Voltage Security Constrained Look-ahead Coordination of Reactive Power Support Devices with High Renewables

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Abstract—Reactive power support devices (RPSDs) such as capacitor banks and Static Var Compensators (SVCs) play a pivotal role in ensuring system voltage security. In large power systems with high penetration of renewables, the variability brought about by the renewables requires more coordination among these reactive power support devices. In this paper, we investigate the benefits of look-ahead coordination of both continuous-state and discrete-state RPSDs across multiple control areas. The objective is to coordinate multiple RPSDs in anticipation of near-term net load variations in order to minimize the costs of RPSD operations and transmission losses. Constraints include ensuring voltage security with respect to a set of possible contingency scenarios. This problem is first formulated as a Mixed Integer Non-Linear Programming (MINLP) problem and then approximated as a Mixed Integer Quadratic Programming (MIQP) problem via power flow Jacobian matrix. The validity of the solution to the MIQP problem is verified by solving AC power flow equations. The proposed approach is examined on the IEEE 24-bus Reliability Test System (RTS) system. Critical discussions on the impacts of wind uncertainties are provided, and a linear approximation approach is proposed to estimate the impacts of wind fluctuations on voltage magnitudes.

I. INTRODUCTION

Renewable energy, which is becoming a major component of power resources, is highly variable with limited predictability. In many regions such as Texas, renewables (primarily wind) are located far from the load centers. The long distance between renewable generation and loads, compounded with the fact that such renewables are highly variable, lead to increasing concerns of maintaining voltage security for operations.

This paper is motivated by the need to coordinate these reactive power support devices (RPSDs) in order to ensure voltage security with deep renewable penetration. The particular set of RPSDs discussed in this paper include both continuous-state devices (e.g. SVCs) and discrete-state devices (e.g. capacitor banks). In practice, many of these devices are operated by multiple transmission owners. The decision of each control area is often not coordinated. This, in turn, leads to unnecessary frequent operations of RPSDs in large power systems. With advances in sensing, communication, and computing, the forecast of renewables in the near-term operation is improving. Such improvement could be leveraged by the system operator for more economic scheduling of RPSDs to ensure security. In this paper, we examine the potential of look-ahead operation of the RPSDs across large geographical areas with high renewables.

There has been a substantial body of literature examining the impact of renewable penetration on system voltage profiles. Reference [1] presents four case studies of European countries and discuss system stability issues due to fluctuating renewables. Many papers formulate an optimal reactive power dispatch (ORPD) problem to solve the voltage problem induced by high penetration of wind power [2], [3], [4]. The ORPD problem often aims at finding optimal settings of current installed RPSDs to ensure system voltage constraints [5].

There are in general two families of voltage constraints: (1) voltage stability constraints; and (2) voltage security constraints. Stability of a power system refers to the continuance of intact operation following a disturbance. It depends on the operating condition and the nature of the physical disturbance [6]. Voltage security constraints often require a particular form of voltage stability index larger or smaller than a threshold. Some typical voltage stability index include the distance to the nose point of the PV curve [7] and smallest singular value of the power flow Jacobian matrix [8]. Security of a power system refers to the degree of risk in its ability to survive imminent disturbances (contingencies) without interruption of customer service. It relates to robustness of the system to imminent disturbances and, hence, depends on the system operating condition as well as the contingent probability of disturbances [6]. Voltage security constraints typically require the voltage magnitudes within desired ranges under a set of plausible contingency scenarios [9].

As an effort to mitigate the impacts of renewables on voltage stability and security, many papers suggest the use of RPSDs [2]. However, [5] points out two major shortcomings of current ORPD literatures:

1) Because of the low computational burden, some ORPD problems only determine the schedules of continuous RPSDs or the discrete decision variables are relaxed to be continuous. This lack of coordination might cause troubles and it is necessary to formulate a comprehensive coordination framework including all the devices of the system;

2) Most of the proposed approaches focus on a single snapshot coordination of RPSDs. The past or future states of the system are not taken into account. Given the
increasing inter-temporal variability from renewables, it becomes more and more important to establish a look-ahead framework to schedule all the RPSDs. Detailed models of the operation costs of RPSDs are necessary in a look-ahead framework.

