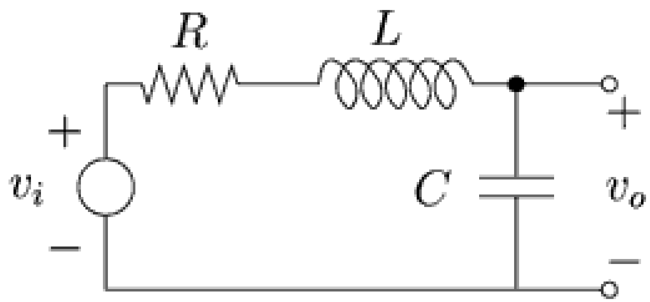
The two-integrator loop

Courtesy of Jorge Zarate

The standard form



$$\frac{v_o}{v_i} = \frac{1/_{sC}}{R + sL + 1/_{sC}} \rightarrow \frac{1/_{LC}}{s^2 + R/_L\, s + 1/_{LC}}$$

- IF the circuit is lossless (R=0), the poles:

$$s_{1,2} = \pm j\frac{1}{\sqrt{LC}} = \pm j\omega_o$$

- Similarly to the Q of an inductor:

$$Q = \frac{\omega_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- Using our definitions: $\frac{v_o}{v_i} = \frac{\omega_o^2}{s^2 + \omega_o/_Q\, s + \omega_o^2}$

The second order circuit

- With an active implementation (gain -H) and normalizing to $s_n = s/\omega_o$.

$$\frac{v_{LP}}{v_i} = \frac{-H}{s_n^2 + {}^1\!/_Q\, s_n + 1} \rightarrow v_{LP}\left(s_n^2 + {}^1\!/_Q\, s_n + 1\right) = -Hv_i$$

- In the time domain:

$$\frac{d^2 v_{LP}(t)}{dt^2} + \frac{1}{Q}\frac{dv_{LP}(t)}{dt} + v_{LP}(t) = -Hv_i(t)$$

- It is clear that we need two integrations to determine v_{LP}. To identify integration:

$$s\left(s + {}^1\!/_Q\right)v_{LP} = -(Hv_i + v_{LP})$$

$$sv_{LP} = -\frac{1}{\left(s + {}^1\!/_Q\right)}(Hv_i + v_{LP}) = vBP$$

- Now, v_{LP} can be obtained if v_{BP} is integrated

$$sv_{LP} = -\frac{1}{\left(s + {}^{1}/_{Q}\right)}(Hv_i + v_{LP}) = vBP$$

$$v_{LP} = \frac{1}{s} v_{BP} = \frac{1}{s}\left[-\frac{1}{\left(s + {}^{1}/_{Q}\right)}(Hv_i + v_{LP})\right]$$

- To further simplify:

$$-\,(Hv_i + v_{LP}) = \left(s + {}^{1}/_{Q}\right) v_{BP}$$

$$v_{BP} = -\frac{1}{s}\,(Hv_i + v_{LP} + {}^{1}/_{Q}\, v_{BP}) = -\frac{1}{s} v_{HP}$$

- Now we can see than v_{BP} can be obtained by integrating with a sign of inversion v_{HP}.

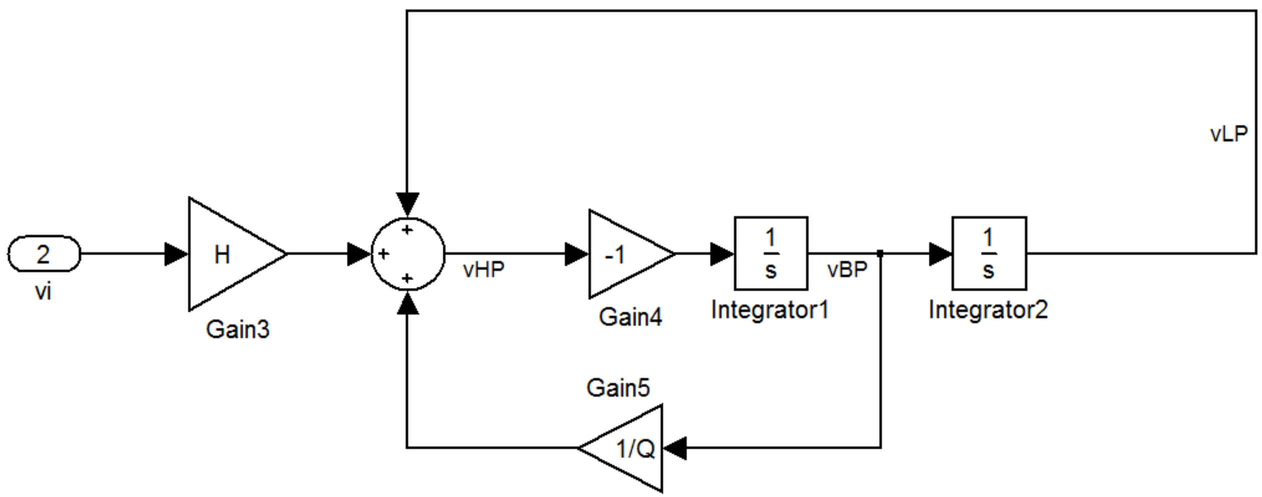
- Where v_{HP} is nothing but the sum of three scaled voltages

