SECOND-ORDER FILTER TYPES

- Using a number of first – order and second – order filter blocks, of any type of approximation. one can built higher – order filters.

- We will focus here in the analysis of second-order filters also known as biquads.

- A design procedure for a biquad filter named Tow- Thomas will be presented.

SECOND-ORDER FILTER TYPES

Second-order blocks are important building blocks since with a combination of them allows the implementation of higher-order filters. The general order transfer function in the s-plane has the form:

$$H(s) = \frac{K_1 s^2 + K_2 s + K_3}{s^2 + \frac{\omega_o s}{Q} + \omega_p^2}$$

Particular conventional cases are:

Lowpass	i.e.,	$K_1 = K_2 = 0$
Bandpass	i.e.,	$K_1 = K_3 = 0$
Highpass	i.e.,	$K_2 = K_3 = 0$
(Notch) Band-Elimination	i.e.,	$K_2 = 0$
Allpass	i.e.,	$K_1 = 1, \quad K_2 = -\frac{\omega_o}{Q} \quad \text{and} \quad K_3 = \omega_o^2$

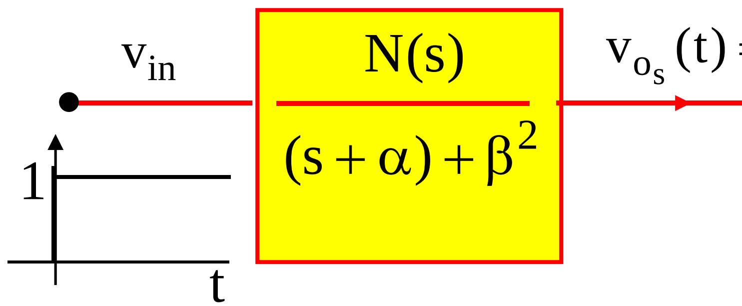
One interesting case used for amplitude equalization is the “equalizer” sometimes referred to as Bump (DIP) Equalizer. In this case, $K_1 = 1 \quad K_3 = \omega_o^2$ and $K_2 = \pm k \frac{\omega_o}{Q}$

Specific structures have different properties. Some structures have enough degrees of freedom to allow them to change independently ω_o, Q (or BW) and a particular gain $|H(\omega_p)|$ where ω_p is a particular frequency, i.e., $\omega_p = 0, \quad \omega_o, \quad \infty$ for the LP, BP and HP cases. Furthermore, some structures have the property to have constant Q or BW while varying f_o..

Properties of Second-Order Systems

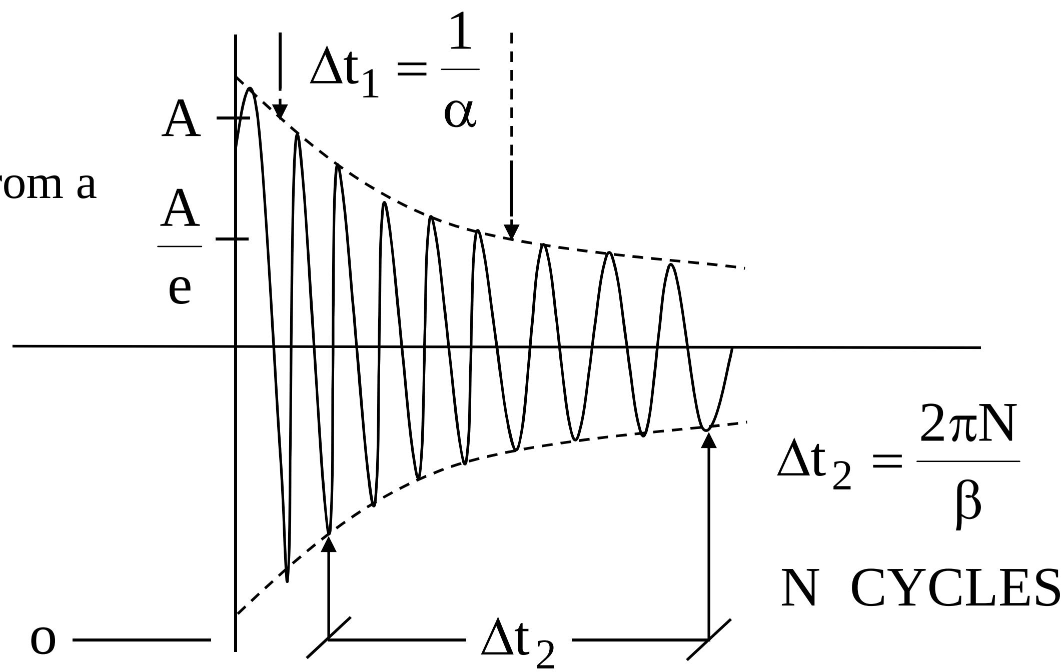
$$s^2 + \frac{\omega_o}{Q}s + \omega_o^2 = (s+\alpha)^2 + \beta^2 = s^2 + 2\alpha s + \omega_o^2$$