There has also been efforts developing multi-period coordination of voltage support devices. Reference [5] formulates a multi-objective mixed integer nonlinear programming (MINLP) problem and utilizes generalized Bender’s decomposition to find out the optimal switching pattern of discrete voltage controllers and ensuring the voltage security of the system. Reference [10] proposes a three-stage coordination framework to minimize total lines loss and number of control actions. The problems solved in both [5] and [10] are MINLP problems.

In this paper, we formulate the problem of look-ahead coordination of RPSDs in line with [5]. The proposed framework considers the operation cost of discrete control devices and transmission losses and guarantees the system voltage security with respect to \( N - 1 \) contingencies. Instead of solving a MINLP problem as in [5] and [10], we solve the linearized problem, which provides insights into the operations of RPSDs and is much more computationally efficient and provides insights into the operations.

The rest of this paper is organized as follows: Section II formulates the look-ahead coordination of RPSDs as a MINLP problem. The main difficulties and linearized problem are presented in Section III. Case studies and further discussions are provided in Section IV. Section V examines the impacts of wind uncertainties and Section VI presents concluding remarks and future works.

II. LOOK-AHEAD ORPD

A. Background

Independent System Operators (ISOs) have the overall responsibility of monitoring and maintaining voltage security over its footprint, which often is comprised of multiple control areas. As an example, the Electric Reliability Council of Texas (ERCOT) manages its voltage issues with multiple entities as elaborated in Fig. 1.

![Fig. 1. Architecture of Voltage Management at ERCOT [11]](image)

ISOs typically perform voltage security screening studies focusing on the next few hours or days by looking at the estimates of voltage obtained from the supervisory control and data acquisition (SCADA) system. If any bus voltage goes beyond the predetermined limits, ISOs will assess the real-time voltage stability and determine necessary corrective operations.

In practice, the control of reactive power support devices such as capacitor banks are often decided at multiple transmission service providers (i.e. control areas). Given their limited information about the entire system, it often leads to unnecessarily volatile switches of these capacitor banks.

In Section II-B, we propose a coordination framework for the operation of reactive devices. The ISO coordinates all the reactive power devices and figure out the most efficient solution to guarantee the security of the system.

B. Problem Formulation

\[
\min \sum_{t=1}^{T} \left( h_B(\text{Q}_B[t]) + \lambda[t] \sum_{t=0}^{n_h} \omega^t \text{P}_L^c[t] \right) \quad (1a)
\]

s.t. \( P^c[t] = A^c_G(\text{P}_G[t] + n^t \text{P}_S^m[t]) + \text{A}_W \text{P}_W[t] - \text{A}_D \text{P}_D[t] \quad (1b) \)

\( Q^c[t] = A^c_CQ^c_C[t] + A^c_CQ^c_C[t] + \text{A}_W \text{Q}_W[t] - \text{A}_D \text{Q}_D[t] \quad (1c) \)

\( P^c_L[t] = \sum_{i=1}^{n_h} |V_i^c[t]| V_j^c[t] Y_{ij}^c \cos(\theta_i^c[t] - \theta_j^c[t] - \phi_{ij}) \quad (1d) \)

\( Q^c_L[t] = \sum_{i=1}^{n_h} |V_i^c[t]| V_j^c[t] Y_{ij}^c \sin(\theta_i^c[t] - \theta_j^c[t] - \phi_{ij}) \quad (1e) \)

\( P^c_i[t] = \sum_{t=1}^{T} \eta_t |V_i^c[t]|^2 + \sum_{t=1}^{T} \eta_t |V_j^c[t]|^2 - 2|V_j^c[t]|V_i^c[t] \cos(\theta_i[t] - \theta_j[t]) \quad (1f) \)

\( |V_i^c[t]| \leq |V_i^{c+}[t] - |V_i^{c-}[t] \quad (1g) \)

\( Q_G^c \leq Q_C^c[t] \leq Q_N^c \quad (1h) \)

\( Q_B^c[t] \in \{0, Q_B^c\} \quad (1i) \)

\( (S_G^c)^2 \leq (P_G^c[t] + n^t \text{P}_S^m[t])^2 + (Q_C^c[t])^2 \leq (S_N^c)^2 \quad (1j) \)

\( i,j = 1,2,\ldots,n_b, \quad c = 0,1,2,\ldots,n_c, t = 1,2,\ldots,T \)