$$v_{HP} = \left(Hv_i + v_{LP} + {}^{1}\!/_{Q}\, v_{BP}\right)$$

- Backing up:
  - We said we could get v_{LP} by integrating v_{BP}
  - And that v_{BP} is obtained by integrating, with a negative sign, v_{HP}
  - And v_{HP} is the summation of three scaled voltages, therefore:

$$v_{LP} = \frac{1}{s}\left[\left(-\frac{1}{s}\right) v_{HP}\right]$$

Block diagram



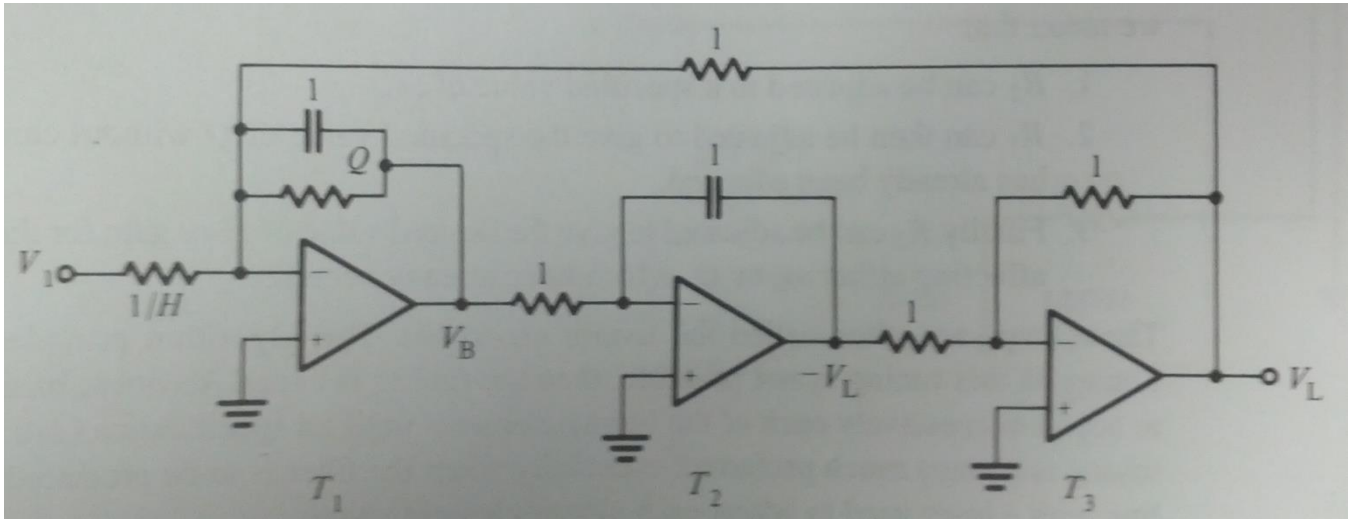
$$v_{LP} = \frac{-H}{s_n^2 + {}^{1}/_{Q}\, s_n + 1} v_i$$

