Risk Measure Based Robust Bidding Strategy for Arbitrage Using a Wind Farm and Energy Storage

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Abstract—This paper proposes the use of a risk measure based robust optimization bidding strategy for dispatching a wind farm in combination with energy storage. Through coordination with energy storage devices, variable wind generators can be utilized as dispatchable energy producers in the deregulated electricity market. The total profit from sale of electricity can be increased by exploiting arbitrage opportunities available due to the inter-temporal variation of electricity prices in the day ahead market. A case study is presented to show that as the forecast error in electricity price increases, the robust optimization based bidding strategy has an increasing probability of yielding better economic performance than a deterministic optimization based bidding strategy. The uncertainty set for robust optimization is selected based on the coherent risk measure conditional value at risk (CVaR). Uncertainties in electricity price forecasting and wind power forecasting are considered. The resulting robust optimization based bidding strategy is evaluated using Monte Carlo simulation for different choices of uncertainty sets.

Index Terms—Bidding strategy, electricity market, energy storage, risk measure, robust optimization, uncertainty set, wind power.

I. INTRODUCTION

T HE INSTALLED capacity of renewable generation resources such as wind and solar in power systems is increasing in many countries. The variability and limited predictability of these renewable resources poses challenges to power system operations. Due to these characteristics, it is difficult for system operators to dispatch wind generators as they dispatch conventional generators. Storage technologies can help in firming the production of wind generation and provide benefits to the system over different time scales. For example, storage would enable wind generators to be dispatched, as well as allow them to participate in ancillary services such as frequency regulation [1].

Utility scale storage such as batteries and flywheels are now being deployed in power systems [2]. An important revenue stream for energy storage is energy arbitrage in short-term electricity markets, such as the day-ahead electricity market. Therefore, the combination of wind farms and energy storage can be dispatched using optimization based methods that maximize the economic returns. The self-scheduling problem of the combination of pumped storage and hydro plants in a market environment has been investigated [3], [4]. Castronuovo and Lopes [5] formulated the operation strategy for a wind generator combined with hydro pumped storage facility as a linear deterministic optimization problem. The value of combining wind farms with energy storage for energy arbitrage in short-term electricity markets has also been analyzed [6]. In addition to the uncertainty in electricity price forecasts, wind farms also experience difficulty in accurately predicting their production in a day-ahead market time-frame. Many researchers have proposed using the stochastic programming approach to deal with uncertainty in generator decision making [7]–[9]. However, the stochastic programming approach is computationally challenging due to the large number of scenarios that have to be considered. Additionally, stochastic programming also requires knowledge of the probability distribution of uncertain variables, which may not be available. More recently Robust Optimization (RO) has received attention as an alternate approach to deal with uncertainty in optimization problems. The RO approach has been applied across a variety of domains including portfolio optimization, supply chain management, network flows, circuit design, wireless networks, and model parameter estimation [10].

The main feature of the RO approach is that it uses a non-probabilistic approach to deal with the uncertainty. Uncertainty is addressed by constructing an uncertainty set and the solutions obtained are robust to all realizations of uncertain data within the defined uncertainty set. This definition of uncertainty leads to a more tractable problem. The question that arises in this regard is as to the selection of these uncertainty sets. One method that has been suggested to determine the uncertainty set is to use risk measures commonly used in finance industry [11]. In financial portfolio optimization the future values of the assets are uncertain, similarly in the generator scheduling problem the market clearing price of electricity in the day-ahead market is uncertain at the time of generator bidding. The uncertainty set can be determined based on a coherent risk measure such as conditional value at risk (CVaR) [12]. Consequently a robust optimization bidding strategy can be obtained based on the risk preference of the wind farm operator. Robust optimization solves for
the worst-case, consequently it will yield conservative results if the forecast errors are low. However, since the robust approach yields solutions that are immunized to all realizations of uncertain data within the uncertainty set, it maybe a suitable approach when forecast errors are high.

The main contributions of this paper are:

• to propose the use of risk measure based robust optimization bidding strategy, for energy arbitrage in the day-ahead electricity market, using the combination of a wind farm and energy storage.

• to verify through a case study that the robust approach has a higher probability of yielding better economic returns compared to a deterministic optimization approach, for a high forecast error in day-ahead electricity market clearing price.

