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COMPARISON OF ENERGY REQUIREMENTS FOR CLASSICAL AND QUANTUM INFORMATION PROCESSING

Julio Gea-Banacloche
Department of Physics, University of Arkansas
Fayetteville, AR 72701, USA

Laszlo B. Kish
Texas A&M University, Department of Electrical Engineering
College Station, TX 77843-3128, USA

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By considering the energy requirements of quantum and classical computers we propose a criterion to separate the classical from the quantum regime and show that the classical scaling laws are much more favorable for conventional, general purpose computation.

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It is sometimes suggested, especially in the popular press, that quantum computers [1, 2] might be, in some sense, the natural successors of today's conventional digital computers, as the current trends in miniaturization reach the atomic level. While it is true that there are a few special tasks (notably, integer factoring [3, 4]) which a quantum computer could, exploiting some unique quantum mechanical effects, perform much faster than a classical computer, this does not address the question of whether it would actually make any sense to push conventional computers into the quantum domain, for anything other than these very special purpose tasks.

The energy dissipation and its relation to error-free operation has recently been identified as one of the most important problems in classical microprocessors [5]. Accordingly, in this note, we address the above issue by comparing the ultimate energy requirements of quantum and classical computers, based on our recent studies on classical [5] and quantum [6] systems, respectively. It was shown in [6] that the minimum error ϵ_q that is to be expected when an elementary logical operation is performed on a quantum computer satisfies

$$\epsilon_q > \frac{h}{E\tau} \tag{1}$$

Here h is Planck's constant, τ is the duration of the operation, and E is an amount of energy that needs to be put into the control system, and removed, after a time τ, in order to "switch" the state of the quantum bit, or qubit. If the control system is an oscillator (such as, e.g., an electromagnetic pulse) of carrier frequency f, typically with $f \gg 1/\tau$, then f must be used in place of $1/\tau$ in Eq. (1). The error in (1) is ultimately due to zero-point ("vacuum") noise in the control system; this is of the order of hf for an oscillator, and a minimum of h/τ for a "static" field, switched on and off over the time τ. (See [7,8] for some relevant discussions on the consequences of zero-point noise in electric circuits.)

By contrast, the fundamental noise limit for classical computers is set by thermal noise [5], which has an associated energy $kT/2$ at a temperature T (k is Boltzmann's constant). To prevent thermal noise from randomly flipping a bit, the excitation energy E needs to be made sufficiently large. Then, under some reasonable asumptions [5], the error probability is given by

$$\epsilon_c > \frac{2}{\sqrt{3}} \exp\left(-\frac{E}{kT}\right) \tag{2}$$

where E is the minimum energy dissipated in the classical circuit in the course of the operation. A comparison of Eqs. (1) and (2) immediately reveals two fundamentally different scaling laws: in the quantum case the error decreases only inversely as the energy used, in the classical case it decreases *exponentially*.

We suggest that these different scalings can be used to effectively characterize a quantum and a classical regime for a digital computer. The quantum regime corresponds to

$$hf > kT \tag{3}$$

and is entered when the vacuum or zero-point noise in the control system exceeds the thermal noise at the applicable temperature. Here f is the frequency of the control oscillator, for an oscillating field, or the gate frequency $1/\tau$ otherwise.

Clearly, by Eqs. (1) and (2), the error rate in the quantum regime will always be greater than in the classical regime, for the same energy. Conversely, the exponential scaling shows that it is energetically much more advantageous, in order to achieve a low error rate, to stay in the classical regime.

Two points need further elaboration. In [6], it was stated that the energy E appearing in Eq. (1) does not necessarily need to be dissipated, only needs to be put into and removed from the control system to switch the desired evolution on and off. This is punctiliously true, but it must be added that there are currently no plans, in any proposed quantum computer architecture, to "recycle" that energy, which is, in fact, effectively lost; nor is it clear how one could actually attempt to recycle it. A static field with a certain energy, for instance, could be switched on and off by charging and discharging a capacitor, and it is hard to see how one could prevent the required energy from being dissipated in the discharging circuit. Similarly, one could think of keeping electromagnetic pulses around for awhile, by bouncing them off of ultra-high reflectivity mirrors, but, in order to effectively reuse them, one would still need some kind of switch to send them off in the proper direction, at the right time, and that switch is again going to require an energy of the order given by Eq. (1) if it is to operate in a time τ with an error not exceeding ϵ_q. In short, there are strong indications that at least an amount of energy of the order of $h/\epsilon_q\tau$ must be dissipated per logical operation, if the switching mechanisms for the control fields in the quantum computer rely ultimately on conventional classical electronics.

