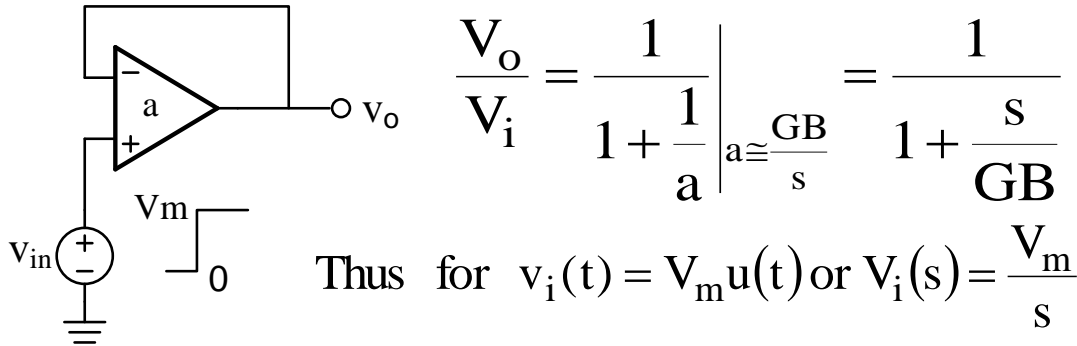


TRANSIENT RESPONSE OF OP AMP CIRCUITS

Let us consider a voltage follower and determine its step response



$$\text{Then } V_o = \frac{V_m}{s} \frac{1}{1 + \frac{s}{GB}} = \frac{V_m}{s} \frac{GB}{s + GB}$$

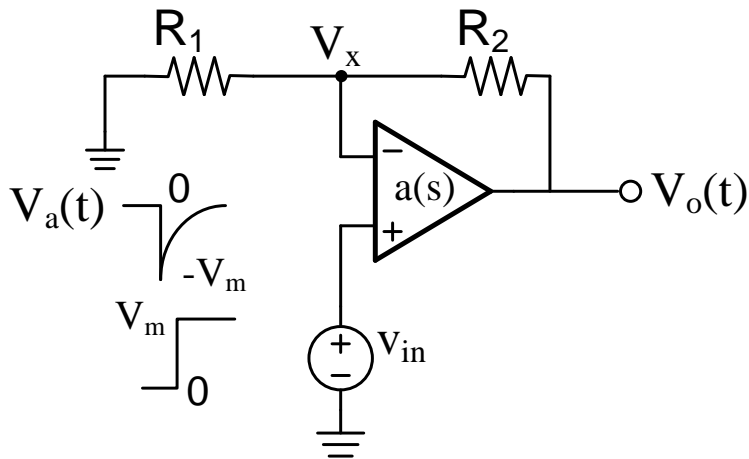
Taking the inverse Laplace it yields

$$v_o(t) = V_m (1 - e^{-t/\tau}) \quad ; \quad \text{where } \tau = \frac{1}{\omega_t} = \frac{1}{GB(\text{rad})} = \frac{1}{2\pi \times f_t}$$

The rising time t_R is defined as the time for the output $v_o(t)$ to reach 90% of V_m and starting at 10% V_m .

$$t_R = \tau (\ln(0.9) - \ln(0.1)) = 2.2 / (2\pi \times f_t)$$

Let us consider next a Voltage Gain $\neq 1$ for the non-inverting amplifier



$$H(s) = \frac{1 + R_2/R_1}{1 + \frac{1}{a(s)} \left(1 + \frac{R_2}{R_1}\right)}$$

Let $a(s) = \frac{GB}{s}$; $-v_a(t) = -v_x(t) + v_{in}$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1 + R_2/R_1}{1 + \frac{s}{GB} \left(1 + \frac{R_2}{R_1}\right)}$$

From Nodal Analysis

$$\left. \begin{aligned} v_x \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{v_o}{R_2} &= 0 \\ \text{and} \\ v_a = v_x - v_{in} \quad , \quad V_o(s) &= -a(s)V_a(s) \end{aligned} \right\} V_a(s) = \frac{-V_i(s)}{1 - \frac{a(s)}{1 + \frac{R_2}{R_1}}}$$

$$V_o(s) = \frac{-a(s)V_i(s)}{1 - \frac{a(s)}{1 + \frac{R_2}{R_1}}}$$

* Reference: Passive and Active Network Analysis and Synthesis, Aram Budak.

Now for $a(s) = \frac{-GB}{s}$, $V_o(s)$ and $V_a(s)$ become

$$V_a(s) = \frac{-V_i(s)}{1 + \frac{G/B}{s\left(1 + \frac{R_2}{R_1}\right)}} = \left(1 + \frac{R_2}{R_1}\right) \frac{-V_i(s)}{\left(1 + \frac{R_2}{R_1}\right) + \frac{GB}{s}} = \frac{-s V_i(s)}{s + GB/(1 + R_2/R_1)}$$

$$V_o(s) = a(s)V_a(s) = \frac{GBV_i(s)}{\left(s + GB/\left(1 + \frac{R_2}{R_1}\right)\right)}$$

then if $V_i(s) = \frac{V_m}{s}$ (a step of magnitude V_m)

$$V_a(s) = \frac{-V_m}{s + GB/(1 + R_2/R_1)} \quad ; \quad V_o(s) = V_m \frac{GB}{s(s + GB/(1 + R_2/R_1))}$$