Problem (1) aims at finding the most smoothed and economic operation schedule of the reactive power support devices (RPSDs) in the upcoming \( T \) snapshots while ensuring voltage security in \( n_c \) contingency scenarios. Variables with \( t \) are in snapshot \( t \), and those with superscript \( c \) belong to contingency scenario \( c \). Decision variables include the operating states of discrete RPSDs (e.g. shunt capacitors) \( Q_B^c[t] \), operating states of continuous RPSDs (e.g. SVCs) \( Q_C^c[t] \) and the voltage set-points of generators (i.e. voltage magnitudes \( |V_i^c[t]| \) of PV buses). The reactive generation \( Q_C^c[t] \) is controlled by the automatic voltage regulators (AVRs) to maintain their bus voltages at desired levels.

Eqn. (1b)-(1c) are the real and reactive power balance equations at each bus. \( P^c_i[t] \) and \( Q^c_i[t] \) denote the real and reactive nodal injection. \( A_D \in \mathbb{R}^{n_b \times n_b}, A_C \in \mathbb{R}^{n_b \times n_C}, A_D \in \mathbb{R}^{n_b \times n_D}, A_C \in \mathbb{R}^{n_b \times n_C} \) and \( A_W \in \mathbb{R}^{n_b \times n_W} \) are adjacency matrices. If component \( k \) is connected with bus \( i \), then \( A(i,k) = 1 \); otherwise \( A(i,k) = 0 \). Eqn. (1e)-(1f) are the power flow equations. \( |V_i^c[t]| \) and \( \theta_i^c[t] \) are the voltage magnitudes and angles of bus \( i = 1,2,\ldots,n_b \). \( Y_{ij}^c \in \mathbb{R}_{ij} \) represents the component related with line \( (i,j) \) (bus \( i \) to bus \( j \)) in the admittance matrix \( Y \).
There are $n_c$ contingency scenarios\(^1\) being considered in Problem (1). More specifically, we focus on the $N - 1$ contingency of losing generators\(^2\). The contingency of losing one generator is modeled through the adjacency matrix of generators $A_G$. Let $A_G^c$ denote the adjacency matrix in the normal condition. The impacts of losing generator $g$ (in scenario $c$) is equivalent with setting the $g$th column of $A_G^c$ to be zeros, and the new matrix is denoted by $A_G^c$ ($c = 1, 2, \ldots, n_c$).

The real power imbalance $P_{G}^c[t]$ due to contingency $c$ is proportionally allocated to each generator (i.e. $P_{G}^c[t] + \eta^c P_{c}^r[t]$). The participating factor $\eta^c$ is a pre-defined vector and determined by the characteristics of generators. It is worth mentioning that $1^T \eta^c = 1$ and $1^T A_G^c \eta^c = 1$, this guarantees the post-contingency balance of real power:

$$1^T(A_G^c P_{G}^c[t] + \eta^c P_{c}^r[t]) + A_W P_W[t] = 1^T A_G^c \eta^c 1^T(A_D P_D[t] - A_G^c P_{G}^c[t] - A_W P_W[t]) = 1^T A_D P_D$$

Eqn. (1) depicts the voltage security constraints. Voltage magnitudes will be maintained within the allowed ranges $[|V^-|, |V^+|]$ for each contingency scenario $c$. In this paper, we use $[0.95, 1.05]$ for normal operation analysis ($c = 0$) and $[0.9, 1.1]$ for contingency analysis ($c = 1, 2, \ldots, n_c$)\(^3\).

Other constraints include the capacity of devices (Eqn. (1i), (1j))\(^4\). Eqn. (1k) represents the generation capacity limits $[13, 14]$. Since $P_{G}^c[t], \eta^c$ and $P_{c}^r[t]$ are all parameters, Eqn. (1k) is equivalent with the following linear inequality:

$$Q_{G}^- \leq Q_{G}^c[t] \leq Q_{G}^+$$

(3)

\[ C. \text{ On Objective Function} \]

The objective function Eqn. (1a) is time-coupled and includes cost of line losses and the operation costs of the RPSDs. The cost of line losses is evaluated at the market energy price $\lambda[t]$. The operation cost of discrete RPSDs $h_B(Q_B[t])$ is proportional to the number of switchings\(^5\):

$$h_B(Q_B[t]) = \sum_{i=1}^{n_B} \sum_{t=1}^{T} \pi_i Q_{B_i}^c \cdot [x_{B_i}[t] - x_{B_i}[t - 1]]$$

