The important thing is not to stop questioning! Curiosity has its own reason for existing. (Albert Einstein)

Noise-based logic

Don’t expect a complete or systematic talk (no time); rather something to challenge/explore

Comments, collaboration are welcome!
The important thing is not to stop questioning! Curiosity has its own reason for existing. (Albert Einstein)

Noise-based logic: Why noise for deterministic logic?

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Although noise-based logic shows potential advantages of reduced power dissipation and the ability of large parallel operations with low hardware and time complexity the question still persist: is randomness really needed out of orthogonality? In this talk after introducing noise-based logic we address this question.

A journal paper about this issue is coming out in the December issue of Fluctuation and Noise Letters
http://www.ece.tamu.edu/~noise/research_files/noise_based_logic.htm

Presented at: ICCAD 2012, SPECIAL SESSION: Computing in the Random Noise: The Bad, the Good, and the Amazing Grace
November 5, 2012, San Jose, CA.
The important thing is not to stop questioning! Curiosity has its own reason for existing. (Albert Einstein)

Why is neural spike transfer stochastic? String verification in the brain

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The 4th International Conference on Cognitive Neurodynamics, 23-27 June 2013, Sigtuna, Sweden
Present and past collaborators on noise-based logic (Alphabetical order).

Sergey Bezrukov (NIH): brain: logic scheme, information processing/routing, circuitry, etc.

Khalyan Bollapalli (former computer engineering PhD student, TAMU): exploration of sinusoidal orthogonal logic

Zoltan Gingl (Univ. of Szeged, Hungary): modeling for circuit realization, etc.

Tamas Horvath (Frauenhofer for Computer Science, Bonn, Germany): string verification, Hamilton coloring problem.

Sunil Khatri, (Computer Engineering, TAMU): hyperspace, squeezed instantaneous logic, etc.

Andreas Klappenecker, (Computer Science, TAMU): quantum-mimicking, large complexity instantaneous parallel operations, etc.

Ferdinand Peper (Kobe Research Center, Japan): squeezed and non-squeezed instantaneous logic, etc.

Swaminathan Sethuraman (former math. PhD student, TAMU): Achilles heel operation.

He Wen (Electrical Engineering, TAMU; Visiting Scholar from Hunan University, China): large complexity instantaneous parallel operations; why noise; complex noise-based logic, etc.

"noise-based logic is one of the most ambitious attempts..."
The microprocessor problem

*Speed-Error-Power* triangle
A switch is a potential barrier which exists (off position) or not (on position). To control/build the potential barrier we need energy.

Maximal clock frequency 
\[ f_0 \approx (RC)^{-1} \]

Dissipation by a single unit 
\[ P_1 \propto f_0 E_1 \propto (RC)^{-1} NU_0^2 \propto \frac{U_0^2}{R} \]

Total dissipation by the chip 
\[ P_N \propto NU_0^2 / R \propto NU_0^2 \propto \frac{U_0^2}{s^2} \]

number of units 
\[ N \propto \frac{1}{s^2} \]

\( s \): characteristic device size

**CMOS drivers' channel resistance** 

**CMOS gate capacitance** 

\[ C \propto s^2 \]

\[ C \propto s \]
**False bit flips.** Gaussian noise can reach an arbitrarily great amplitude during a long-enough period of time and the \( \textit{rms} \) noise voltage grows with miniaturization: \[ U_n = \sqrt{\frac{kT}{C}} \]

For band-limited white noise, frequency band \((0, f_c)\), the \textit{threshold crossing frequency} is:

\[ \nu(U_{th}) = \frac{2}{\sqrt{3}} \exp\left(\frac{-U_{th}^2}{2U_n^2}\right) f_c \]

where \[ U_n = \sqrt{S(0)f_c} \]

### Energy dissipation of single logic operation at $\epsilon$ error probability:

$$E > kT \ln \frac{1}{\epsilon}$$

### Speed-Max-Error-Power

$$\epsilon < 10^{-25} \Rightarrow E \approx 60kT$$

Practical situation is much worse; prediction in 2002-2003:

It was supposed that:

- The bandwidth is utilized;
- The supply voltage is reduced proportionally with size (to control energy dissipation and avoid early failure due to hot electrons.)
End of Moore’s law: thermal (noise) death of integration in micro and nano electronics

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Received 16 July 2002; received in revised form 19 September 2002; accepted 19 September 2002
Communicated by C.R. Doering

Abstract

The exponential growth of memory size and clock frequency in computers has a great impact on everyday life. The growth is empirically described by Moore’s law of miniaturization. Physical limitations of this growth would have a serious impact on technology and economy. A thermodynamical effect, the increasing thermal noise voltage (Johnson–Nyquist noise) on decreasing characteristic capacitances, together with the constrain of using lower supply voltages to keep power dissipation manageable on the contrary of increasing clock frequency, has the potential to break abruptly Moore’s law within 6–8 years, or earlier.