The rest of this paper is organized as follows. Section II presents the background on robust optimization, risk measures used in finance industry, incorporating decision maker’s risk aversion and the risk measure based construction of uncertainty sets. Section III presents the formulation of the robust optimization based bidding strategy. Further, the decision making process for selecting the optimum bidding strategy, from the point of view of the wind farm operator, is presented. In Section IV, the economic performance of the robust optimization approach is compared to a deterministic optimization approach, using Monte Carlo simulation, for increasing forecast error in day-ahead electricity market price. Also, a case study is presented to illustrate the risk measure based robust bidding strategy for an energy arbitrage application using the combination of a wind farm and a generic energy storage. This is followed by concluding remarks and future work in Section V.

II. BACKGROUND

A. Robust Optimization

Robust optimization is a relatively recent technique for optimization under uncertainty. RO offers a non-probabilistic approach to deal with uncertainty through the use of an uncertainty set. The optimization solves for the worst case of uncertainty within the uncertainty set, hence the solution is feasible for all realizations of uncertain variables within the given uncertainty set. The other advantage is that for many classes of optimization problems the RO formulation is tractable [10].

Over the past few years researchers have proposed robust optimization approaches for power system applications [13]. Due to higher penetration of generation from variable renewable sources such as wind and solar, as well as increasing demand side participation enabled by smart grid technologies, unit commitment in deregulated electricity markets has become more challenging. Bertsimas et al. [14] propose a two-stage adaptive robust optimization model for the security constrained unit commitment problem in the presence of nodal net injection uncertainty. The method used is based on Benders’ decomposition and the level of conservatism of the solution is controlled by an uncertainty budget. Similarly [15], [16] and [17] also apply robust optimization for the unit commitment problem, with the uncertainty set determined by an uncertainty budget. Jiang et al. [18] propose a method to provide a robust unit commitment schedule for thermal generators in the day-ahead market with wind power fluctuations. Zheng et al. [19] propose applying robust optimization to economic dispatch in addition to the unit commitment problem.

The uncertainty set can be based on some historical information about the values of the uncertain parameters. One way that has been suggested is based on the correspondence of the uncertainty set with risk measures commonly used in finance [11], [12]. In this paper we apply the risk measure approach to determine the uncertainty set for robust optimization based bid scheduling for a wind farm combined with energy storage.

B. Risk Measures

In the field of finance the portfolio allocation problem is an optimization problem where the uncertain coefficients are the future asset returns. Risk measures are used to quantify the likelihood and size of potential losses. A risk measure is effectively a mapping from a set of random variables (e.g., portfolio returns) to the set of real numbers. The aim of the portfolio optimization is to find the minimum risk portfolio in the set of feasible portfolios. Analogous to this, the bid scheduling problem for the combination of energy from wind farm and storage device can also be framed as an optimization problem. The aim is to maximize the profit from sale of electricity under uncertainties in electricity price forecasting and wind power forecasting.

1) Value at Risk (VaR): VaR gives the monetary risk associated with the bid schedule of the combination of the wind farm and energy storage, due to uncertainties in the forecasts. VaR is computed as the maximum profit over a target time horizon such that the probability of the profit being less than or equal to this value is less than or equal to $1 - \beta$ [20].

Given a confidence level $\beta \in (0, 1)$ and the normally distributed random variable, profit, Value at Risk for the operating day is defined as

$$ VaR_\beta(\text{profit}) = \max \{ t | Pr(\text{profit} \leq t) \leq 1 - \beta \} \quad (1) $$

The main disadvantages of VaR are that (i) it does not capture tail cases, and (ii) VaR is not coherent.

2) Coherent Risk Measures: A risk measure $\rho(\cdot)$ is coherent if it satisfies the following four conditions [21].

1) Sub-additivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$

2) Homogeneity: For any $\xi \geq 0$, $\rho(\xi X) = \xi \rho(X)$

3) Monotonicity: $\rho(X) \leq \rho(Y)$ if $X \geq Y$

4) Translational invariance: $\rho(X + c) = \rho(X) - c$, for any constant $c$

Convexity, defined as $\xi \in [0, 1]$, $\rho(\xi X + (1 - \xi) Y) \leq \xi \rho(X) + (1 - \xi) \rho(Y)$ follows from above conditions. The main consequence of coherency is the preservation of convexity, which in turn implies computational tractability of optimization [12], [22].