The corresponding time-domain expressions become

$$v_a(t) = -V_m e^{-tGB/(1+R_2/R_1)}$$

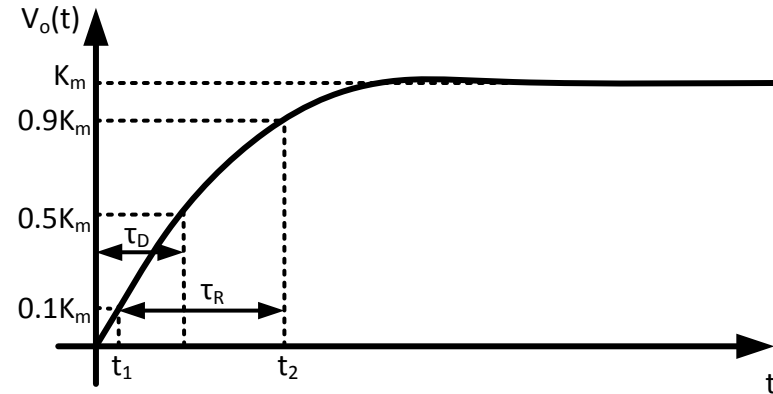
$$v_o(t) = (1 + R_2/R_1)V_m \left[1 - e^{-tGB/(1+R_2/R_1)}\right]$$

TRANSIENT RESPONSE EXPRESSION DERIVATIONS

- Finding delay time τ_D :

$$0.5K_m = K_m(1 - e^{-\tau_D\omega_{3dB}})$$

$$\frac{1}{2} = e^{-\tau_D\omega_{3dB}}; \tau_D = \frac{0.693}{\omega_{3dB}}$$



- Finding rising time $\tau_R = t_2 - t_1$

$$0.1K_m = K_m(1 - e^{-t_1\omega_{3dB}}); t_1 = \frac{0.1}{\omega_{3dB}}$$

$$0.9K_m = K_m(1 - e^{-t_2\omega_{3dB}}); t_2 = \frac{2.3}{\omega_{3dB}}$$

Thus

$$\tau_R = t_2 - t_1 = \frac{2.2}{\omega_{3dB}} = \frac{2.2}{2\pi f_{3dB}} = \frac{0.35}{f_{3dB}}$$

$$\max \frac{dv_o(t)}{dt} = V_m GB = \text{amplitude step} \times \text{gain - bandwidth product}$$

Note that the Gain Effect has been cancelled for the non-inverting case.

GB must be in r/s units

In order to keep the circuit operating in the linear region the following inequality must be satisfied.

$$V_m GB < \text{Slew Rate (SR)}$$

where GB is expressed in r/s. If this condition is not satisfied the step response is slewing-rate-limited.

Furthermore, if we consider a sinusoidal input $V_o \cos \omega t$, an output $v_o = V_{om} \sin \omega t$

Then

$$\max \frac{dv_o}{dt} = \omega V_{om}$$

$$2\pi f V_{om} \leq SR$$

Aside, V_{om} for op amp becomes the saturation V_{SAT} , then

$$\text{full power bandwidth (FPB)} = \frac{SR}{2\pi V_{SAT}}$$

$$\text{i.e. } V_{SAT} = 13V, SR = 0.5 \times 10^6$$

$$\text{then FPB} = 6.1 \text{ KHz}$$

For the Inverting Amplifier

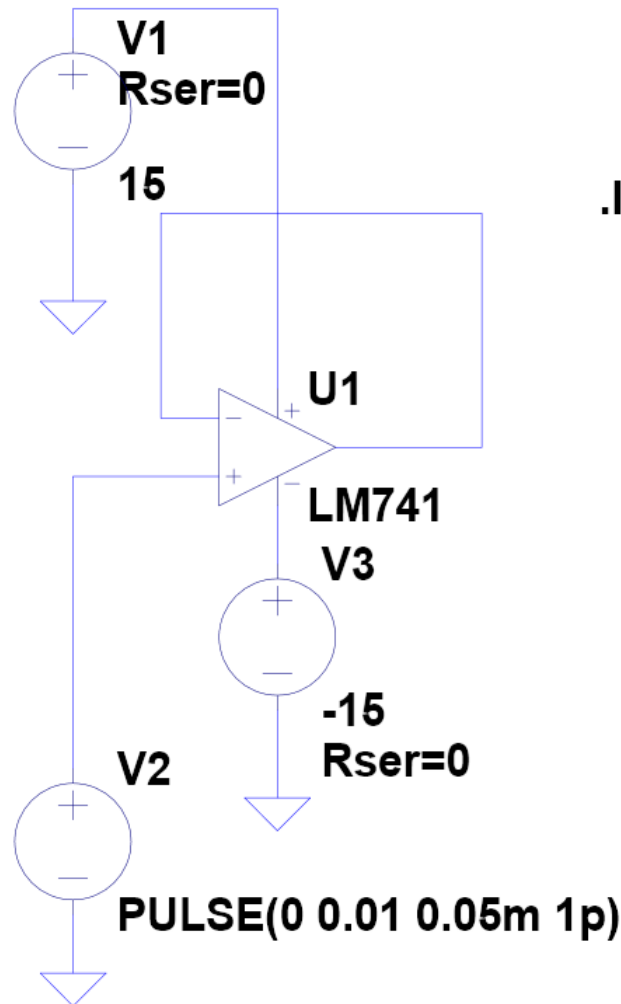
$$v_o(t) = -\frac{R_2}{R_1} v_m \left(1 - e^{-\frac{tGB}{1+\frac{R_2}{R_1}}} \right)$$