where $\qquad \alpha = \dfrac{\omega_o}{2Q}, \quad \beta = \omega_o\sqrt{1 - \dfrac{1}{4Q^2}}$



$$v_{o_s}(t) = K_1 + K_2 e^{-\alpha t}\sin(\beta t + \theta)$$

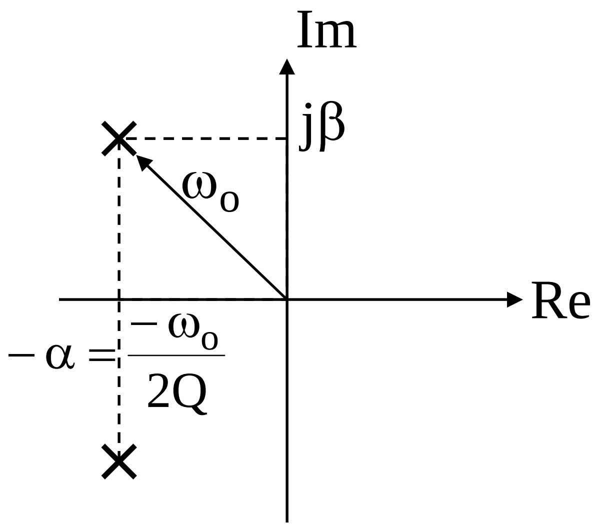
How to determine the pole location from a step response?