In fact, the situation is similar in a classical microprocessor. The electric energy, which is stored in the MOSFET gate capacitors and is controlling the CMOS logic gate, could, in principle, be extracted by a proper LC circuitry and timed switches (the same technique that is applied in the "switching" power supplies of today's computers). The exception to this is a small part, of the order of kT, of the controlling electric energy. However, while energy dissipation problems are extremely serious in today's microprocessors [5], no initiative or realistic hope exists for extracting this energy in order to use it again. That would need an overhead circuitry of LC elements and switches which would be more complex than the original microprocessor, and its L elements would make it extremely bulky. Moreover, very similarly to the quantum case, because all the switches in this overhead circuitry would need similar controlling energy as the original CMOS gates, the efficiency of the energy-recycling operation would be strongly questionable. In conclusion, in both the classical and the quantum computers, it is only reasonable to consider the controlling energy pulses of gate operations as irretrievably lost, as we have done above.

The other point concerns quantum error correction [9], which can, in principle, reduce the overall failure probability for a logical operation on an encoded qubit, provided the individual logical operations can be carried out with an error probability smaller than a certain threshold ϵ_{th}. This reduction is superexponential for concatenated codes, n levels deep: $\epsilon_n = \epsilon_{th}(\epsilon_0/\epsilon_{th})^{2^n}$, whereas the energy cost of concatenation is only exponential in n: $E_n \geq (hf/\epsilon_0)N_c^n$, where N_c is the number of qubits in the base code (e.g., $N_c = 7$ for a popular code introduced by Steane [10]). This modifies the scaling (1), so that now one might have, for instance, for the 7-qubit code,

$$\epsilon_n = \epsilon_{th} \exp\left[-\left(\ln\frac{\epsilon_{th}}{\epsilon_0}\right)\left(\frac{\epsilon_0 E_n}{hf}\right)^{0.36}\right] \tag{4}$$

This is typically better than Eq. (1), but it is still worse than Eq. (2) because of the large upfront energy cost: one must start out with $E \geq E_0 = hf/\epsilon_0$, which is very large compared to kT (since $hf > kT$ in the quantum regime and $\epsilon_0 < \epsilon_{th} \leq 10^{-4}$). A further disadvantage is the sublinear growth of the argument of the exponential with E; in the above example, the number 0.36 in Eq. (4) comes from $\ln 2/\ln 7$, where 2 is the number of total single-bit errors that cause the code to fail, and 7 the number of physical qubits used to encode a logical qubit. It must be noted, however, that more efficient codes exist, and concatenation may not even be the best strategy for special-purpose applications; see [11] and references therein for very thorough discussions of these points. Nonetheless, the general qualitative differences between quantum and classical cases remain, broadly speaking, as indicated above.

Yet, even this fails to tell the whole story. The quantum error correcting codes considered in [9–11] are necessarily complex because they are designed to prevent errors fatal to *quantum* algorithms; but if all one wants to do is to run a *classical* algorithm on quantum hardware, one only needs protection against one kind of error (namely, bit flips), and this can be provided comparatively easily by the quantum generalization of classical repetition codes. Such codes would use $2n+1$ qubits to encode a single logical qubit, and provide protection against n bit-flip errors; if ϵ is the probability of a single-bit flip, the total failure probability would then scale as $\epsilon^{n+1} \sim \exp((n+1)\ln\epsilon)$, where n is of the order of the total energy, so for these codes one again would have an exponential decrease of the error probability with the energy dissipated, just as in the classical regime.

Nonetheless, and even though the threshold for these codes is very moderate, one still has to pay an upfront energy penalty, relative to the classical case, and this would still be substantial. For instance, according to Eq. (2), to achieve the current error rates of about 10^{-25} of conventional computers, it would theoretically be sufficient to dissipate an energy of about $58kT$, in the classical regime. In contrast, in the quantum regime, we find it would be near-optimum in this case to start out with a single-qubit failure probability ϵ of the order of 0.1 (and therefore a minimum energy, per logical operation on a single physical qubit, of about $10hf$), which requires a scale-up in computer size of about $2n+1 = 107$ (this follows from a careful calculation, keeping track of combinatorial factors), and thus a total energy of at least $1070hf$ per logical operation on an encoded qubit. With $hf > kT$, this is easily seen to be at least two orders of magnitude larger than the classical result.

There are, of course, other inconveniences that would arise from having to rely so heavily on error correction, such as the scale-up in the physical size of the computer, already mentioned, and additional cycles (and energy) lost to error diagnostic and correction.

All in all, our conclusion is that, from an energy-efficiency point of view, there seems to be no reason why one might want to run a conventional computation on "quantum hardware;" on the contrary, it appears to be energetically favorable to keep conventional, general-purpose computers away from the quantum regime whose characteristic scaling we have identified here. We should stress, however, that this logic need not apply to *special-purpose* quantum computers, such as those that might be built solely to, for instance, run the quantum factoring algorithm [3, 4]. We conjecture that whether such machines are ever built will probably depend, ultimately, on whether that specific application is still of strategic importance a decade or two from now.

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