$$\max \frac{dv_o(t)}{dt} = -\frac{R_2}{R_1 + R_2} v_m GB$$

Then the inequality becomes

$$\left| \frac{R_2}{R_1 + R_2} v_m GB \right| < SR$$

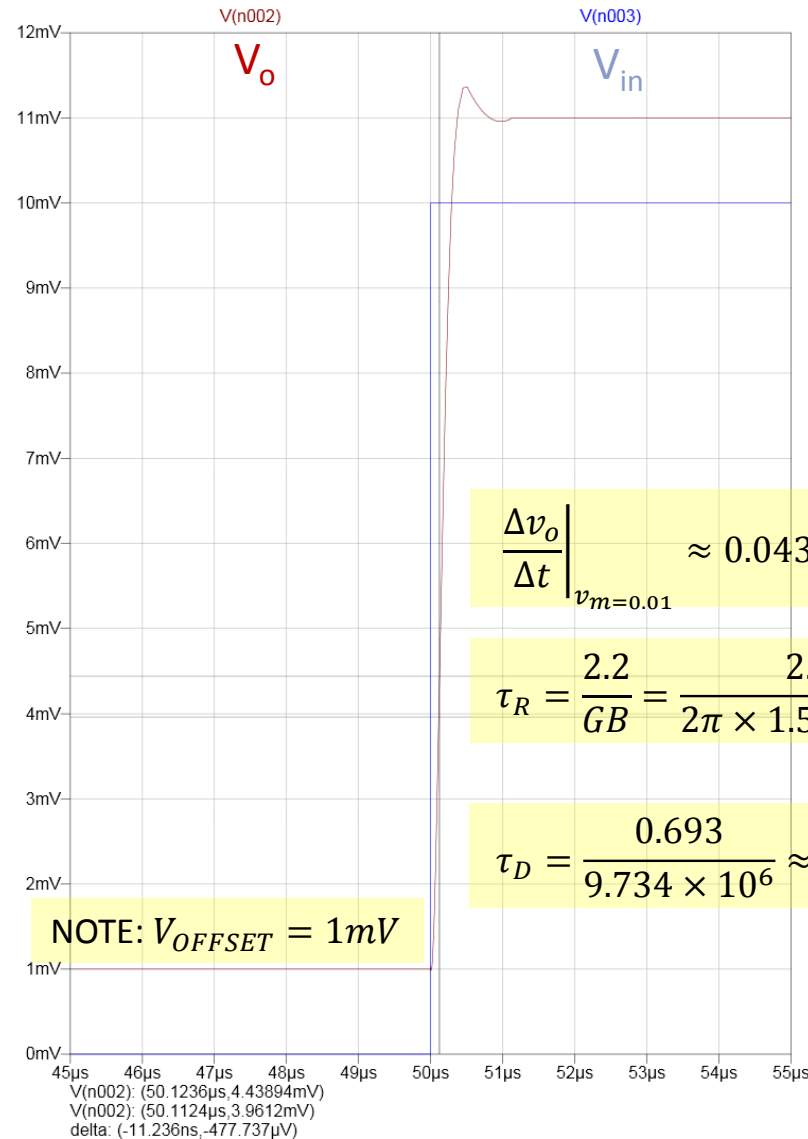
Unity Gain Voltage Buffer Example



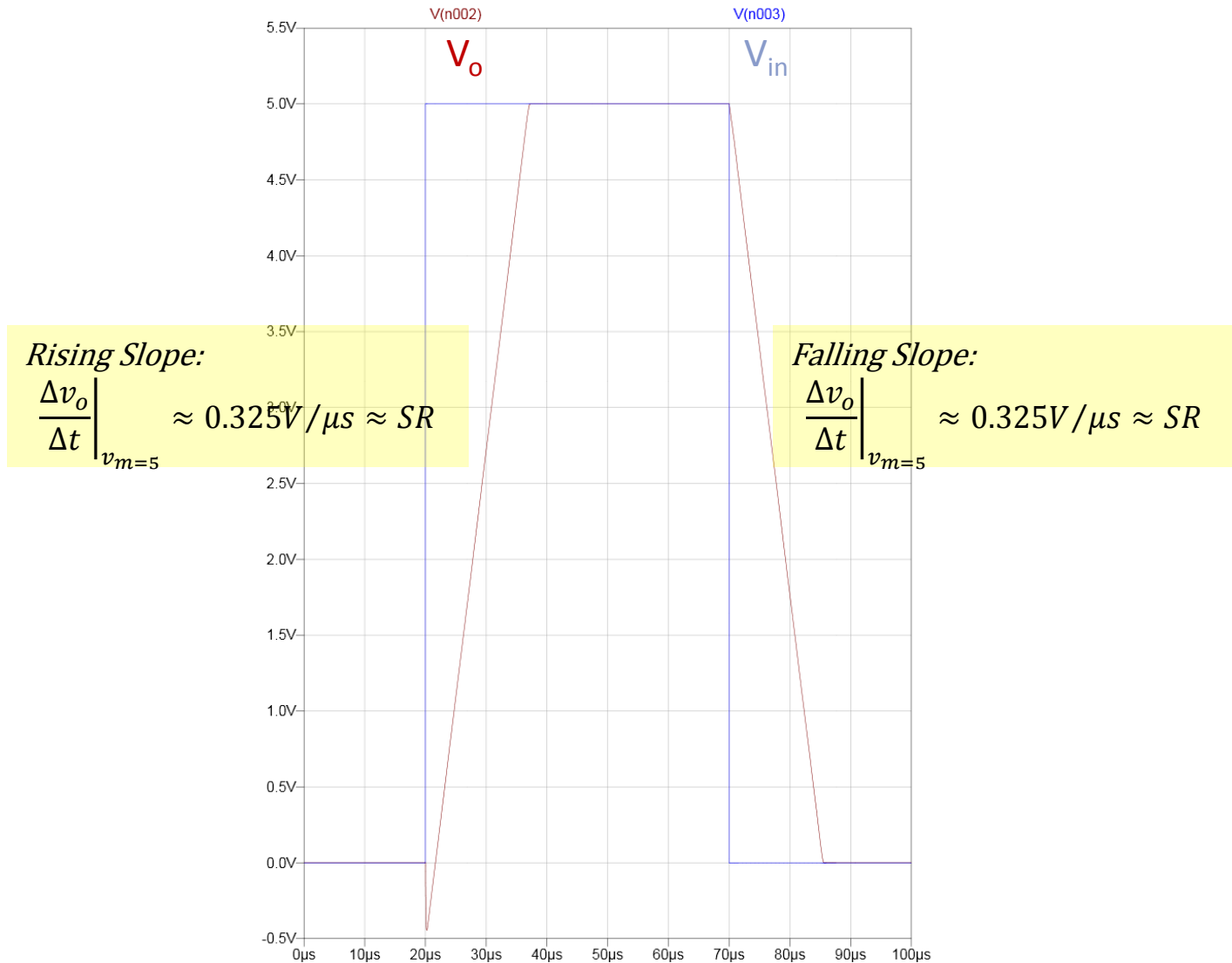
`.lib LM741.sub`

`.tran 0 0.1m 0`

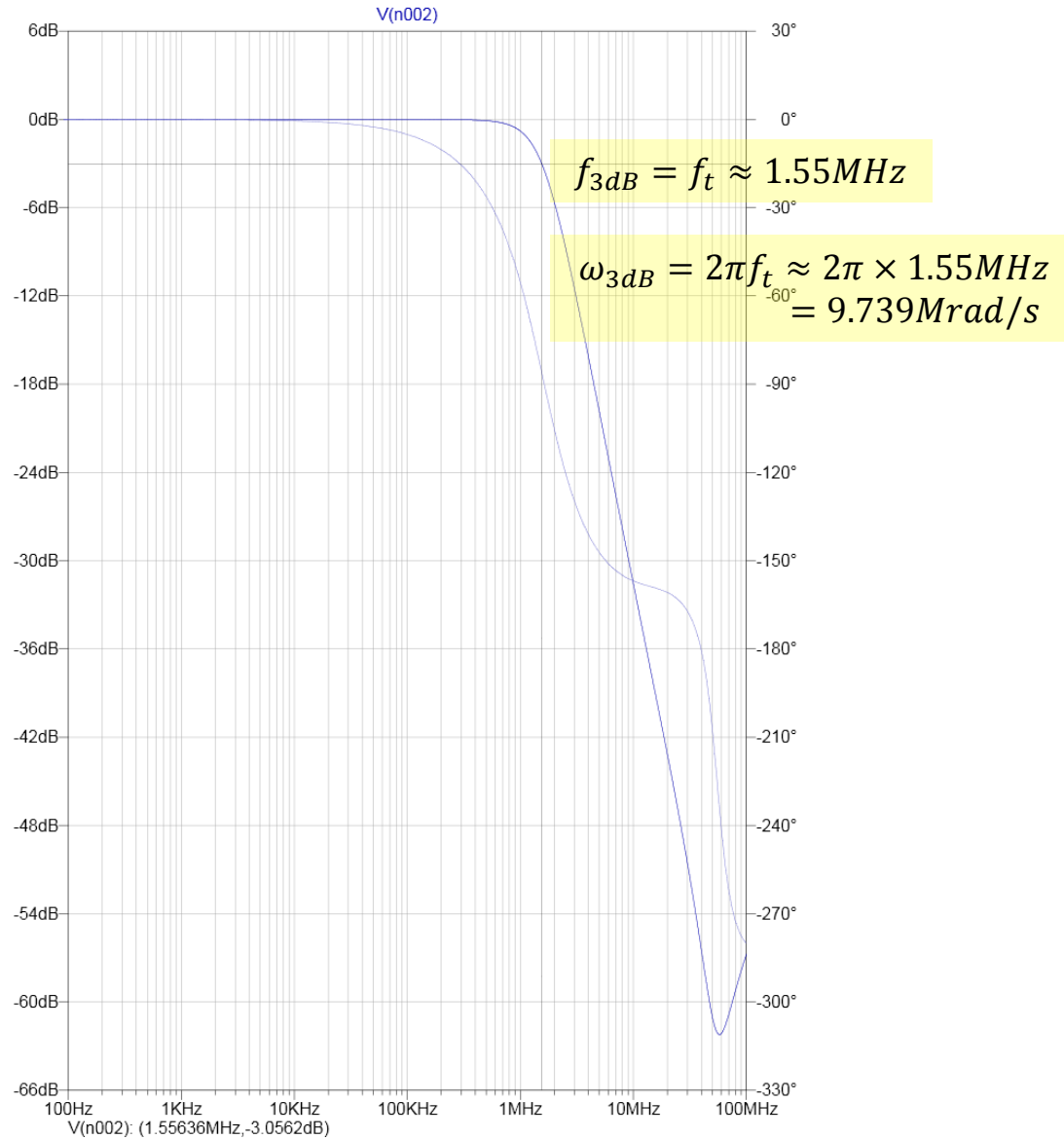
(a) 10 mV Input Step



(b) 5 V Input Square Wave



(c) 3dB Cut Off Frequency



EXAMPLE: NON-INVERTING AMPLIFIER

- An operational amplifier with $SR = 0.325 \text{ V}/\mu\text{s}$ and $\omega_t = GB = 9.739M \text{ rad/s}$, is used to implement a non-inverting amplifier of gain 10, which will limit the rate of rise of the output, the slewing or the bandwidth when
 - a) The input $V_i = 10\text{mV}$ step
 - b) The input $V_i = 0.6 \text{ V}$ step
 - c) The input is a sinusoidal $0.5 \sin(2\pi \times 5k \times t)$
 - d) The input is a sinusoidal $1.5 \sin(2\pi \times 5k \times t)$

Solution

a) Determin $V_m GB = 0.01V \times 9.739Mr/s = 0.097V/\mu s$

And $SR = 0.325V/\mu s$

Since SR is the larger of the two, the rate of the output is limited by the BANDWIDTH

b) In this case, $V_m GB = 0.6V \times 9.739Mr/s = 5.84V/\mu s$

Thus, $SR < V_m GB$. The rate of rise is limited by the slewing rate. Note that the output will change linearly rather than exponentially.