3) Conditional Value at Risk (CVaR): CVaR is defined as the conditional expectation of the profit, given that the profit is less than or equal to the VaR value. Thus given a confidence level $\beta \in (0, 1)$,

$$ CVaR_\beta(\text{profit}) = E[\text{profit} | \text{profit} \leq VaR_\beta] \quad (2) $$
Therefore CVaR is the expected value of the worst \(1 - \beta\) cases of profit. Compared to VaR, CVaR is a conservative risk measure since it captures the tail of the probability distribution of profit. The main advantage of CVaR is that it is coherent [23]. This addresses the motivation of using the CVaR risk measure in this paper. CVaR being a coherent risk measure preserves the convexity of the robust counterpart to the linear optimization problem with uncertain data. Therefore, the resulting robust optimization problem is tractable.

C. Decision Maker’s Risk Aversion

CVaR captures the tail of the bidding profit scenarios as specified by a confidence level \(\beta\). Therefore for a given confidence level CVaR can be used as a performance measure to compare different bidding strategies. By changing the confidence level the conservatism level of this performance measure can be adjusted. For instance if we take \(\beta = 100\%)\) then CVaR gives the mean for all profit scenarios. Thus the confidence level \(\beta\) can be used to represent the decision maker’s risk aversion. A more risk averse decision maker may choose a larger value of \(\beta\), while a risk tolerant decision maker may choose a smaller value of \(\beta\).

D. Risk Measures and Uncertainty Sets

Expressing the decision maker’s risk preference as a coherent risk measure, allows us to formulate the optimization problem with uncertain data as a robust optimization problem with a convex uncertainty set. Example 3.2 in Bertsimas and Brown [11] specifies the link between CVaR risk measure and polyhedral uncertainty sets, as follows.

Given \(N\) samples of data i.e., \(\{a_1, \ldots, a_N\}\), the uncertainty set for the uncertain vector \(\tilde{a}\) corresponding to CVaR is

\[
\mathcal{U} = \text{conv} \left( \left\{ \frac{1}{1 - \beta} \sum_{i \in I} p_i a_i + \left( 1 - \frac{1}{1 - \beta} \sum_{i \in I} p_i \right) a_j : I \subseteq \{1, \ldots, N\}, j \in \{1, \ldots, N\} \setminus I, \sum_{i \in I} p_i \leq 1 - \beta \right\} \right)
\]  

(3)

If we assume the probability distribution of data samples \(a_i\) as \(p_i = 1/N, \forall i\) and also take \(1 - \beta = j/N\), then for some \(j \in \mathbb{Z}_+\) this has the interpretation of the convex hull of all \(j\)-point averages of matrix \(A = [a_1, \ldots, a_N]\).

Without loss of generality this result can also be used to find the uncertainty set for cost coefficients of the optimization problem [11]. Thus we can find the uncertainty set for price \(\hat{\lambda}\) based on wind farm operator’s choice of \(\beta\) and the historical data of prices represented by the vectors \(\lambda_i, i = 1, \ldots, N\). The choice of \(\beta\) affects the size of the uncertainty set. Namely a larger \(\beta\) yields a larger uncertainty set, while a smaller value of \(\beta\) yields a smaller uncertainty set.

III. FORMULATION

The bid scheduling for the combination of a wind farm and energy storage device is formulated as a robust linear optimization problem which aims to maximize the total profit from sale of electricity in the day ahead market [24]. The inputs are the forecasts of electricity prices and wind farm power production, whereas the outputs are the hourly power injection profiles of the wind farm and the energy storage. The bids comprising of the hourly power injection totals of the wind farm and energy storage device are submitted to the market. Upon market clearing the system operator sends the dispatch signal comprising of successful injection bids, to the wind farm operator (Fig. 1). The wind farm and energy storage inject power to the grid to match the dispatch commands. The bidding strategy leverages the storage device by exploiting the arbitrage opportunities, available due to inter-temporal price variations. The abbreviation \(DKK\) is used for Danish Kroner. The nomenclature used is given in Table I.