(4)

$$= \sum_{i=1}^{n_B} \sum_{t=1}^{T} \pi_i Q_{B_i}^c \cdot (x_{B_i}[t] - x_{B_i}[t - 1])^2$$

(5)

$\pi_i$ is the unit operation cost of device $i$ for switching one time. According to [15], $\pi_i$ is evaluated by the total unit installation cost divided by the total number of switchings in its lifetime. Typical values of $\pi_i$ is 0.41/($\text{MVar}$-times). For a 50$\text{MVar}$ capacitor bank, its operation cost is $\pi Q_{B_i}^c = 0.41 \times 50 = 20.5 \text{$/times}$ [15].

More discussions on choosing proper objective functions are provided in Section IV-D1.

\[ III. \text{LINEARIZED \ LOOK-AHEAD \ ORPD} \]

\[ A. \text{Computational Complexity} \]

Formulation (1) is a Mixed Integer Non-Linear Programming (MINLP) problem. It is challenging to solve and there is no guarantee on the global optimal solution. The intractability issue is mainly due to the non-linearity of power flow equations (Eqn. (1e)-(1f)). A common approach to tackle this issue is to linearize power flow equations [16], [17], [18]. The non-linear relationship between $P, Q$ and $|V|$, $\theta$ is approximated by a linear sensitivity matrix $A$:

$$\frac{\Delta P}{\Delta \theta} = A \Delta \theta$$

(6)

Many different approaches based on different forms of matrix $A$ have been proposed, e.g. [15], [16], [19], [20], [21]. Among all the proposed approaches, power flow Jacobian matrix is the most popular choice and has relative good linearization accuracy [16].

\[ B. \text{Linearization} \]

We follow the idea in [19], [20], [21], the original problem (Problem (1)) is linearized to be Problem (10) using power flow Jacobian matrix.

Let $t = 0$ denote current snapshot, and $\cdot[0]$ (e.g. $P_{G}[0], Q_{G}[0], Q_{B}[0], Q_{C}[0]$) denote the current operating states of the devices. Based on the power flow solutions of current snapshot $t = 0$, we can calculate the power flow Jacobian matrix, and the power flow equations (Eqn. (1e)-(1f) can be approximated as:

$$\begin{align*}
\frac{P_{G}[t] - P_{G}[0]}{Q_{G}[t] - Q_{G}[0]} & \approx \frac{\partial P_{G}^c}{\partial \theta} \left(\frac{\partial P_{G}^c}{\partial V^+} + \frac{\partial P_{G}^c}{\partial V^-}\right)\left[\theta[t] - \theta[0]\right] \left[|V[t]| - |V[0]|\right] \tag{7} \\
\Delta P_{G}^c[t] & = \left[\frac{\partial P_{G}^c}{\partial \theta} \left(\frac{\partial P_{G}^c}{\partial V^+} + \frac{\partial P_{G}^c}{\partial V^-}\right)\right]_{t=0} \left[\theta[t] - \theta[0]\right] \left[|V[t]| - |V[0]|\right] \tag{8} \\
\Delta Q_{G}^c[t] & = \left[\frac{\partial Q_{G}^c}{\partial \theta} \left(\frac{\partial Q_{G}^c}{\partial V^+} + \frac{\partial Q_{G}^c}{\partial V^-}\right)\right]_{t=0} \left[\theta[t] - \theta[0]\right] \left[|V[t]| - |V[0]|\right] \tag{9}
\end{align*}$$

Problem (10) is obtained by replacing Eqn. (1e)-(1f) with Eqn. (7).

$$\min \sum_{t=1}^{T} \left(h_B(Q_B[t]) + \lambda[t] \sum_{c=0}^{n_c} w^c (P_{G}[0] + \Delta P_{G}^c[t])\right) \tag{10a}$$

s.t. Eqn. (1)h, (1)\(c\), (1)d:

$$\begin{align*}
\Delta P_{G}^c[t] & = P_{G}^c[t] - P_{G}[0] \tag{10e} \\
\Delta Q_{G}^c[t] & = Q_{G}^c[t] - Q[0] \tag{10f} \\
\Delta|V^+|^c[t] & = |V^+|^c[t] - |V[0]| \tag{10g} \\
\Delta|V^-|^c[t] & = |V^-|^c[t] - |V[0]| \tag{10h} \\
P_{G}[0] & = Ac P_G[0] + A_W P_W[0] - AD P_B[0] \tag{10i} \\
Q[0] & = Ac Q_G[0] + Ac Q_C[0] + A_D Q_B[0] - A_D Q_D[0] \tag{10j}
\end{align*}$$