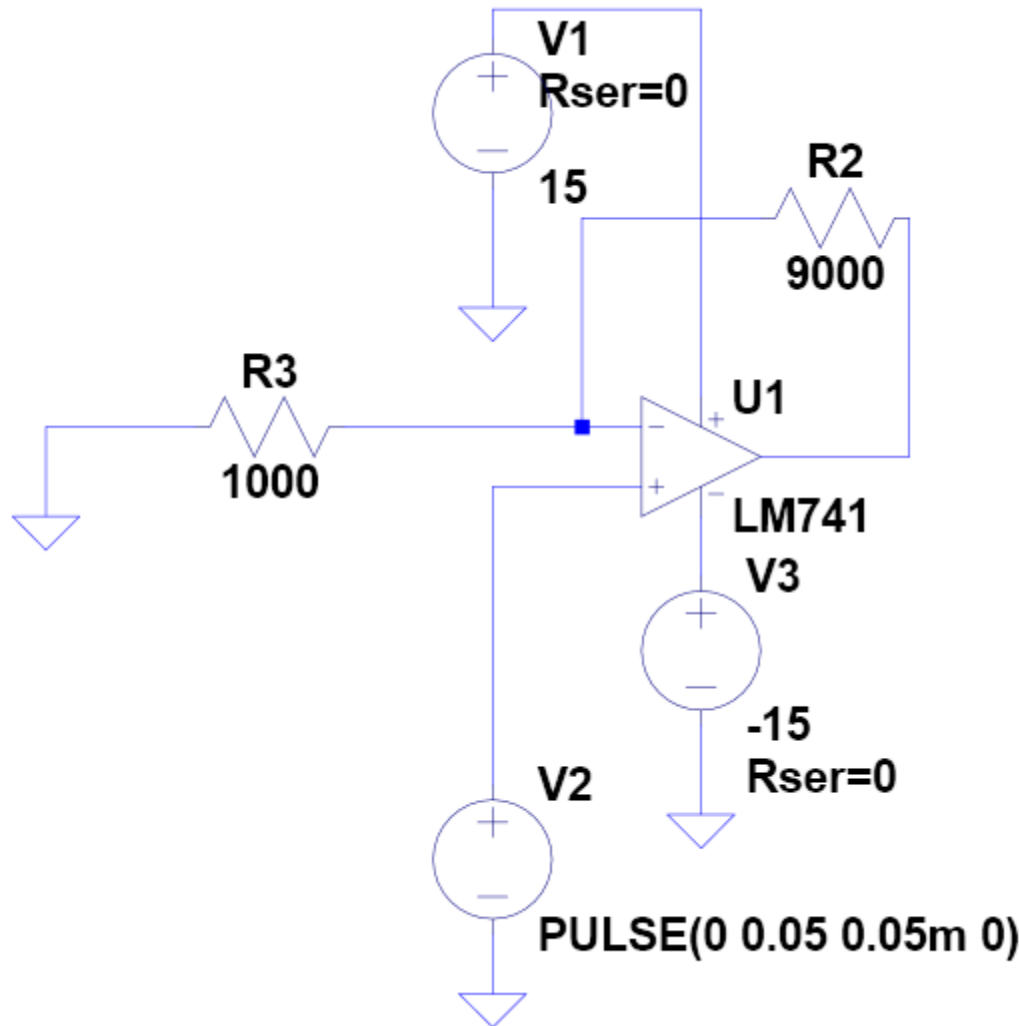
Solution

- c) Determine maximum change in closed loop. (V_m is the peak value)

$$\begin{aligned}V_{om} \omega_{in} &= V_m \left(1 + \frac{R_2}{R_1} \right) \omega_{in} = 0.5 \times 10 \times 2\pi \times 5 \times 10^3 \\ &= 0.157 V / \mu s < SR\end{aligned}$$

Since $V_m \omega_{in}$ is smaller than the SR, the rate of rise of the output is limited by the BANDWIDTH