A. Robust Optimization Bidding Strategy

The objective function and constraints for the robust optimization bidding strategy are as follows.

\[
\begin{align*}
\min_{P_s^d(k), P_s^e(k), P_e^d(k), P_e^c(k), E_s(k)} & \sum_{k=1}^{N} \left[ -\hat{\lambda}(k) (P_e^d(k) - P_s^d(k)) + \lambda(k) (P_s^e(k) - P_e^c(k)) \right] \\
+ C_u P_u^d(k) + C_s \left( P_s^d(k) + P_e^c(k) \right) + C_e E_s(k) 
\end{align*}
\]  

(4)
s.t.

\[-P^r_w \leq \hat{P}^a_w(k) - \bar{P}^a_w(k - 1) \leq P^r_w \] (5)

\[0 \leq \bar{P}^a_w(k) \leq \hat{P}_w(k)\] (6)

\[E^\text{max}_s(k) \leq E_s(k) \leq E^\text{min}_s \] (7)

\[0 \leq P^r_s(k) \leq P^\text{max}_s\] (8)

\[0 \leq P^d_s(k) \leq P^\text{max}_s\] (9)

\[E_s(k) = E_s(k - 1) - \frac{1}{\eta_d} P^d_w(k) + \eta_c P^r_s(k) \]

\[-\left(\hat{P}_w(k) - \bar{P}^a_w(k)\right)\] (10)

where \(\lambda\) is the uncertain electricity price variable, \(\hat{P}_w\) is the uncertain available wind power, \(\mathcal{U}\) is the uncertainty set for electricity price, and \(\mathcal{V}\) is the uncertainty set for available wind power.

The objective function (4) consists of revenue from electricity sales from both the wind farm and storage, marginal cost of wind generation, degradation costs associated with charging and discharging of storage device, and energy storage operation costs. By minimizing the negative of the total profit, (4) effectively maximizes the total profit. The inter-temporal ramping constraint for the wind farm is given by (5). The upper limit on the wind farm's power injection to the grid is specified by (6). The energy, charging and discharging power limits of the storage device are specified by (7)–(9). The dynamic equation for the storage device is (10). In the implementation code, in order to maintain tractability, this equality constraint is converted into two inequality constraints with small tolerances. The term \(\hat{P}_w(k) - \bar{P}^a_w(k)\) is the firming power provided by storage to compensate for wind power forecast errors. It is assumed that the storage device can only be charged using the wind power production and not by the grid (Fig. 1). This assumption is used since we are interested in analyzing the impact of storage on the utilization of wind resource. The power loss in storage charging and discharging is a function of \(\eta_d\) and \(\eta_c\), the discharging and charging efficiencies of the storage device respectively. The roundtrip efficiency of the storage device can be taken as the product of these two values, i.e., \(\eta = \eta_d\eta_c\).

The problem (4)–(10) is of the form

\[\min \; \tilde{c}^T x \]

s.t. \(Ax \leq \tilde{b}\)

\(x \in X, \tilde{c} \in \mathcal{U}, \tilde{b} \in \mathcal{V}\) (11)

where \(x\) is the vector of decision variables, \(\tilde{b}\) and \(\tilde{c}\) are vectors of uncertain data. The uncertain problem (11) can be reformulated as

\[\min \; t \]

s.t. \(t - \tilde{c}^T x \leq 0\)

\(Ax - \tilde{b}y \leq 0\)

\(x \in X, y = 1, \tilde{c} \in \mathcal{U}, \tilde{b} \in \mathcal{V}\) (12)

When the uncertainty sets are polyhedral they can be represented by matrix inequalities. Thus (12) can be written in the min-max form as

\[\min \; f^T w \]

s.t. \(\max g^T w \leq h\)

\(D_j g_j \leq d_j\) (13)

Taking the dual of the inner maximization subproblem we get

\[\min \; f^T w \]

s.t. \(p^T d_i \leq h_i\)

\(p^T D_i - w \)

\(p_i > 0\) (14)

Thus the original problem is transformed into a tractable linear programming formulation.

B. Decision Making Process for Wind Farm Operator

In this section the decision making process from the perspective of a price taking wind farm operator for participating in the day ahead electricity market is discussed. We propose an off-line decision to choose the optimization model based on Monte Carlo simulation which makes use of historical data on the uncertain variables, followed by an online decision which uses the chosen optimization model and the forecasts of wind power and electricity price to yield the bidding strategy. The flowchart for this decision making process is shown in Fig. 2.