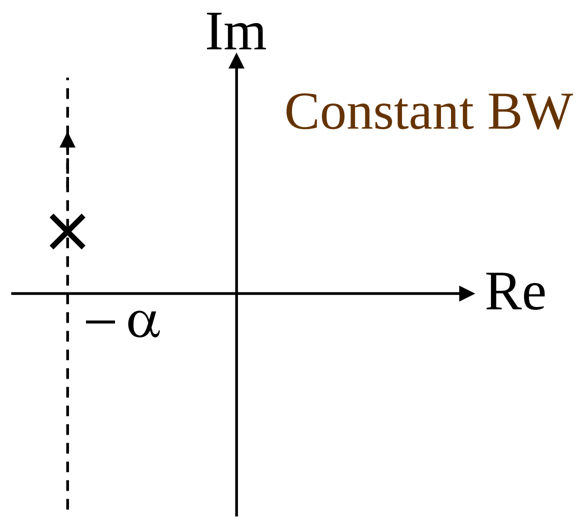


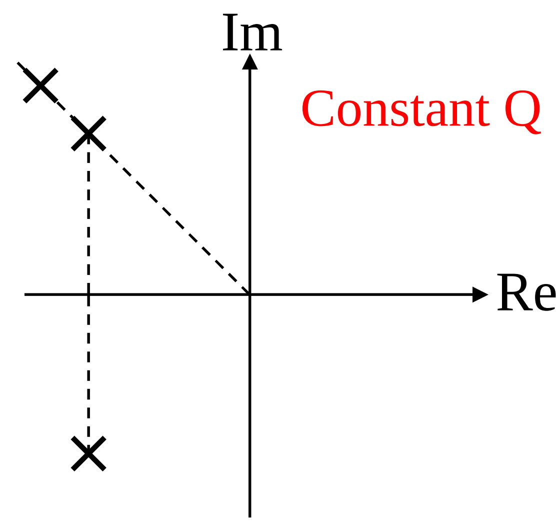
$$K_1,\ K_2 = f(N(s))$$

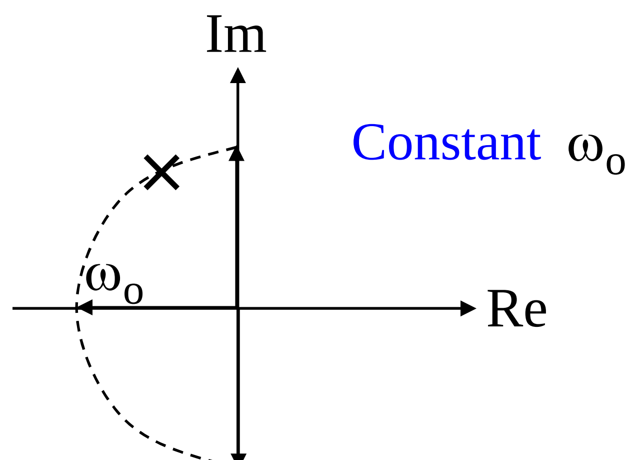
Pole locations and properties

$$\alpha = \frac{1}{\Delta t_1}$$ and $$\beta = N\frac{2\pi}{\Delta t_2}$$









POLE-ZERO LOCI

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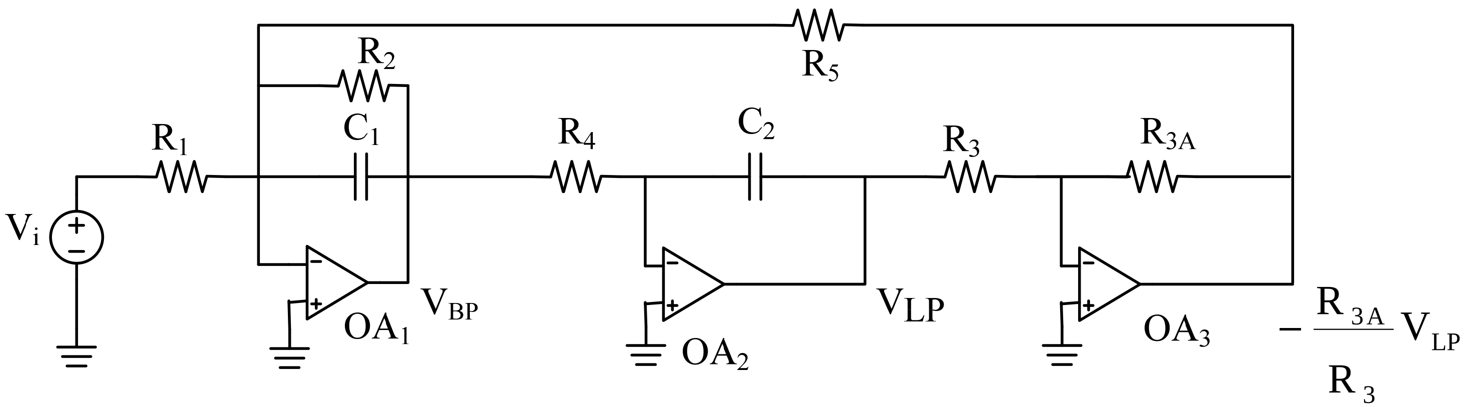
The Biquad Filter

This is a two integrator loop that can implement two real or complex poles. Zeros can be also achieved if an additional summer is added.



Using KCL at the inverting – input of OA_1 can write

$$\frac{V_i}{R_1} + V_{BP}\left(\frac{1}{R_2} + sC_1\right) - \frac{\frac{R_{3A}}{R_3}V_{LP}}{R_5} = 0 \qquad (1)$$

$$V_{LP} = -V_{BP}\frac{1}{sC_2R_4} \qquad (2)$$

Solving (1) and (2) yields:

$$H_{BP} = \frac{-Ks}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \qquad (3)$$

Where

$$K = 1/C_1R_1$$

$$\frac{\omega_o}{Q} = \frac{1}{C_1R_2} \quad \text{or} \quad Q = R_2\sqrt{\frac{R_{3A}C_1}{R_3C_2R_4R_5}}$$