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Conclusion was (2002): if the miniaturization is continuing below 30-40 nm, then the clock frequency cannot be increased.

No increase since 2003! Prophecy fulfilled much earlier!

Even though Moore’s law has seemingly been followed, the speed of building elements are not utilized. Supply voltage has been kept high.
Comparison of single quantum gates with single classical logic gates
The brain vs computer

dreams and reality
In the "Blade Runner" movie (made in 1982) in Los Angeles, at 2019, the Nexus-6 robots are more intelligent than average humans.

2019 is only 6 years from now and nowadays we have been observing the slowdown of the evolution of computer chip performance.

We are simply nowhere compared a Nexus-6.

Have we missed the noisy neural spikes in our computer developments???
Isaac Asimov (1950's): *The Three Laws of Robotics:*

1. A robot may not injure a human being, or, through inaction, allow a human to come to harm.
2. A robot must obey orders given to him by human beings except where such orders would conflict with the First Law.
3. A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

*Not even the best supercomputers are able to address such refined perception of situations!*

*We have great problems even with the most elementary necessities, such as recognition of natural speech of arbitrary people or speech in background noise.*
How does biology do it?? A quick comparison.

Note: Average power consumption of a supercomputer in the worldwide TOP-10 list (2012) is 1.32 million Watts.

This Laptop

- Power dissipation: about 12 W
- Number of switches (transistors): $10^{13}$
- Very high bandwidth (GHz range)
- Signal: deterministic, binary voltage
- Deterministic binary logic scheme, *general-purpose (?)*
- Potential-well based, addressed memory
- High speed of fast, primitive operations
- Low probability of errors
- Sensitive for operational errors (freezing)

Human Brain

- Brain dissipation: about 12 W
- Number of switches (neurons): $10^{11}$
- Extremely low bandwidth (< 100 Hz)
- Signal: *stochastic spike train, noise*
- Unknown logic scheme, *special-purpose (???)*
- Unknown, associative memory
- Slow but intelligent operations
- High probability of errors, even with simple operations
- Error robust (no freezing) (?)
Often a Poisson-like spike sequence. The relative frequency-error scales as the reciprocal of the square-root of the number of spikes.

\[ \Delta = \frac{1}{\sqrt{n}} \]

Supposing the maximal frequency, 100 Hz, of spike trains, 1% error needs to count \(10^4\) spikes, which is 100 seconds of averaging!

Pianist playing with 10 Hz hit rate would have 30% error in the rhythm at the point of brain control. Parallel channels needed, at least 100 of them.

(Note: controlling the actual muscles is also a problem of negative feedback but we need an accurate reference signal).

Let's do the naive math: similar number of neurons and transistors in a palmtop, but 30 million times slower clock; plus a factor of \(10^4\) slowing down due to averaging needed by the stochastics. The brain should perform about 300 billion times slower than our palmtop computer!
The brain is using different logic and computational schemes than computers and it is using a lot of *special-purpose, noise-based operations*, not like our general-purpose computers.

It must do that because its "brute force" abilities are much weaker than that of a laptop.

*Try to multiply the first 100 integer numbers and check how long does it take. For a laptop computer, it takes less than a millisecond.*
The brain is using different logic and computational schemes than computers and it is using a lot of *special-purpose, noise-based operations*, not like our general-purpose computers.

It must do that because its "*brute force*" abilities are much weaker than that of a laptop.

**Or try to memorize this text:**

```
vyicshrgoeeiakcleDMntsstnaoatii
```
The brain is using different logic and computational schemes than computers and it is using a lot of *special-purpose, noise-based operations*, not like our general-purpose computers.

It must do that because its "brute force" abilities are much weaker than that of a laptop.

**Or try to memorize this text:**

vyicshrgoeieiakcleDMntsstnaoatii

**Michel Dyakonov is a great scientist**

For the brain, the second version is much easier, while for a computer, it does not matter; more precisely, *the first version is easier* because of the lack of blank characters.
Another major difference between the brain and a computer:

To discover the difference between the two strings the computer must compare each bit in the two strings. The brain does not have to do that.