1) We assume that historical data on wind power forecast error and electricity price forecast error is available to the decision maker. This data can be used to estimate the type of probability distribution in forecast error and the worst case of uncertainty that may be experienced by the wind farm operator.

2) Next the performance of the optimization based bidding for multiple values of the parameter \(\beta\) are compared using Monte Carlo simulation. The considered optimization models may range from deterministic (\(\beta = 0\%\)) to worst-case robust (\(\beta = 100\%\)). Based on the profits obtained in the Monte Carlo runs CVAR can be estimated for each model in order to compare their performance.

3) The decision maker selects the optimization model based on both the relative performance as obtained from Monte Carlo simulations as well as the decision maker's confidence in the forecast. The decision maker may use historical data on forecast error to decide the confidence level in a given forecasting method. If the decision maker has a high level of confidence in the forecast it may not make sense to use a more robust approach since it would be too conservative. However, if the decision maker believes that the price forecast error is likely to be high, choosing a more robust approach, i.e., a higher value of \(\beta\) may be appropriate. A risk averse decision maker may choose a more robust approach even when the price forecast error is anticipated to be medium, in order to minimize potential loss of revenue.
in case the actual price forecast error is higher than anticipated. The uncertainty set associated with the particular choice of $\beta$ can be obtained as described in Section II-D.

4) Once the optimization model is chosen, the decision maker can use the dashboard tool to determine the optimum bidding strategy. The uncertainty set for wind power can be selected based on probabilistic forecasts. The dashboard uses the forecasts of the day-ahead electricity prices and wind power production for the operating day as inputs in the optimization.

Thus the optimization based bidding strategy for the combination of wind farm and energy storage can be obtained.

C. Dashboard Tool for Bidding Strategy Selection

In order to facilitate bidding strategy selection, a software tool with a dashboard GUI will be made available to the decision maker (Fig. 3). This software tool allows the decision maker to use historical data in an offline study to estimate the type of probability distribution of the forecast error as well as the worst case of uncertainty that the wind farm operator may experience. The tool then compares the performance of the optimization model for different values of the parameter $\beta$, based on Monte Carlo simulation, and determines the uncertainty set. Thus the uncertainty set is selected through an offline process. Next in the online phase, the optimization routine is run to determine the bidding strategy for the wind farm and energy storage device for the day-ahead electricity market. The inputs are the wind power forecast and electricity price forecast for the operating day.

IV. CASE STUDY

In this section, a case study is presented in which the economic performance of the robust optimization based bidding strategy is evaluated. For simplicity, we assume that all the hourly bids submitted by wind farm and energy storage combination to the market operator are successful. The characteristics of the wind farm and a generic energy storage device are presented in Table II.

A. Performance of Robust Optimization Bidding Strategy

The performance of the robust optimization bidding strategy relative to the deterministic approach is analyzed for different levels of electricity price forecast error. The deterministic model, which is presented in [24], uses the point forecasts of the inputs in the optimization to find the bid schedule. Both the robust and deterministic optimization based strategies are obtained for a set of price forecasts with error measured in Mean Absolute Error (MAE). MAE is the unweighted average of the absolute values of the forecast errors.

$$\text{Mean Absolute Error (MAE)} = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i|$$

To evaluate the performance of each bidding strategy, Monte Carlo simulation is used. A number of scenarios of actual electricity price (1000 scenarios) are considered. The result we are interested in is the number of scenarios where the profit from robust optimization (RO) based bidding strategy is greater than that from the deterministic optimization (DO) based bidding strategy. The results for several days are analyzed (Fig. 4). In all the cases for an increase in price forecast error there is a corresponding increase in the probability of getting better results.
using robust optimization compared to using deterministic optimization.

It has been observed that day-ahead wind power forecast error as a percentage of installed capacity has MAE in the range of 15%–25% for a single wind farm [25], [26]. Wind farms that want to bid into day-ahead market have a serious problem of dealing with the uncertainty due to high wind power production forecast errors. Further the electricity price forecast for the day ahead time horizon can have an error of up to around 15% [27]. The robust optimization approach can be used to manage the uncertainty due to high day-ahead forecast error, and obtain better economic performance compared to the deterministic optimization approach.