$$\omega_o^2 = \frac{(R_{3A}/R_3)}{C_1C_2R_4R_5}$$

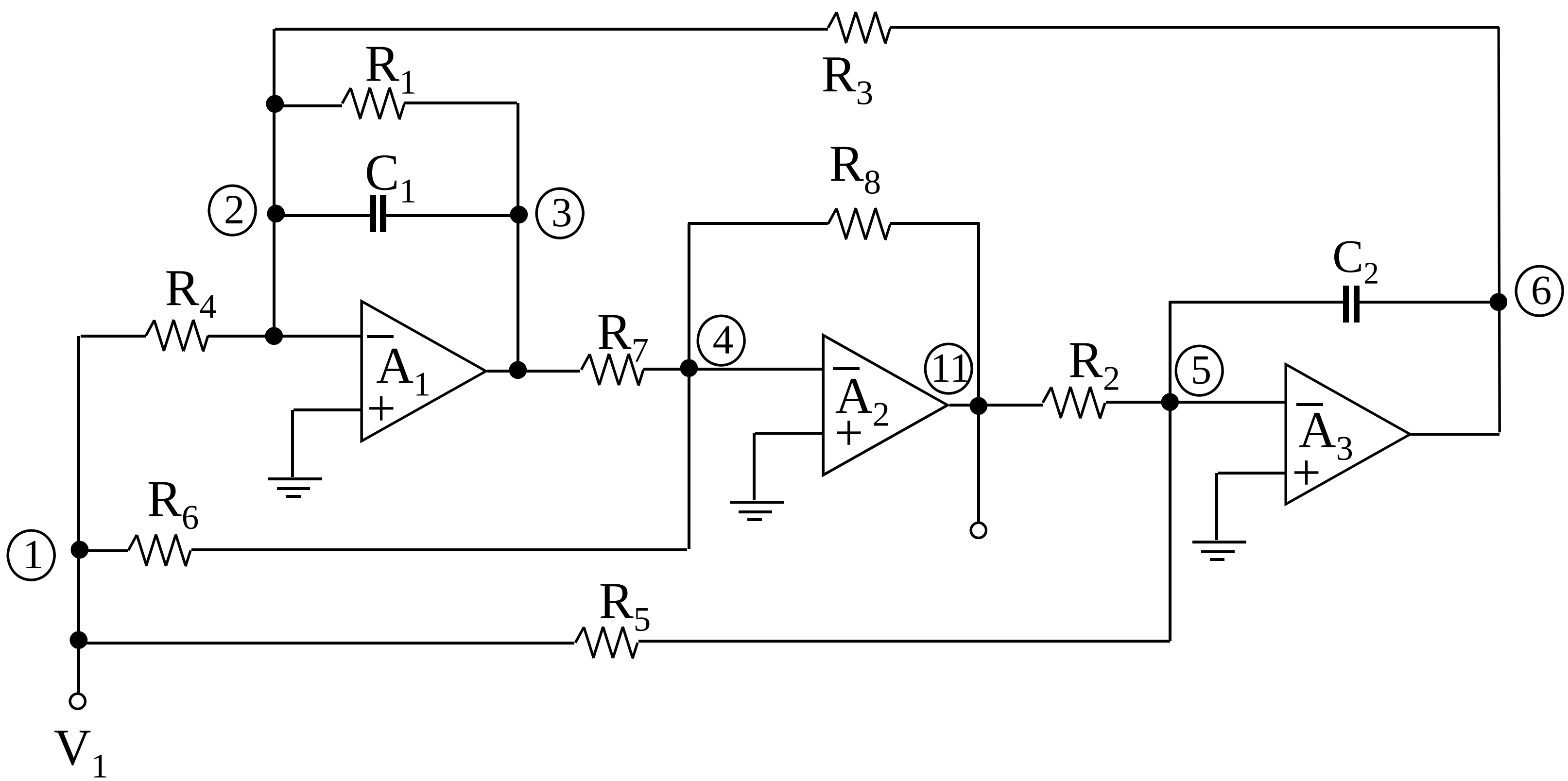
Using (3) and (2) becomes

$$H_{LP}(s) = \frac{V_{LP(s)}}{V_i(s)} = \frac{\dfrac{-K_s}{sC_2R_4}}{s^2 + \dfrac{\omega_o}{Q}s + \omega_o^2} = \frac{\dfrac{1}{C_1R_1C_2R_4}}{s^2 + \dfrac{\omega_o}{Q}s + \omega_o^2}$$

$$H_{LP}(o) = \frac{R_5}{R_1(R_{3A}/R_3)}$$

$$H_{BP}(\omega_o) = -\frac{R_2}{R_1}$$

Feedforward Tow-Thomas Biquad Circuit



Analysis using KCL at the input of A1 plus the simple relations of V11/V3 and V6/V11 yields the following transfer function.