At proper conditions very easily discovers difference of patterns without detailed investigations of all the details.
Noise-based logic
What is Noise-based logic:

- *Noise carries the logic information.*
- *The logic base, which is a reference signal system, consists of uncorrelated (orthogonal) stochastic signals (noises). These are orthogonal vectors. Superpositions are possible: vector space.*
- *This reference system is needed to identify these vectors in a deterministic way. Deterministic logic.*

What **noise-based logic** is certainly not: *It is not noise-assisted signal transfer*, for example:

- *It is not stochastic resonance*
- *It is not dithering*
- *It is not linearization by noise*

None of these schemes use the noise as information carrier.

Note: because noise-based logic is **deterministic logic**:

- *It is not stochastic computing*
- *It is not randomized algorithm but it can be a natural hardware for that*
How does a noise-based logic hardware look like?

**Generic noise-based logic outline**

Reference Noise System
orthogonal stochastic time functions

logic information (noise)

Noise-based Logic Gate

logic information (noise)

Noise-based Logic Gate

logic information (noise)

Reference signals
(noises)

Reference signals
(noises)
1. Logic signals are noises that are orthogonal on the noise. Base: $N$ orthogonal noises: noise-bits.

2. Multivalued logic.


4. Hyperspace vectors. Product of two or more different base noises: orthogonal to each base noise. Their superpositions represent $2^N$ bits simultaneously in a single wire.

Quantum computers: $N$ qubits represents $2^N$ classical bits

(Note: sinusoidal functions can also do this, see below, but there is a price)
But, periodic functions, like sinusoidals can also do this! Why noise ??
But why noise?

At least three major aspects of noise compared to periodic:

- **Physics: Entropy production (energy dissipation):**

  *Simple wording: noise is freely available; generated by the system without power requirement.*

  Deeper: Brillouin's negentropy law. The deterministic signal has negative entropy (negentropy) due to its information entropy $I_s$ (logarithm of amplitude resolution; reduced relative uncertainty). Due to the Second Law of Thermodynamics, the entropy of the whole closed system cannot decrease thus, at least, the same amount if positive entropy (in this case, heat) will be produced. If a resonator circuit is used on the oscillator, this heat production will be repeated within the passive relaxation time (Q-times the period) of the resonator thus a continuous heating power will be generated:

  \[
  P_{\text{heat}} = \frac{TS_s}{\tau} \geq \tau^{-1} kT I_s \ln(2)
  \]

  In a resonator-free oscillator the situation is worse because the same heat is produced at each period of oscillation, which means the dissipation is Q-times higher.

- **Resilience** of distinguishability of time series, compare periodic/stochastic.

- **Computational complexity** at certain (quantum-mimics) special-purpose operations.
Example - 1 for entropy generation:

**Correlator-based noise-based logic**
Basic structure of correlator-based noise-based logic with continuum noises:

Theoretically much less power dissipation.
But that needs special devices (may not exist yet).
Slower: longer time.


Note: analog circuitry but *digital accuracy* due to the threshold operation in the DC part!
Example: XOR gate comparing two logic vectors in a space of arbitrary dimensions (binary, multi-value, etc), with binary output giving "True" value only when the two input vectors are orthogonal. Even though the equation contains four multiplications, two saturation nonlinearities, one inverter, and two time averaging, the hardware realization is much simpler. It requires only one multiplier, one averaging unit and two analog switches. Realizations of the other gates also turns out to me simpler than their mathematical equations.

$$Y(t) = \langle X_1(t)X_2(t) \rangle H(t) + \langle X_1(t)X_2(t) \rangle L(t)$$

Analog circuitry but digital accuracy!

Theoretically much less power dissipation. But that needs special devices (may not exist yet). Slower: longer time.

The real potential would be due to multivalued aspects.
Examples - 2 for resilience:

**Instantaneous noise-based logic:**

a) **Brain: Random unipolar spike sequence based noise-based logic**


b) **With random telegraph waves: Boolean**


c) **With random telegraph waves: String verification**


d) **With random telegraph waves: product strings in superposition (quantum-mimic)**

Introducing the neuro-bit


- Neural spikes. Using set-theoretical operations. The $A$ and $B$ sets below represent two partially overlapping neural spike trains.
- $AB$ is the overlapping part.
- $A \bar{B}$ is the spike train $A$ minus the overlapping spikes.
- $\bar{A}B$ is the spike train $B$ minus the overlapping spikes.

$A \bar{B}$, $\bar{A}B$ and $AB$ are orthogonal, they do not have common part! The partially overlapping spike trains can be use as neuro-bits in the same was as it was with the noise bits.

*N neuro bits will make $2^N-1$ orthogonal elements, that is a $2^N-1$ dimensional hyperspace.*

The very same multidimensional hyperspace as it was obtained with the coninuoum noise-bits.