Bertsimas et al. [28], [29] address the issue of the performance guarantee of the robust optimization model versus the budget of uncertainty. Based on their approach we consider the problem (12) and assume that each of the \( n \) elements of the vector of uncertain cost coefficients \( \bar{c}_i \) belongs to a symmetric interval \([\bar{c}_i - \delta_i, \bar{c}_i + \delta_i] \) centered at the point forecast value \( \bar{c}_i \) with maximum deviation \( \delta_i \). The parameter \( \Gamma \in [0, n] \) is defined by \( \sum_{i=1}^{n} (\delta_i / \bar{c}_i) \leq \Gamma \), and is called the budget of uncertainty of the cost coefficients. \( \Gamma \) is the upper bound of the aggregate scaled deviations of the actual values of the coefficients from their point forecast values, and thus can be used to represent the accuracy of forecasting. The key result from [28] can be summarized as follows. For the robust optimization problem let \( x^* \) be an optimal solution and \( t^* \) the optimal objective function value, then \( P_r(t^T x^* < t^*) \leq \epsilon \) if \( \Gamma \) is chosen to be \( 1 + \Phi^{-1}(1 - \epsilon) \sqrt{n} \), where \( \epsilon \in (0, 1) \), \( n \) is the number of uncertain variables (here \( n = 24 \) hours since we consider one day) and \( \Phi \) is the cumulative distribution function of the standard Gaussian random variable. Thus the actual value of the profit from bidding will exceed the predicted value with probability at least equal to \( 1 - \epsilon \), and this value is the performance guarantee. Fig. 5 shows the theoretical performance guarantee of the robust optimization model, under above assumptions on uncertainty in coefficients, for different values of the budget of uncertainty [29].

B. Results of Robust Optimization Bidding Strategy

The robust optimization given in Section III yields the power injection profile for the wind farm and storage device for each hour of the entire day. Based on these injection profiles the bids, which are the hour-by-hour total power injection of the wind farm and energy storage combination, are obtained. The bids for the day-ahead electricity market have to be submitted 12 hours before the beginning of the operating day. Electricity price data from Nordpool for West Denmark is used for the simulations. It is assumed that during actual operation any excess wind generation is sold in the hour-ahead market, whereas any deficit has to be purchased from the hour-ahead market at the clearing price for that hour. The total profit from electricity sales is calculated based on the actual values of market clearing price. This settlement is done after the end of the operating day. The robust optimization problem is implemented and solved in MATLAB using linprog solver and the YALMIP toolbox [30].

In order to analyze the performance of the optimization based bidding strategy, Monte Carlo simulation method is used for a what-if type analysis based on assumptions about forecast errors. The Cauchy distribution is considered to be a reasonable model for the distribution of wind power forecast error [31] and electricity price forecast error [32]. Based on historical data consisting of 3 months of hourly forecasts and actual values of market clearing price and wind power, we also find that using Cauchy distribution to model the errors is a reasonable assumption (Figs. 6 and 7). Therefore for both the wind farm power production and the electricity price an error is generated at random for each hour of the day by sampling a Cauchy distribution within bounds defined by 90% confidence interval. For each hour of the day the realization of the actual value of the input quantity (i.e., wind farm power production and electricity price) is obtained by subtracting the error from the forecast value. In this manner \( M = 1000 \) scenarios of actual wind farm power production and electricity price are generated for the given day using random sampling. For each scenario based on the bids obtained from the robust optimization the profit for the operating day can be calculated. These 1000 values are assumed...
to be independent identically distributed (i.i.d.) observations of the profit. Based on the order statistics, the VaR and the CVaR can be estimated from these observations [33].

In this case study the robust bidding strategy is analyzed for one operating day. The forecasts and actual data for the electricity price and wind power production are provided by Vestas. Fig. 8 shows the forecast and actual hourly electricity prices for the day-ahead market, as well as the actual prices in the hour-ahead market for the given day. The error between forecast and actual day-ahead price is high in hours 17 and 21. Table III shows the results of the evaluation of the robust bidding strategy using Monte Carlo simulation, for different uncertainty sets based on wind farm operator’s choice of parameter \( \beta \). Table IV shows the 95% CVaR values for different uncertainty sets and forecast errors in prices measured in MAE%. The robust bidding strategy for the combination of the wind farm and storage corresponding to \( \beta = 10\% \) and \( \text{MAE} = 7.34\% \) is shown in Fig. 9. In hours 3–5 when the forecast price is low, part of the wind energy is used to charge the storage device. In hour 11 when the forecast price is high the stored energy is injected into the grid. Therefore the storage device can be used to take advantage of arbitrage opportunities that result from temporal variations in the electricity price.