Non-inverting Amplifier of Gain = 10



`.lib LM741.sub`

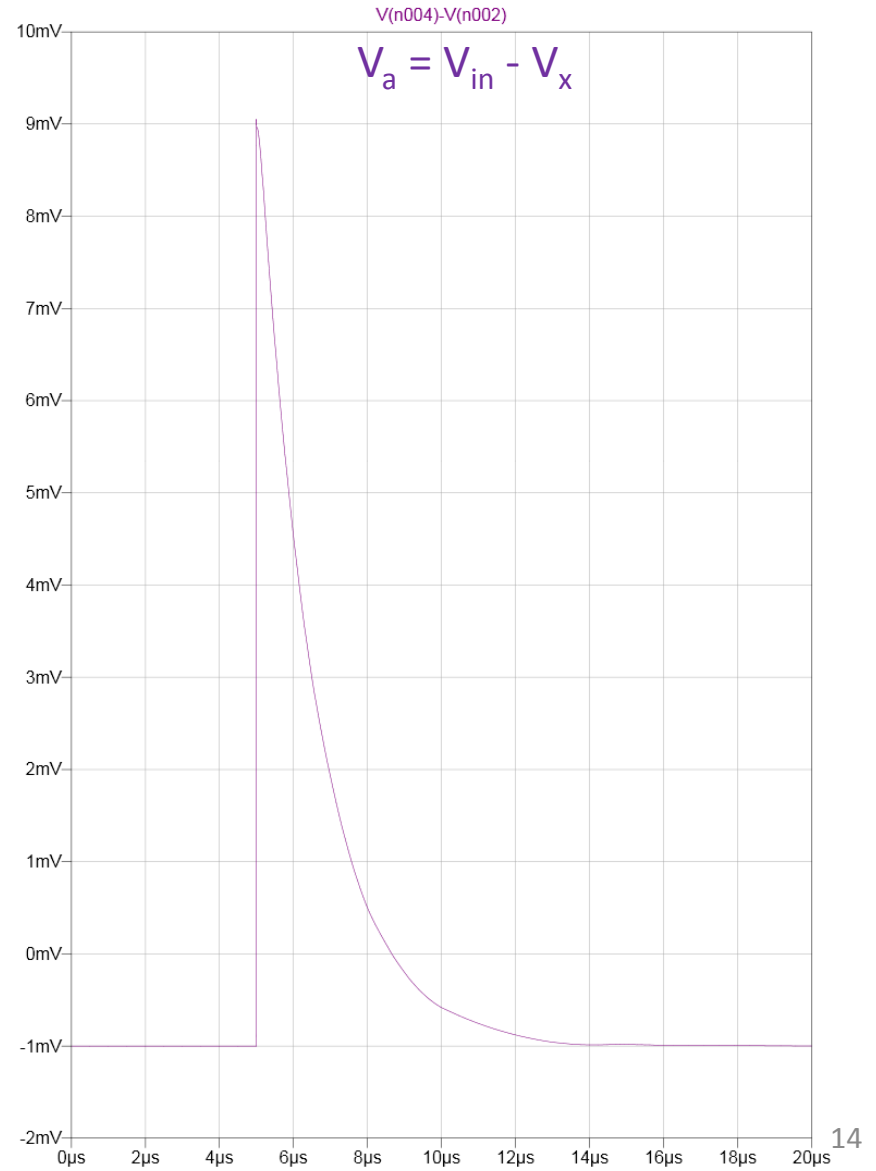
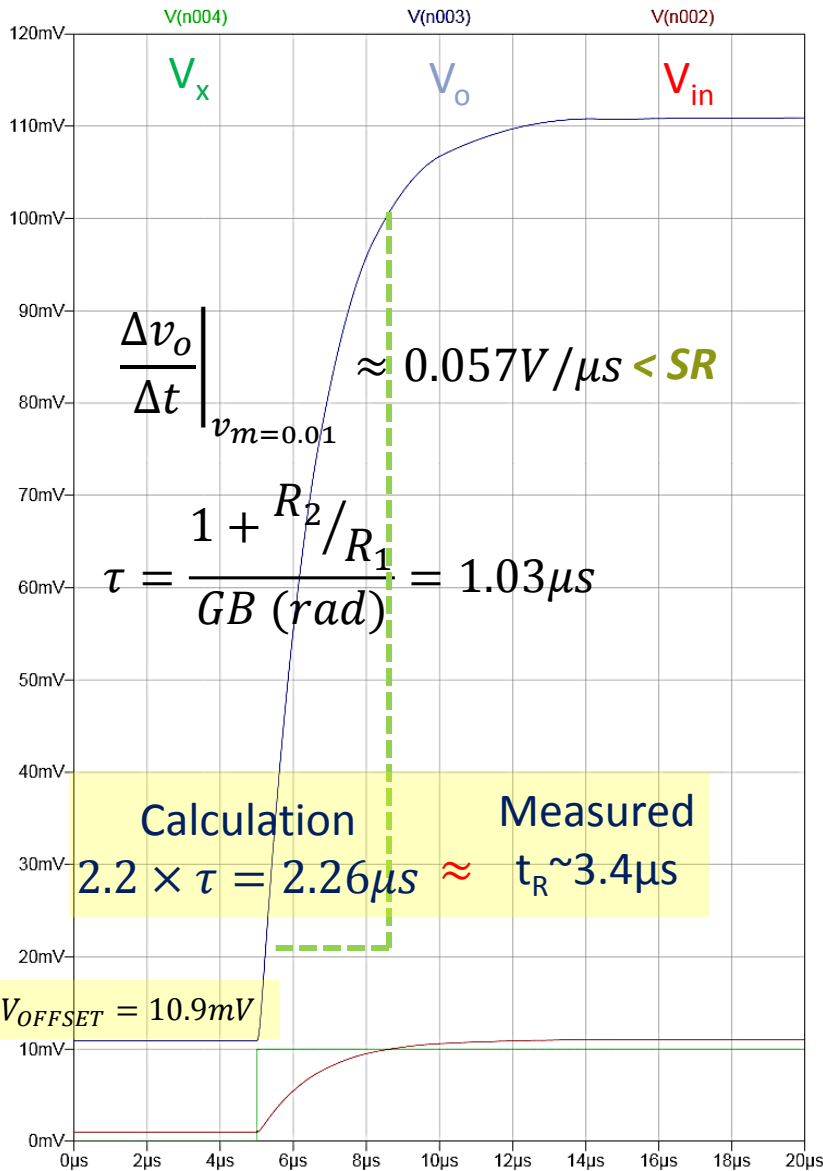
`.tran 0 0.1m 0`