Eqn. (1)h, (1)i, (1)j, (3).

$$\Delta \theta^c \leq \Delta \theta^+ \leq \Delta \theta^- \leq \Delta \theta^+ \tag{10s}$$

$$\begin{align*}
\Delta|V^+|^c[t] & \leq \Delta|V^+|^c[t] \leq \Delta|V^+|^c[t] \tag{10m} \\
c = 1, 2, \ldots, n_c, t = 1, 2, \ldots, T
\end{align*}$$

Since this linear approximation is only accurate within the neighborhood of current operating point, Eqn. (10i) and (10m) are necessary. After solving Problem (10), its solution will be verified by solving the power flow equations of each contingency scenario and each snapshot.

\[ IV. \text{CASE STUDIES} \]

In this section we provide case studies on the IEEE RTS 24-bus system [22].
A. Settings

We make the following changes to the IEEE RTS 24-bus system:

- one wind farm with capacity 250 MW is added to bus 4;
- there is a synchronous condenser at bus 14 in the original system; three capacitor banks are added to bus 1, 10 and 15 with capacities 50 MVar, 50 MVar and 100 MVar. As discussed in Section II-C, the operation costs of the three capacitor banks are: 20.5$/times, 20.5$/times and 41$/times.
- the load profiles and generation of wind farms are shown in Fig. 2. Different colors represent different bus numbers.
- each one of the two 400 MW generators at bus 18 and 21 is replaced with four 100 MW generators, other settings of the generators remain the same as the original 24-bus system.
- the market energy price $\lambda[t], t = 1, 2, \ldots, T$ is assumed to be 60$/MWh and the operation interval is 15 minutes.
- all contingency scenarios have equal weights $!c$.

Given the wind and load profiles in Fig. (2), $T$ security-constrained economic dispatch problems are solved to get the real generation schedule $P_G[t]$, which is shown in Fig. (3).

![Fig. 2. Wind And Load Profiles](image)

Fig. 2. Wind And Load Profiles

Matpower 6.0 [23] is used to calculate the power flow Jacobian matrix, line loss sensitivity matrix and power flow solutions. The test system is simulated using Matlab R2016b on a PC with Intel i7-2600 8-core CPU@3.40GHz and 16GB RAM memory.

B. Simulation Results

Problem (10) is solved using Gurobi 7.0 [24]. The barrier method found the optimal solution (with 0.0% gap) in 3.79 seconds. The optimal total cost is $6411.3, and the optimal schedules of RPSDs and voltage set-points of generators are presented in Fig. (4).

![Fig. 3. Generation Schedule](image)

Fig. 3. Generation Schedule

![Fig. 4. Optimal Schedules of RPSDs and Voltage Set-points of Generators](image)

Fig. 4. Optimal Schedules of RPSDs and Voltage Set-points of Generators

2) Accuracy: Since voltage magnitudes are critical for system security assessment, we measure the largest distance between the voltage profiles from the optimal solution and those of the power flow solutions. Fig. (5) provides a qualitative comparison of the voltage profiles. Quantitative analysis is presented in Table I. Let $|V^c[t]| \in \mathbb{R}^{n_b}$ denote the voltage magnitudes of scenario $c$ at time $t$ from AC power flow solution, and $|\hat{V}^c[t]|$ denotes the voltage magnitudes from solving Problem (10). The distance between $|V^c[t]|$ and $|\hat{V}^c[t]|$ is defined as:

$$ d(|V^c|, |\hat{V}^c|) = \max_{c,t} ||V^c[t] - \hat{V}^c[t]||_1 \tag{11} $$

Table I presents the largest errors $d(|V^c|, |\hat{V}^c|)$ in normal condition ($c = 0$) and contingency scenarios ($c = 1, 2, \ldots, 38$). All the errors are less than 0.5% in normal condition and less than 1%
in all contingency scenarios. It is worth mentioning that the 38th contingency scenario often leads to the largest approximation error. The reason is that the 38th contingency scenario represents losing the largest generator in the system, which causes a substantial deviation from normal operating point.