The simulation is conducted on a laptop with Intel Core 2 Duo 2.2 GHz CPU with 4 GB of RAM. It takes 13.3 seconds to generate the Monte Carlo scenarios. For each value of \( \beta \) the robust optimization and evaluation takes on average 7.2 seconds.

In order to determine uncertainty sets for wind power production around the deterministic forecast we use probabilistic forecasts. Percentiles of a probabilistic forecast are usually defined such that the probability of wind power production being
TABLE IV
CVaR for Combinations of $\beta$ and MAE

<table>
<thead>
<tr>
<th>MAE (%)</th>
<th>$\beta=10%$</th>
<th>$\beta=5%$</th>
<th>$\beta=1%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.34</td>
<td>46,958</td>
<td>46,959</td>
<td>46,960</td>
</tr>
<tr>
<td>8.35</td>
<td>47,000</td>
<td>47,001</td>
<td>47,003</td>
</tr>
<tr>
<td>10.26</td>
<td>47,077</td>
<td>47,078</td>
<td>47,078</td>
</tr>
<tr>
<td>10.93</td>
<td>47,267</td>
<td>47,268</td>
<td>47,269</td>
</tr>
</tbody>
</table>

Fig. 10. Wind deterministic and percentile forecast.

<table>
<thead>
<tr>
<th>Uncertainty Set for Wind (percentile bands)</th>
<th>95% CVaR (DKK)</th>
<th>95% Mean Profit (DKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% – 60%</td>
<td>44,581</td>
<td>43,865</td>
</tr>
<tr>
<td>30% – 70%</td>
<td>41,249</td>
<td>40,507</td>
</tr>
<tr>
<td>20% – 80%</td>
<td>36,012</td>
<td>35,300</td>
</tr>
</tbody>
</table>

TABLE V
Results of Monte Carlo Simulation for Wind Uncertainty Sets

less than the value given by the $\beta$ percentile forecast is $\beta$ percent [34]. Fig. 10 shows the deterministic forecast, actual wind power production, and the probabilistic forecasts for 40 and 60 percentiles for the given day. Pairs of probabilistic forecasts are taken as the lower and upper bounds for the uncertainty set for wind power. Table V shows the results of the Monte Carlo simulation, with different uncertainty sets for wind power, based on pairs of probabilistic forecasts. Thus the optimal robust bidding strategy is obtained which considers uncertainty in both electricity price and wind power (Fig. 11).

Finally we present a comparison of the robust optimization approach to the stochastic optimization approach. For the stochastic model we solve the expected value problem using the sample average approximation method [35]. This involves solving a large deterministic problem using the Monte Carlo method. With $N_s = 100$ samples the stochastic approach yields a mean profit of DKK 51,164 and takes 714.2 s to solve. Whereas for $\beta = 10\%$ the robust approach yields a mean profit of DKK 50,775 and takes 20.4 s to solve. The problem size of the stochastic approach increases linearly with $N_s$ and the computation time would be very large for a large sample size, whereas the robust approach is comparable to the deterministic in computational effort required.

V. CONCLUSION

This paper presents a robust optimization approach, with uncertainty set based on risk measure used in finance industry, to obtain optimum bidding strategy for a wind farm and energy storage in a day-ahead electricity market. Robust optimization based strategy is seen to have increasing probability of yielding better economic performance than the deterministic approach as the forecast error in electricity price increases. This is important because wind producers who bid into day-ahead electricity markets have to deal with uncertainty due to large forecast errors. The combination of wind power and energy storage leads to better utilization of the uncertain wind resource and increased economic performance through use of energy arbitrage. The economic performance of the bidding strategy is evaluated using Monte Carlo simulation by making suitable assumptions about the probability distribution of the electricity price and wind power forecast errors.

Future work could include the integration of robust optimization with a model predictive approach for hour-ahead and real-time markets. In order to improve the utilization of the renewable resource other applications of the wind and energy storage combination, including ancillary services such as frequency regulation could also be considered.

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THATTE et al.: RISK MEASURE BASED ROBUST BIDDING STRATEGY FOR ARBITRAGE USING A WIND FARM AND ENERGY STORAGE


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