A

B
With 3 neuro-bits ($N=3$). It makes $2^N-1 = 7$ hyperspace vectors. Using these vectors in a binary (on/off) superposition, we can represent 127 different logic values in a synthesized neural spike train in a single line. (Bezrukov, Kish, *Physics Letters A* 373 (2009) 2338-2342)

The very same hyperspace as it was obtained with the noise-bits. (Kish, Khatri, Sethuraman, *Physics Letters A* 373 (2009) 1928-1934)
A key question: can we make these set theoretical operations with neurons? Yes: the key role of inhibitory input of neurons becomes clear then.

(Bezrukov, Kish, Physics Letters A 373 (2009) 2338-2342)

\[ A(t)B(t) \]
\[ A(t)\overline{B}(t) \]
\[ \overline{A}(t)B(t) \]

\( N \)-th order orthogonator: \( N \) inputs for partially overlapping spike trains and \( 2^N - 1 \) output ports with orthogonal spike trains.

The second-order orthogonator gate circuitry utilizing both excitatory (+) and inhibitory (-) synapses of neurons. The input points at the left are symbolized by circles and the output points at the right by free-ending arrows. The arrows in the lines show the direction of signal propagation.
The orthon building element and its symbol.

Resilience: Brain signal scheme utilizing stochastic neural spikes, their superpositions and coincidence detection

Coincidence detector utilizing the reference (basis vector) signals.

Very fast. No statistics/correlations are needed.

Neural circuitry utilizing coincidences of neural spikes.
The basic building element orthon (left) and its symbol (right).


\[ A(t)B(t) \]

excitatory

\[ A(t)\bar{B}(t) \]

inhibitory

Spike times make sense

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Many behavioral responses are completed too quickly for the underlying sensory processes to rely on estimation of neural firing rates over extended time windows. Theoretically, first-spike times could underlie such rapid responses, but direct evidence has been lacking. Such evidence has now been uncovered in the human somatosensory system. We discuss these findings and their potential generalization to other sensory modalities, and we consider some future challenges for the neuroscientific community.


\[ AB = |1,1\rangle \]

\[ A\bar{B} = |1,0\rangle \]

Neurons Tune to the Earliest Spikes Through STDP

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*Neural Computation* 17, 859–879 (2005) © 2005 Massachusetts Institute of Technology
Examples - 3 for computational complexity:

resilience example c:

**c) With random telegraph waves: String verification**


resilience example d: Quantum mimicking

**d) Instantaneous NBL with random telegraph waves: product strings in superposition**


Hyperspace-based *instantaneous noise-based logic*
Instantaneous NBL. Example: Random Telegraph Waves. Their products: hyperspace

Random Telegraph Wave (RTW) taking +1 or -1 with 50% probability at the beginning of each clock period.

\[ \text{RTW}^2 = 1 \quad \text{RTW}_1 \cdot \text{RTW}_2 = \text{RTW}_3 \]

all orthogonal

Advantage: can be realized with digital circuitry and physical random number generators. Free of parasitic errors.

Arbitrary \( N \)-long bit strings can be represented by \( 2N \) independent waves; 2 waves for each bit, to represent the 2 possible values

\[
\begin{align*}
V_2^0, V_1^1; & \quad V_2^0, V_2^1; & \quad V_3^0, V_3^1; & \quad V_4^0, V_4^1; & \quad V_5^0, V_5^1; & \quad V_6^0, V_6^1
\end{align*}
\]

The actual string is represented by the product of the \( N \) waves that correspond to the bit values, for example:

\[
\begin{align*}
V_1^1 \cdot V_2^0 \cdot V_3^1 \cdot V_4^1 \cdot V_5^0 \cdot V_6^1 &= Y_1 \\
V_1^1 \cdot V_2^0 \cdot V_3^0 \cdot V_4^1 \cdot V_5^0 \cdot V_6^1 &= Y_2
\end{align*}
\]

Application example: string (-difference) verification with low communication complexity. The probability that \( Y_1 \) and \( Y_2 \) go together for \( M \) steps is \( 0.5^M \). 83 time steps result in for less than \( 10^{-25} \) error probability


In the brain with unipolar spikes, XOR operations do the same.
XOR brain logic circuitry

\[ Y(t) = X_1(t) \text{ XOR } X_2(t) \]
Random Telegraph Wave (RTW) taking +1 or -1 with 50% probability at the beginning of each clock period.

