OPERATIONAL AMPLIFIERS

ELEN - 457

Kamran Entesari

Spring 2006
Outline

- Op Amp fundamentals and ideal macro model
- Non-ideal properties and macro models
- Stability and Noise analysis in Op Amp circuits
- OTA fundamental properties
- Multipliers and nonlinear applications
- Active filters
- Signal generators
- Oscillators
- Switched capacitor techniques
- D/A and A/D converters
- Phase-locked loops
Section 1

1) Op Amp fundamentals and ideal macro model

2) Circuits with resistive feedback
Op Amp Fundamentals

Different Amplifier Types:

1. **Voltage Amplifier**

2. **Current Amplifier**

3. **Trans-conductance Amplifier**

4. **Trans-resistance Amplifier**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Amplifier Type</th>
<th>Gain</th>
<th>$R_i$</th>
<th>$R_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_i$</td>
<td>$V_o$</td>
<td>Voltage</td>
<td>$V/V$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>$i_i$</td>
<td>$i_o$</td>
<td>Current</td>
<td>$A/A$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_i$</td>
<td>$V_o$</td>
<td>Trans-conductance</td>
<td>$A/V$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>$i_i$</td>
<td>$i_o$</td>
<td>Trans-resistance</td>
<td>$V/A$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
Op Amp Fundamentals

The Operational Amplifier:

• Op Amp is a voltage amplifier with extremely high gain (741, Gain: 200,000 (V/V), Op-77, Gain: 12 (V/uV)).

• \( r_d, a, r_o \) are open-loop parameters.

• \( v_P \): Non-inverting

• \( v_N \): Inverting

• \( v_0 = a \cdot v_D = a (v_P - v_N) \)

The Ideal Op Amp:

• The virtual input short does not draw any current.

• For voltage purposes: Input appears as a short circuit.

• For current purposes: Input appears as an open circuit.
Op Amp Fundamentals

Basic Op Amp Configurations:

- **Non-inverting Amplifier**

  ![Non-inverting Amplifier Diagram]

  \[ A \text{ (Closed Loop Gain)} = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{1 + \left(\frac{R_2}{R_1}\right)/a} \]

  \[ R_i \text{ (Closed Loop)} = \infty \]

  \[ R_o \text{ (Closed Loop)} = 0 \]

- **The Voltage Follower (Unity Gain Amplifier)**

  ![Voltage Follower Diagram]

  \[ R_1 = \infty \]

  \[ R_2 = 0 \]

  \[ A = 1 \]

  \[ R_i = \infty \]

  \[ R_o = 0 \]

  - A resistance transformer
  - The source does not deliver any current to load
Op Amp Fundamentals

Basic Op Amp Configurations:

- **Inverting Amplifier**

  ![Inverting Amplifier Diagram]

  \[ A \text{ (Closed Loop Gain)} = \left(-\frac{R_2}{R_1}\right) \cdot \frac{1}{1 + \left(\frac{R_2}{R_1}\right)/a} \]

  \[ A \text{ (Ideal)} = \left(-\frac{R_2}{R_1}\right) \]

  \[ R_i \text{ (Closed Loop)} = R_1 \]

  \[ R_o \text{ (Closed Loop)} = 0 \]

- **The Summing Amplifier (Popular Application: Audio Mixing)**

  ![Summing Amplifier Diagram]

  \[ v_o = -\left(\frac{R_F}{R_1} v_1 + \frac{R_F}{R_2} v_2 + \frac{R_F}{R_3} v_3\right) \]

  \[ R_{ik} = R_k \quad k = 1, 2, 3 \]

  \[ R_o = 0 \]

  The output is the weighted sum of the inputs.
Op Amp Fundamentals

Basic Op Amp Configurations:

• **The Difference Amplifier** (Popular Application: Instrumentation)

\[ v_o = \left( \frac{R_2}{R_1} \right) \left[ \frac{(1 + \frac{R_1}{R_2})}{(1 + \frac{R_3}{R_4})} (v_2 - v_1) \right] \]

\[ \frac{R_1}{R_2} = \frac{R_3}{R_4} \rightarrow v_o = \left( \frac{R_2}{R_1} \right) \left[ v_2 - v_1 \right] \]

\[ R_{i1} = R_1, \quad R_{i2} = R_3 + R_4, \quad R_o = 0 \]

• **The Differentiator**

\[ i_C = C \frac{dV_I}{dt} \]

\[ i_R = -\frac{v_o}{R} \]

\[ v_o = -RC \frac{dV_I}{dt} \]

• Tends to oscillate (We see the reason later)

• By putting the \( R_s \) in series with \( C \), the oscillation problem is solved

• The circuit still provides differentiation function over the limited bandwidth
Op Amp Fundamentals

Basic Op Amp Configurations:

- **The Integrator** (Popular Applications: Function generators, Active filters, A/Ds, Analog (PID) controllers)

  ![Integrator Circuit Diagram]

  \[ \frac{v_i}{R} = -C \frac{dv_o}{dt} \]

  \[ \frac{dv_o}{dt} = -\frac{v_i}{RC} \]

  - Due to the input offset error of the op amp, the output drifts until it saturates at the value close to one of the supplies.

  - By putting \( R_p \) in parallel with \( C \), we can prevent saturation and have integration over a limited bandwidth.