Description of the Parameters for the Tow-Thomas Filter

General Transfer Function

$$T(s) = -\frac{R_8}{R_6}\,\frac{s^2 + \left(\frac{1}{R_1 C_9} - \frac{1}{R_4 C_9}\frac{R_6}{R_7}\right) + \frac{R_6}{R_7}\frac{1}{R_3 R_5 C_9 C_{10}}}{s^2 + s\left(\frac{1}{R_1 C_9}\right) + \frac{R_8}{R_7}\frac{1}{R_3 R_2 C_9 C_{10}}}$$

where

$$\omega_p^2 = \frac{R_8}{R_7 R_2 R_3 C_9 C_{10}}, \qquad \omega_z^2 = \frac{R_6}{R_3 R_5 R_7 C_9 C_{10}}$$

$$Q_p = R_1\sqrt{\frac{R_8 C_9}{R_2 R_3 R_7 C_{10}}}, \qquad Q_z = \sqrt{\frac{R_6 C_9}{R_3 R_5 R_7 C_{10}}} \Bigg/ \left(\frac{1}{R_1} - \frac{R_6}{R_4 R_7}\right)$$

and

$$|H_{HP}| = \frac{R_8}{R_6}, \qquad \text{for} \qquad R_1 = \frac{R_4 R_7}{R_6}, \quad R_5 \to \infty$$

$$|H_{BP}| = \frac{R_1 R_8}{R_4 R_7}, \qquad \text{for} \qquad R_5,\ R_6 \to \infty$$

$$|H_{LP}| = \frac{R_2}{R_5}, \qquad \text{for} \qquad R_4, R_6 \to \infty$$

For the bandstop (notch)

$$|H_{notch}| = \frac{R_8}{R_6}, \qquad \text{for} \qquad R_1 = \frac{R_4 R_7}{R_6}, \quad R_5 = \frac{R_6 R_2}{R_8}$$

Design Equations for the Tow-Thomas Filter

Let

$$R_3 = R$$

$$R_2 = a^2 R_3$$

$$R_7 = R_8 = R'$$

$$C_1 = C_2 = C$$

$$\omega_p = \frac{1}{aRC}, \quad \omega = \frac{1}{C}\sqrt{\frac{R_6}{R_5 R R'}}$$

$$Q_p = \frac{R_1}{aR}, \quad Q_z = \sqrt{\frac{R_6}{R_5 R R'}} \Bigg/ \left(\frac{1}{R_1} - \frac{R_6}{R_4 R'}\right)$$

$$|H_{HP}| = \frac{R'}{R_6}, \qquad \text{for} \qquad R_1 = \frac{R_4 R'}{R_6} = R_4 |H_{HP}|, \quad R_5 \to \infty$$