RTW\(^2\) = 1 ; \quad RTW_1*RTW_2 = RTW_3

all orthogonal

Note: simplified to save presentation time. Modified timing and/or complex waves are needed for best performance

When the binary values of a bit are represented by waves \(V_0\) and \(V_1\) then the **NOT operator is multiplication by** \(V_0*V_1\)

proof: \((V_0*V_1)*V_1 = V_0\) \quad (V_0*V_1)*V_0 = V_1
Random Telegraph Wave (RTW) taking +1 or -1 with 50% probability at the beginning of each clock period.

$$RTW^2 = 1; \quad RTW_1 \cdot RTW_2 = RTW_3$$

all orthogonal

Universe: superposition of all the possible product strings

$$Y_N = \left[ V_1^0(t) + V_1^1(t) \right] \left[ V_2^0(t) + V_2^1(t) \right] \ldots \left[ V_N^0(t) + V_N^1(t) \right]$$

Example-2: Large, parallel operations in hyperspace

The second noise-bit in the superposition of $2^N$ binary numbers is inverted by an $O(N^0)$ hardware complexity class operation!

$$\left( V_2^0 \ast V_2^1 \right)^*$$

Single wire

- $V_1^0 V_2^0 V_3^0 = |1,0,0\rangle$
- $V_1^0 V_2^0 V_3^0 = |0,0,0\rangle$
- $V_1^1 V_2^1 V_3^0 = |1,1,0\rangle$
- $V_1^0 V_2^1 V_3^1 = |1,0,1\rangle$
- $V_1^0 V_2^1 V_3^0 = |0,1,0\rangle$
- $V_1^0 V_2^0 V_3^1 = |0,0,1\rangle$
- $V_1^1 V_2^1 V_3^1 = |1,1,1\rangle$
- $V_1^0 V_2^1 V_3^1 = |0,1,1\rangle$
Random Telegraph Wave (RTW) taking +1 or -1
with 50% probability at the beginning of each clock period.

\[ RTW^2 = 1; \quad RTW_1 \cdot RTW_2 = RTW_3 \]

all orthogonal

\[(V_0 \cdot V_1) \cdot V_1 = V_0\]
\[(V_0 \cdot V_1) \cdot V_0 = V_1\]

**Example-2: Large, parallel operations in hyperspace**

Can be done with sinusoidal signals, too! Isn't that better? Then a Fourier-series analysis over the base period would serve with the full result!
The signal system with sinusoidals: Linear vs Exponential harmonic (sinusoidal) bases:

\[ L_r(t) = e^{j2\pi(2r-1)f_0t} \]
\[ H_r(t) = e^{j2\pi 2rf_0t} \]

\[ L_r(t) = e^{j2\pi 2^{2r-2}f_0t} \]
\[ H_r(t) = e^{j2\pi 2^{2r-1}f_0t} \]

**Time complexity:** \( f_{max}/f_{min} \)

<table>
<thead>
<tr>
<th>Bit</th>
<th>Logic Value</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear Representation</td>
<td>Exponential Representation</td>
</tr>
<tr>
<td><strong>1st</strong></td>
<td>( L_1 )</td>
<td>( f_0 )</td>
</tr>
<tr>
<td></td>
<td>( H_1 )</td>
<td>( 2f_0 )</td>
</tr>
<tr>
<td><strong>2nd</strong></td>
<td>( L_2 )</td>
<td>( 3f_0 )</td>
</tr>
<tr>
<td></td>
<td>( H_2 )</td>
<td>( 4f_0 )</td>
</tr>
<tr>
<td><strong>...</strong></td>
<td><strong>...</strong></td>
<td><strong>...</strong></td>
</tr>
<tr>
<td><strong>Nth</strong></td>
<td>( L_N )</td>
<td>((2N-1)f_0)</td>
</tr>
<tr>
<td></td>
<td>( H_N )</td>
<td>( 2Nf_0 )</td>
</tr>
</tbody>
</table>

Hyperspace (product) vector, \( \prod_{r=1}^{N} X_r \) time complexity: \( O(N^2) \) \( O(2^{2N}) \)

Degenerate example: \( L_1 H_2 = H_1 L_2 \)

OK
Conclusions (why noise)

• Orthogonal noises form a freely available logic signal system (e.g. \( N \) resistors).

• In the brain logic scheme noise provides extraordinary resilience compared to periodic spikes.

• In (quantum-mimic) setting up the instantaneous hyperspace a sinusoidal hyperspace requires \( O(2^N) \) time complexity while the RTW-based scheme \( O(1) \).

• The FFT analysis of the sinusoidal hyperspace vector requires \( O(2^N) \) time complexity while the corresponding analysis of the RTW-based ones will require only an \( O(N) \) time complexity (Stacho, 2012).

• And a lot of open questions, including:

• Tamas Horvath (Fraunhofer, IAIS, Germany): "The connection between the expressive power of NBL and that of probabilistic Turing machines is an interesting open question for further research."
end of talk