mm²</td>
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<tr>
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<td>70%</td>
<td>5.0V</td>
<td>4Ω</td>
<td>4.0mA</td>
<td>87dB</td>
<td>77dB</td>
<td>250KHz</td>
<td>1000mW</td>
<td>-</td>
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<td>[10]*</td>
<td>0.40%</td>
<td>87%</td>
<td>2.7V</td>
<td>4Ω</td>
<td>2.8mA</td>
<td>-</td>
<td>-</td>
<td>125KHz</td>
<td>700mW</td>
<td>-</td>
</tr>
<tr>
<td>[11]</td>
<td>0.04%</td>
<td>79%</td>
<td>3.6V</td>
<td>8Ω</td>
<td>2.5mA</td>
<td>-</td>
<td>84dB</td>
<td>250KHz</td>
<td>500mW</td>
<td>2.25 mm²</td>
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<tr>
<td>[12]</td>
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<td>12.0V</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>180KHz</td>
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<tr>
<td>[13]</td>
<td>0.19%</td>
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</tr>
<tr>
<td>[14]</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>20MHz</td>
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<td>-</td>
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<tr>
<td>SE</td>
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<td>75dB</td>
<td>450KHz</td>
<td>250mW</td>
<td>2.48 mm²</td>
</tr>
</tbody>
</table>

* Commercial product.
Thank you
References