$$SR \approx 0.325V/\mu s$$

$$GB = 2\pi \times 1.55Mr/s$$

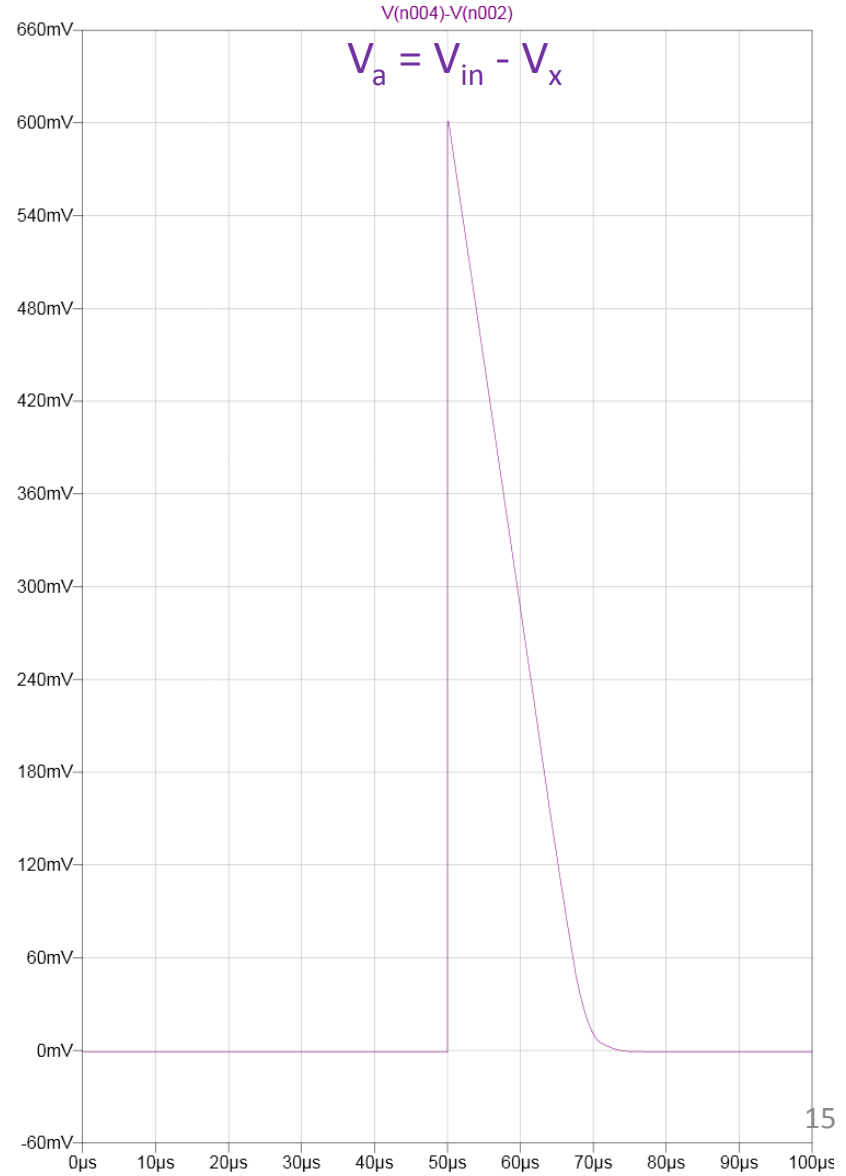
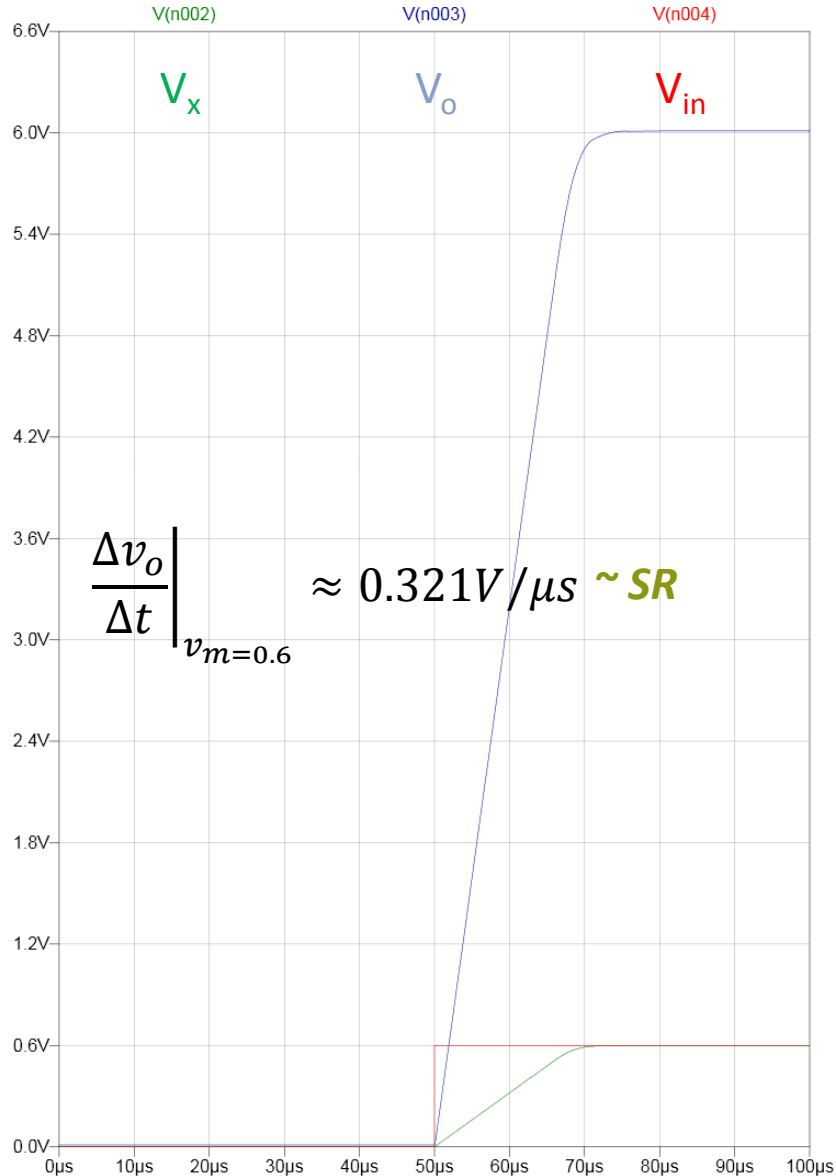
(a) 10 mV Input Step