**Table I**

**Comparison of Voltage Magnitudes** ($d(|V^c|, |\hat{V}^c|)$)

<table>
<thead>
<tr>
<th>Time</th>
<th>Normal Condition</th>
<th>Contingency Scenario</th>
<th>Worst Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>0.00010</td>
<td>0.00177</td>
<td>$c = 38$</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>0.0015</td>
<td>0.0085</td>
<td>$c = 38$</td>
</tr>
<tr>
<td>$t = 3$</td>
<td>0.0040</td>
<td>0.0072</td>
<td>$c = 38$</td>
</tr>
<tr>
<td>$t = 4$</td>
<td>0.0011</td>
<td>0.0084</td>
<td>$c = 38$</td>
</tr>
<tr>
<td>$t = 5$</td>
<td>0.0023</td>
<td>0.0075</td>
<td>$c = 10$</td>
</tr>
<tr>
<td>$t = 6$</td>
<td>0.0041</td>
<td>0.0079</td>
<td>$c = 5$</td>
</tr>
<tr>
<td>$t = 7$</td>
<td>0.0024</td>
<td>0.0077</td>
<td>$c = 9$</td>
</tr>
<tr>
<td>$t = 8$</td>
<td>0.0042</td>
<td>0.0083</td>
<td>$c = 38$</td>
</tr>
</tbody>
</table>

From simulation results (Table I and Fig. 5), the approximation seems satisfying. We want to emphasize that there is **no guarantee** on the approximation accuracy until rigorous theoretical analysis is conducted.

**D. Discussions**

1) **On Objective Functions:** Different from most of the literatures, where minimizing losses is the only objective, we include the operation costs of the control devices. There have been some studies on modeling the cost related with reactive power. For example, [15] provides a method to calculate the operation cost of reactive generations and continuous RPSDs (e.g. compensators). [25] provides more comprehensive modeling of the cost of discrete control devices.

2) **On the Impacts of Operation Costs of RPSDs:** By setting $\pi_i = 0$ for $i = 1, 2, \cdots, n_E$, the objective of Problem (1) becomes the same as most of the literatures: only minimizing the total line losses. The optimal solution to Problem (10) with $\pi = 0$ is shown in Fig. (7). Comparing Fig. (4) with Fig. (7), we found that some unnecessary operations of RPSDs may happen when their operation costs are not being considered.

The cost of line losses for the IEEE 24-bus system is about 60$/\text{MWh} \times 55\text{MW} \times 0.25\text{h} = 8258$. And the cost of switching a 100MVar capacitor bank once is around 40 dollars, which is about 5% of the line loss costs. With lower energy price, the operation cost will possess a higher portion of the overall cost. Only minimizing line losses might lead to frequent switchings of RPSDs, which could increase the overall costs and lead to suboptimal solutions.

**V. Estimate the Impacts of Wind Uncertainties**

We focus on the influences of temporal variations of wind generations in the look-ahead coordination of RPSDs. The uncertainty of wind is another critical issue to be discussed. Because of the non-linearity of power flow equations, the relationship between voltage magnitudes and wind fluctuations is also non-linear. In this section, we approximate this non-linear relationship using the modified Jacobian matrix, and simulation results show that this approximation is quite accurate.

**A. A Linear Approximation**

To estimate voltage magnitude changes, we need to reformulate the power flow Jacobian matrix Eqn. (7). Let $y_1$ denote the control variables related with real/reactive power (i.e. $\Delta P$ of PV buses, $\Delta P$ and $\Delta Q$ of PQ buses), and $y_2$ denote the state variables (i.e. $\Delta Q$ of PV buses, $\Delta P$ and $\Delta Q$ of slack buses):

$$y_1 := \begin{bmatrix} \Delta P_{PV} \\ \Delta P_{PQ} \\ \Delta Q_{PV} \\ \Delta Q_{PQ} \end{bmatrix}, \quad y_2 := \begin{bmatrix} \Delta P_{REF} \\ \Delta Q_{REF} \end{bmatrix}$$