$$v_{BP} = \frac{-Hs}{s_n^2 + {}^{1}/_{Q}\, s_n + 1} v_i$$

$$v_{HP} = \frac{Hs^2}{s_n^2 + {}^{1}/_{Q}\, s_n + 1} v_i$$

Tow-Thomas

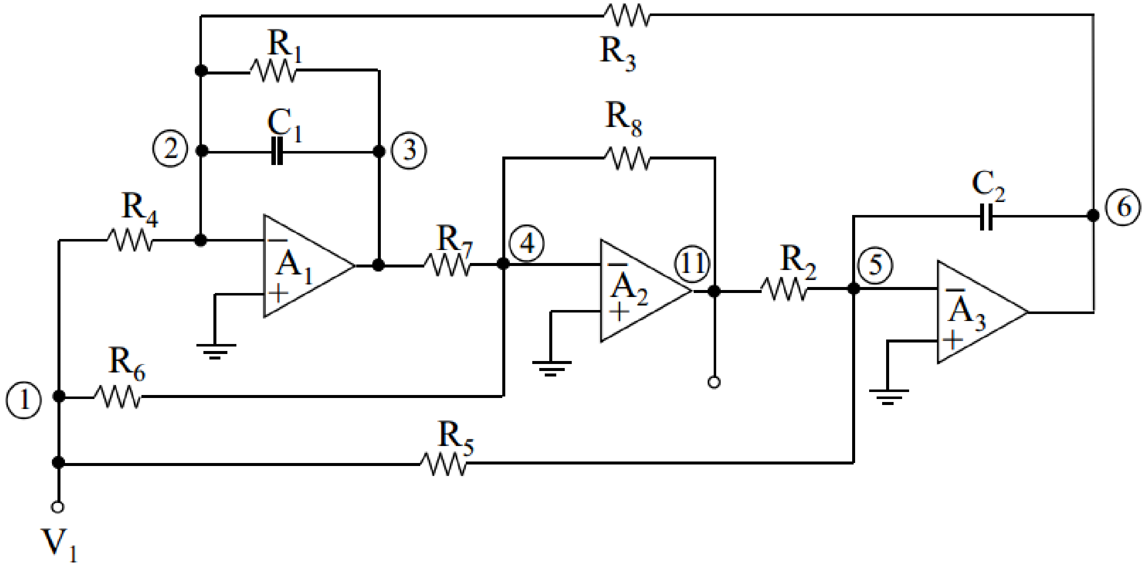
- Block diagram calls for a lossy inverting integrator and for an ideal non-inverting integrator
- Replace the ideal non-inverting for an ideal inverting + inverting amplifier



- Where is the high-pass output?

Feed-forward Tow-Thomas

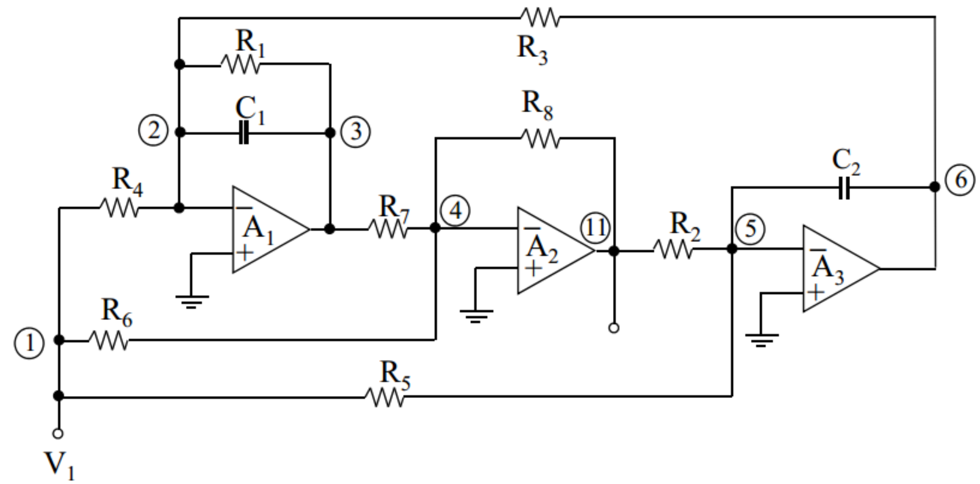
- Single-point output, multiple functions can be achieved