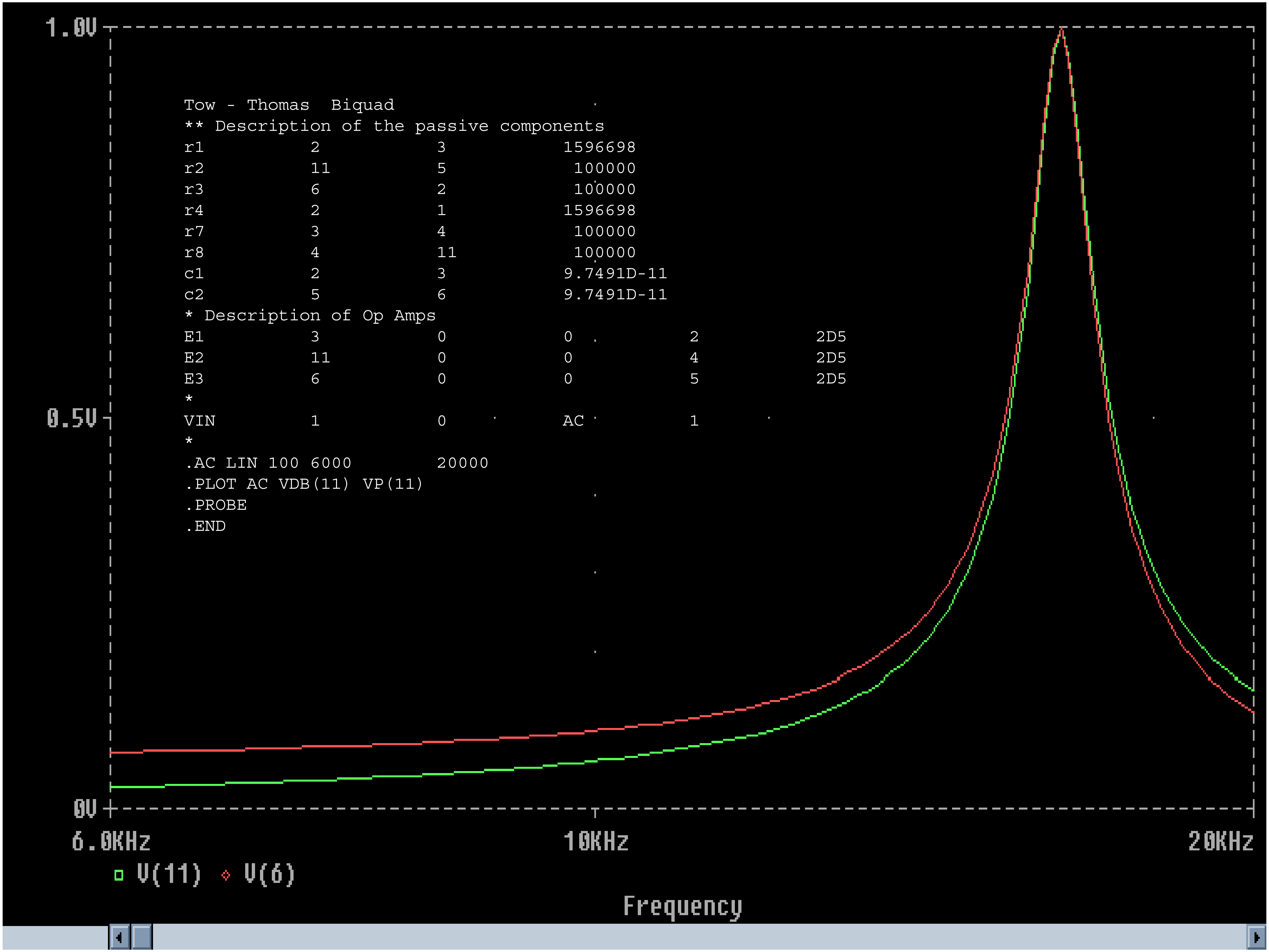
$$|H_{HP}| = \frac{R_1}{R_4}, \qquad \text{for} \qquad R_5, R_6 \to \infty$$

$$|H_{LP}| = \frac{a^2 R}{R_5}, \qquad \text{for} \qquad R_4, R_6 \to \infty$$

$$|H_{notch}| = \frac{R'}{R_6}, \qquad \text{for} \qquad R_1 = \frac{R_4 R'}{R_6}, \quad R_5 = \frac{a^2 R R_6}{R'}$$

PSPICE Input file of Tow Thomas Filter

```
Tow - Thomas  Biquad
** Description of the passive components
r1      2       3       1596698
r2      11      5        100000
r3      6       2        100000
r4      2       1       1596698
r7      3       4        100000
r8      4       11       100000
c1      2       3       9.7491D-11
c2      5       6       9.7491D-11
* Description of Op Amps
E1      3       0       0       2       2D5
E2      11      0       0       4       2D5
E3      6       0       0       5       2D5
*
VIN     1       0       AC      1
*
.AC LIN 100 6000        20000
.PLOT AC VDB(11) VP(11)
.PROBE
.END
```


1.0V
0.5V
0V
6.0KHz
10KHz
20KHz
V(11)
V(6)
Frequency
Tow - Thomas Biquad
** Description of the passive components
r1 2 3 1596698
r2 11 5 100000
r3 6 2 100000
r4 2 1 1596698
r7 3 4 100000
r8 4 11 100000
c1 2 3 9.7491D-11
c2 5 6 9.7491D-11
* Description of Op Amps
E1 3 0 0 2 2D5
E2 11 0 0 4 2D5
E3 6 0 0 5 2D5
*
VIN 1 0 AC 1
*
.AC LIN 100 6000 20000
.PLOT AC VDB(11) VP(11)
.PROBE
.END