BW LIMIT



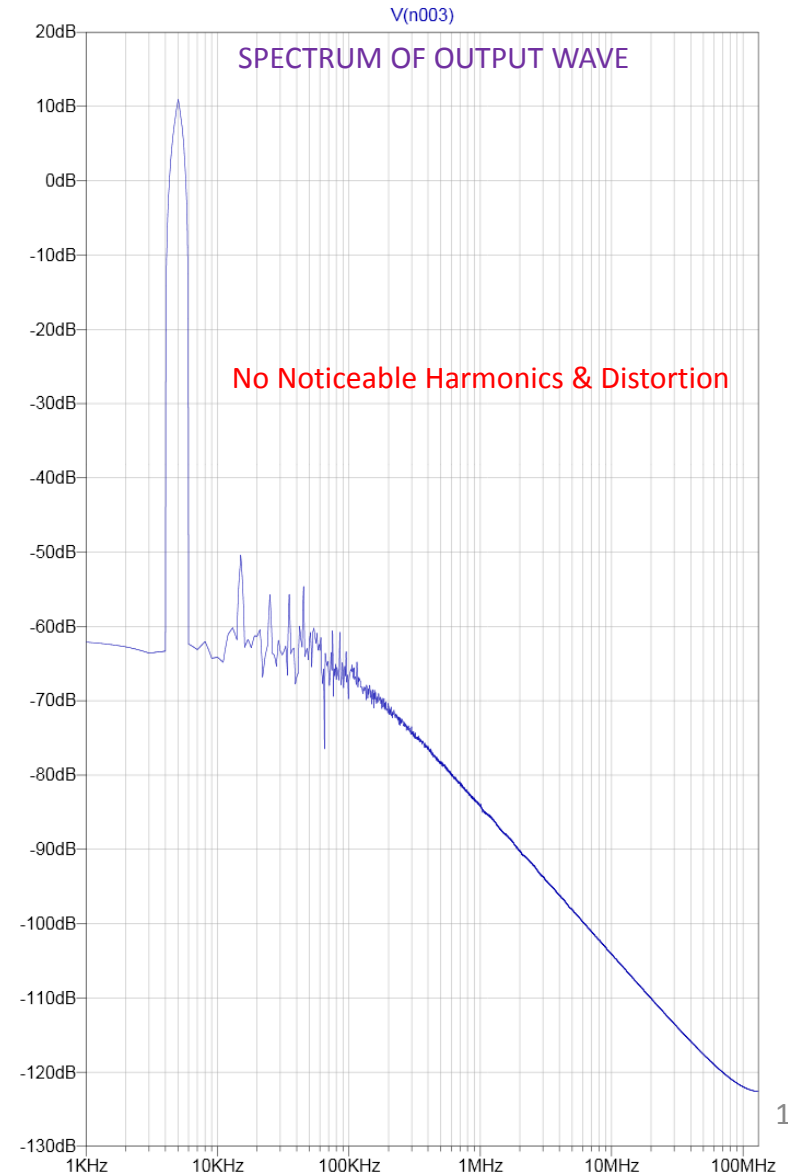
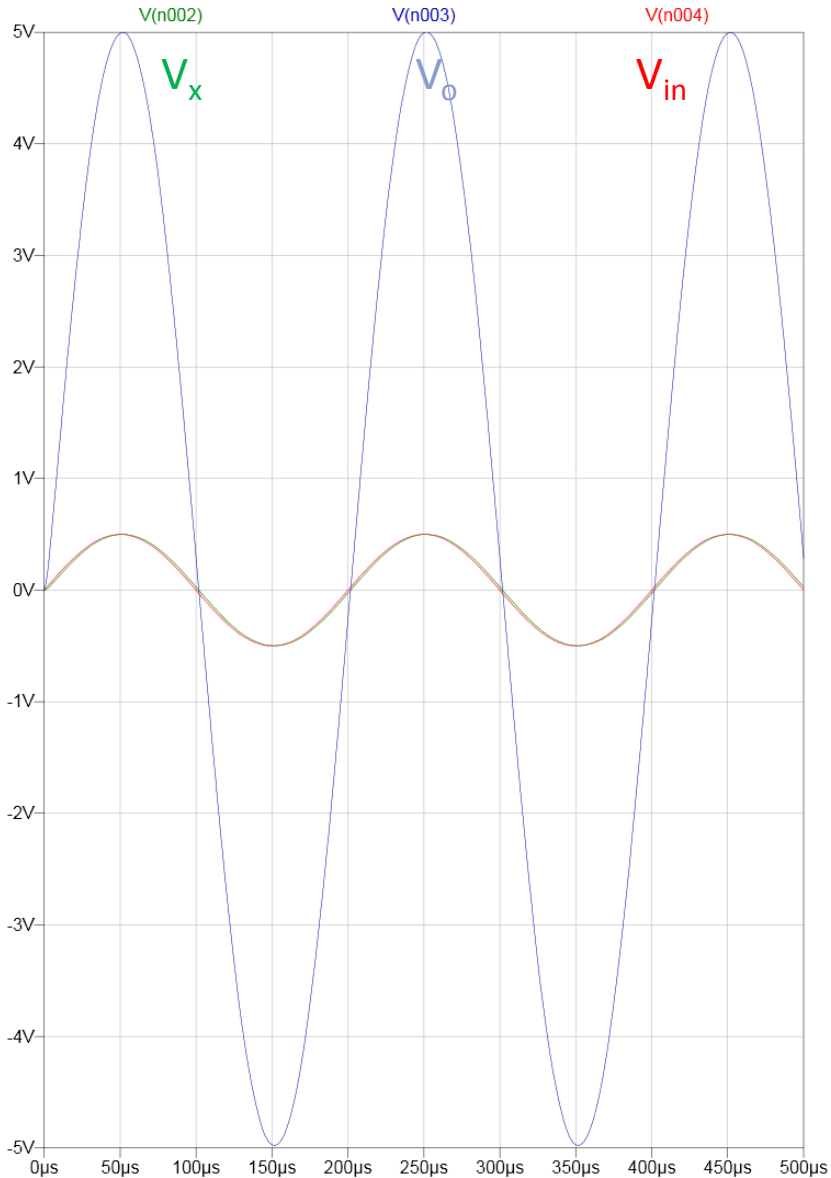
(b) 600 mV Input Step

SR LIMIT



(c) 5 kHz 0.5V Input Sin Wave

BW LIMIT



(d) V_{SAT} @ 5 kHz 1.5V Input Sin

SR LIMIT

$$V_m \omega_{in} = 1.5 \times 10 \times 2\pi \times 5k = 0.47 V/\mu s > SR = 0.325 V/\mu s$$

$$V_{SAT} = \frac{27.8}{2} = 13.9 V$$