Let $x_1$ represent the control variables related with voltages (i.e. $\Delta |V|$ of PV buses, $\Delta |V|$ and $\Delta \theta$ of slack buses), and $x_2$ denote the state variables (i.e. $\Delta \theta$ of PV buses, $\Delta |V|$ and $\Delta \theta$ of PQ buses):

$$x_1 := \begin{bmatrix} \Delta |V|_{PV} \\ \Delta |V|_{PQ} \\ \Delta \theta_{PV} \\ \Delta \theta_{PQ} \end{bmatrix}, \quad x_2 := \begin{bmatrix} \Delta |V|_{REF} \\ \Delta \theta_{REF} \end{bmatrix}.$$  

$x_1$ and $y_1$ represent the variables we can directly control in the power flow equations, the state variables $x_2$ and $y_2$ are implicitly determined by $x_1$ and $y_1$. If the power flow equation could be approximated as Eqn. (14), where all the control variables are on the right-hand side and the state variables are on the left-hand side, then we can easily estimate the changes of state variables from the changes of control variables.

$$\begin{bmatrix} y_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ x_1 \end{bmatrix}$$

Unfortunately, the original Jacobian matrix in Eqn. (7) has the form$^3$:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which mixes the control and state variables on both sides of the equation.

By substituting $y_2$ and $x_2$ in Eqn. (15) using Eqn. (14), we get:

$$(J_{12}C - I)y_1 + (J_{11} + J_{12}D)x_1 = 0$$

$$(A - J_{22}C)y_1 + (B - J_{21} - J_{22}D)x_1 = 0$$

Therefore:

$$A = J_{22}^{-1}J_{12}, \quad B = J_{21} + J_{22}J_{12}^{-1}J_{11}, \quad C = J_{12}^{-1}, \quad D = J_{12}^{-1}J_{11}$$

$^3$Please notice that $J_{12}$ and $J_{21}$ are square matrices, and $J_{11}$ and $J_{22}$ are rectangular matrices.
**B. Simulation Results**

Two approaches are compared: (1) calculate voltage magnitude changes by solving power flow equations; and (2) estimate the changes using the method in Section V-A. We modify the settings of wind generations in Section IV and focus on only one snapshot \((t = 4)\). The wind uncertainties here is mainly due to its unpredictability. Reference [26] analyzes the wind data from ERCOT and concludes that the Cauchy distribution is a better choice than the Gaussian distribution. Beta distribution or Weibull distribution to fit the persistence forecast errors of wind generation. In this study, the wind forecast error \(\%\) is modeled as a Cauchy distribution with location parameter \(x_0 = 0\) and scale parameter \(\gamma = 5\%\). With the optimal solutions (states of RPSDs and voltage set-points of generators) solved in Section IV, 1000 wind scenarios are generated and 1000 corresponding power flow problems are solved. Fig. 8 presents the scatter plot and estimated probability density functions of wind generation and voltage magnitudes of bus \(4\) (the location of wind farm). The blue dots in Fig. 8 are obtained by solving power flow equations, and the red dots are our linear approximations using the modified Jacobian matrix in Eqn. (14). This linear approximation is quite accurate, it has error less than 0.1% with moderate wind fluctuations \((\pm 30\%)\).

![Fig. 8. Impacts of Wind Uncertainties](image)

As shown in Fig. 8, wind uncertainties result in uncertainties of voltage magnitudes. With deeper penetration of renewables or a heavy-loaded system, voltage security issue due to wind fluctuations could be quite severe. Since Problem (1) and (10) are deterministic optimization problems, they are not able to handle the voltage security issue with wind uncertainties. We need to formulate the ORPD problem using stochastic optimization frameworks such as robust optimization and stochastic programming.

**VI. CONCLUDING REMARKS**

In this paper, we present a look-ahead coordination framework for reactive power support devices across multiple control areas to ensure voltage security. The proposed framework considers the global needs on reactive support and determines efficient cooperation schedules of the reactive devices. Given the fact that the original problem is non-convex, we reformulate the problem using power flow Jacobian matrix. Case studies demonstrate that the approximation is satisfying and the solution enforces the voltage security given all the contingency scenarios. Future work will consider uncertainties from renewables, as well as analyze the performance gap between linearized solution and the global optimal solution.

**REFERENCES**