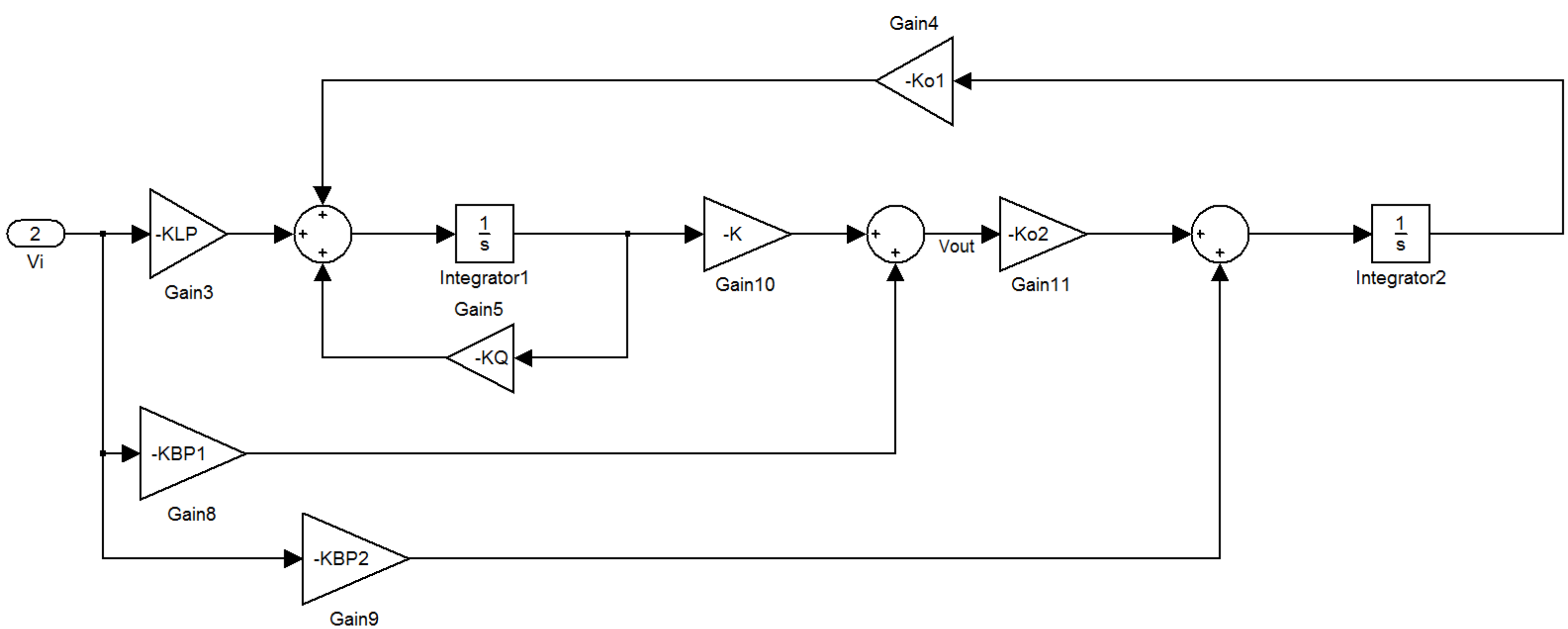


$$\frac{v_o}{v_i} = -\frac{R_8}{R_6}\frac{s^2 + \left(\frac{1}{R_1C_1} - \frac{R_6}{R_4R_7C_1}\right)s + \frac{R_6}{R_3R_5R_7C_1C_2}}{s^2 + \left(\frac{1}{R_1C_1}\right)s + \frac{R_8}{R_2R_3R_7C_1C_2}}$$

Simulink

- Obtain V_{out}/Vi or (V11/Vi), whatever is easier for you, also, map the blocks to gain/integrators time constants





Mason's Rule

- Direct paths:

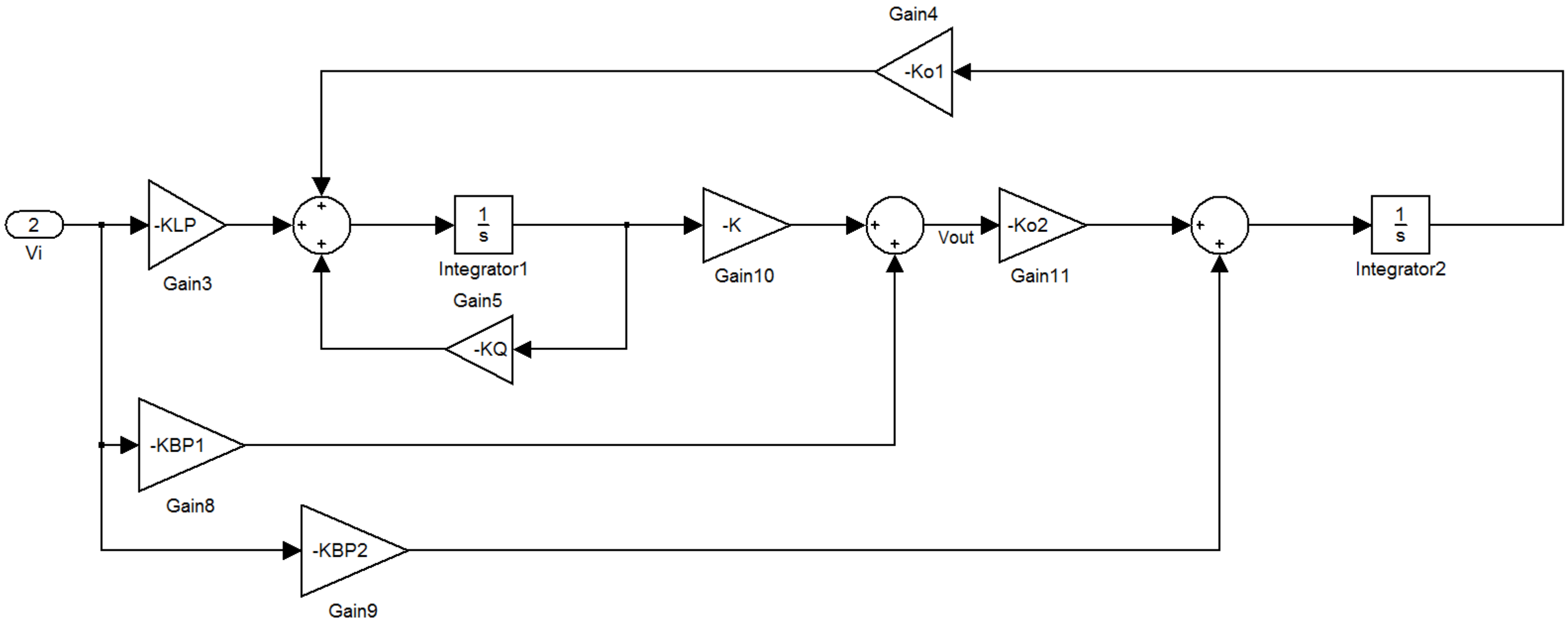
$$1) -\frac{KLP}{s}\cdot -K \qquad 2) -KBP1 \qquad 3) -KBP2\cdot -\frac{K}{s}\cdot -\frac{Ko1}{s}$$

- Loops:

