A Four Transmit Antenna Orthogonal Space-Time Block Code with Full Diversity and Rate

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1 Introduction

The Alamouti code [1], remarkable for having an elegant linear receiver, is now a paradigm in space-time block coding. Orthogonal designs [2] generalize Alamouti's scheme to use more antennas. Unfortunately, the Hurwitz-Radon theorem shows that complex orthogonal designs can not achieve full diversity and rate simultaneously for *all* symbol constellations, except in the two transmit antenna case [2]. Other codes show promise, including the STTD-OTD code [3], which is orthogonal but not full diversity, and constellation rotating codes [4], which achieve full rate and diversity but are not orthogonal.

In this work we use carefully tailored rotated PSK constellations to design a full rate, full diversity complex orthogonal space-time block code for 4 transmit antennas.

2 The New Code

We assume the standard slow, flat Rayleigh fading channel model. The goal is to design a $N_t = 4$ transmit antenna orthogonal space-time block code with M-PSK constellations. The new code has the same form as the orthogonal STTD-OTD code [3] defined by,

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_1 & s_2 & -s_3 & -s_4 \\ -s_2^* & s_1^* & s_4^* & -s_3^* \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{A} & -\mathbf{B} \end{bmatrix}, \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are Alamouti blocks. The symbols of the data matrix $\mathbf{S}$ are redefined to be linear combinations of complex "base" symbols, d_1, d_2, d_3, and d_4. We use the following encoding scheme which normalizes the energy transmitted.

$$s_1 = \frac{d_1 + d_2}{\sqrt{2}}, \qquad s_2 = \frac{d_3 + d_4}{\sqrt{2}}, \qquad s_3 = \frac{d_1 - d_2}{\sqrt{2}}, \qquad s_4 = \frac{d_3 - d_4}{\sqrt{2}}. \tag{2}$$

A measure of the quality of a square space-time code is the diversity product [5], $\zeta_v = \frac{1}{2} \min_{\mathbf{S}_1 \neq \mathbf{S}_2 \in V} |\det(\mathbf{S}_1 - \mathbf{S}_2)|^{\frac{1}{N_t}}$, where V is the set of all data matrices $\mathbf{S}$. We observe that $0 \leq \zeta_v \leq 1$ and any square code with $\zeta_v > 0$ is said to achieve full diversity.

To find ζ_v for the new code, notice that $\det(\mathbf{S}_1 - \mathbf{S}_2) = 4 \det(\mathbf{A}_1 - \mathbf{A}_2) \det(\mathbf{B}_1 - \mathbf{B}_2)$, where $\mathbf{S}_1$ and $\mathbf{S}_2$ are matrices of the same form as in(1) and $\mathbf{A}_i$ and $\mathbf{B}_i$ are Alamouti blocks. For full diversity we need $\det(\mathbf{A}_1 - \mathbf{A}_2) \neq 0$, or equivalently $d_1^1 - d_1^2 + d_2^1 - d_2^2 \neq 0$ and $d_3^1 - d_3^2 + d_4^1 - d_4^2 \neq 0$, where d_j^i is the j-th base symbol of $\mathbf{A}_i$. A similar result follows from $\det(\mathbf{B}_1 - \mathbf{B}_2) \neq 0$. Note that rotating the M-PSK base constellations of d_2 and d_4 by (for example) $\frac{\pi}{M}$ with respect to d_1 and d_3 gives the code full diversity. Thus, the new code with modified M-PSK constellations is an orthogonal full diversity and full rate space-time code for 4 transmit antennas. At first, it appears the new code violates the Hurwitz-Radon theorem, which states that such orthogonal designs cannot exist for all possible symbol constellations. However, they do exist for *some* specific constellations.

3 Performance

The symbol error rates (SER) for the new code with QPSK symbols and other comparable codes in known quasi-static fading are presented in Fig. 1. The plot shows the new code performance against a rate 1/2 STTD orthogonal design with 16-QAM [2] and a constellation rotating code with QPSK [4]. Performance of maximal ratio combining is also presented with SNR normalized by 6 dB ($N_r = 4$). The new code outperforms all other transmit diversity codes compared.

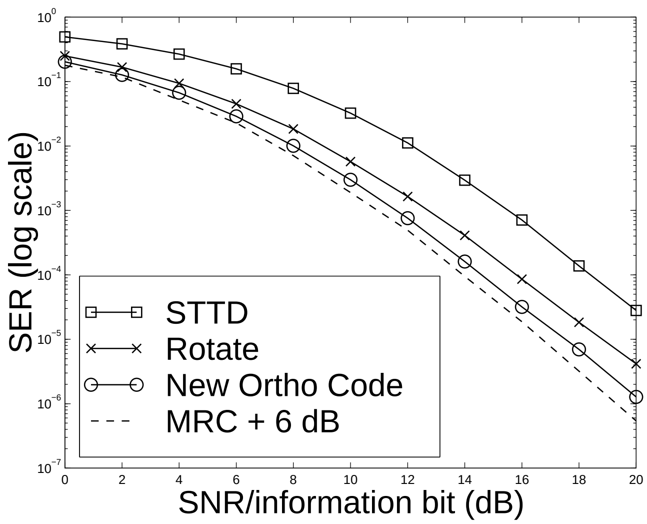

Figure 1: Comparison with other 4-transmit antenna codes at 2 bits/sec/Hz.

References

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