$$1) -\frac{KQ}{s} \qquad 2)\ \frac{-K\cdot -K\text{o}1\cdot -K\text{o}2}{s^2}$$

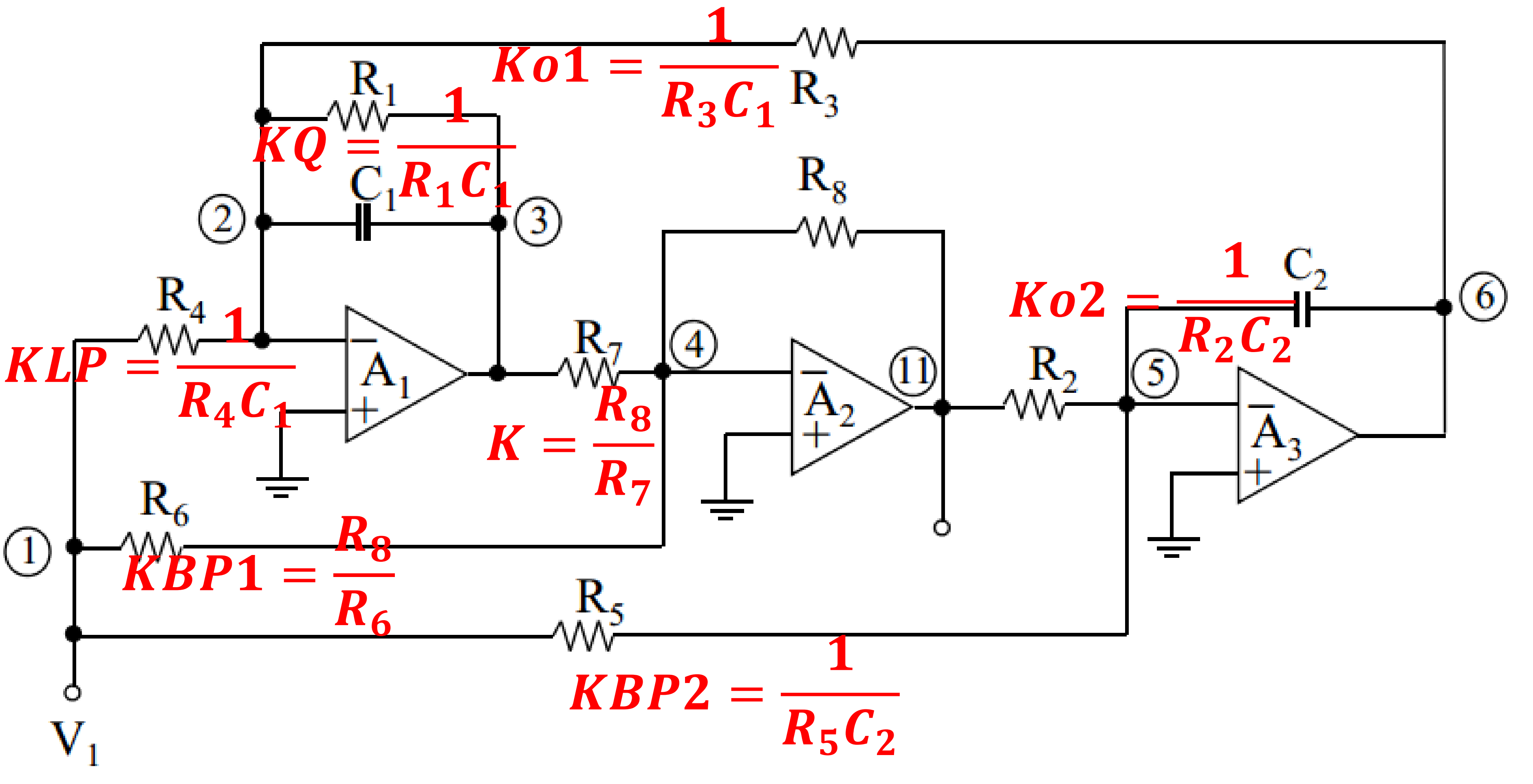
$$\frac{v_o}{v_i}=\frac{\frac{(KLP)K}{s}-\frac{(KBP2)K(Ko1)}{s^2}-KBP1\left(1+\frac{KQ}{s}\right)}{1+\frac{KQ}{s}+\frac{KKo1Ko2}{s^2}}$$

$$\frac{v_o}{v_i}=-KBP1\frac{s^2+\left(KQ-\frac{(KLP)K}{KBP1}\right)s+KBP2(Ko1)K}{s^2+KQs+KKo1Ko2}$$



$$KQ = \frac{1}{R_1 C_1} \qquad KLP = \frac{1}{R_4 C_1} \qquad KBP1 = \frac{R_8}{R_6} \qquad KBP2 = \frac{1}{R_5 C_2} \qquad K = \frac{R_8}{R_7}$$

$$Ko1 = \frac{1}{R_3 C_1} \qquad Ko2 = \frac{1}{R_2 C_2}$$



R1
R3
R8
C1
R4
R7
A1
A2
A3
R2
C2
R6
R5
V1
1
2
3
4
5
6
11
$Ko1 = \frac{1}{R_3C_1}$
$KQ = \frac{1}{R_1C_1}$
$KLP = \frac{1}{R_4C_1}$
$K = \frac{R_8}{R_7}$
$Ko2 = \frac{1}{R_2C_2}$
$KBP1 = \frac{R_8}{R_6}$
$KBP2 = \frac{1}{R_5C_2}$