- **The Negative Resistance Converter (NIC)**

  ![NIC Circuit Diagram]

  \[ R_{eq} = \left( -\frac{R_2}{R_1} \right) R \]

  - Current is floating toward the source

  - Negative resistance releases the power.

  - Applications:
    1) Neutralization of unwanted resistances in the design of current source
    2) Control pole location (Oscillators)
Op Amp Fundamentals

Negative Feedback; A Systematic Approach:

Building Blocks:

1) Error Amplifier; \( x_o = a \cdot x_d \)
2) Feedback Network; \( x_f = \beta \cdot x_o \)
3) Summing Network; \( x_d = x_i - x_f \)

\[
A \text{ (Closed Loop Gain)} = \frac{x_o}{x_i} = \frac{a}{1+a\beta}
\]

\[
T \text{ (Loop Gain)} = a\beta
\]

\[
A = \frac{1}{\beta} \cdot \frac{T}{1+T} = A_{\text{ideal}} \cdot \frac{1}{1 + (1/T)} = A_{\text{ideal}} \cdot [1 - \frac{1}{1+T}] = A_{\text{ideal}} \cdot [1 - \varepsilon]
\]

Gain Error = \( \frac{A - A_{\text{ideal}}}{A_{\text{ideal}}} \approx \frac{1}{T} \)

• Price for a tight closed loop accuracy: \( a \gg A \)

• The smaller the closed-loop is, the smaller the percentage from deviation is.
Feedback Properties:

1) Gain De-sensitivity
   - The negative feedback desensitizes the open loop gain
   - Components in $\beta$ should have very good quality

2) Nonlinear Distortion Reduction
   - As long as $a$ is sufficiently large and to make $T >> 1$, $A$ will be fairly constant and close to $1/\beta$ in spite of the decrease of $a$ away from the origin

3) Effect on Disturbance and Noise

\[ A = \frac{a}{1 + a\beta} \]
\[ \frac{dA}{da} = \frac{1}{(1 + a\beta)^2} \]
\[ \frac{\Delta A}{A} \sim \frac{1}{1 + T} \]
\[ \frac{\Delta a}{a} \]
\[ \frac{\Delta A}{A} \sim -\frac{\Delta \beta}{\beta} \]

\[ x_1 : \text{Input offset errors} \]
\[ x_2 : \text{Power supply hum} \]
\[ x_3 : \text{Output load changes} \]
\[ x_o = \frac{a_1 a_2}{(1 + a_1 a_2 \beta)} \left[ x_1 + x_1 + \frac{x_2}{a_1} + \frac{x_3}{a_1 a_2} \right] \]
Feedback in Op Amp Circuits:

**Negative feedback topologies**

- **Input Series FB**
- **Input Shunt FB**
- **Output Shunt FB**
- **Output Series FB**

- Input + Feedback enter the amplifier at different nodes: Input Series FB
- Input + Feedback enter the amplifier at the same nodes: Input Shunt FB
- If we short the output load and still there is FB signal at the input: Output Series FB
- If we short the output load and still there is FB signal at the input: Output Series FB

**AT THE INPUT/OUTPUT PORT, A SERIES TOPOLOGY RAISES AND A SHUNT TOPOLOGY LOWERS THE CORRESPONDING PORT RESISTRANCE**
Op Amp Fundamentals

Analysis of Basic Op Amp Configurations Using Feedback Theory:

• **Non-inverting Amplifier**
  (Input Series – Output Shunt FB)

\[
A \sim \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{T}{1+T}
\]

\[
\beta = \frac{R_1}{R_2 + R_1}
\]

\[
R_i \sim r_d \cdot [1 + T]
\]

\[
R_o \sim r_o / [1 + T]
\]

• **Non-inverting Amplifier**
  (Input Shunt – Output Shunt FB)

\[
A \sim \left(-\frac{R_2}{R_1}\right) \cdot \frac{T}{1+T}
\]

\[
\beta = \frac{R_1}{R_2 + R_1}
\]

\[
R_i \sim R_1 + \frac{R_2}{1+a} \quad \text{(Miller Effect)}
\]

\[
R_o \sim r_o / [1 + T]
\]
Finding The Loop Gain (T) Directly:

- Suppress all *input* sources,
- Break the loop at some convenient point
- Inject the test signal ($v_T$)
- Find the return signal ($v_R$) at the breaking point using the feedback path:

$$v_R = a \cdot b \cdot (-1) \cdot v_T$$
$$T = a \cdot b = \left. -\frac{v_R}{v_T} \right|_{x_I = 0}$$

$$A = A_{\text{ideal}} \cdot \frac{1}{1 + (1/T)}$$
$$R = r \cdot (1 + T), \ r: \text{open-loop resistance} \ (a \to 0)$$

Finding the Feedback Factor ($\beta$) Directly:

By finding $\beta$, using datasheet we can find $a$ and calculate $T = a \cdot \beta$

- Suppress all *input* sources,
- Disconnect the op amp
- Replace the op amp with its terminal resistances ($r_d, r_o$)
- Apply a test source $v_T$ via $r_o$, find the difference voltage $v_D$ across $r_d$, then:

$$\beta = \left. -\frac{v_D}{v_T} \right|_{x_I = 0}$$
Op Amp Fundamentals

Op Amp Powering:

- **0.1 \( \mu \text{F} \) capacitance:**
  1) Prevents the AC noise coming from non-zero impedance between the supply and the ground.
  2) Neutralizes spurious feedback loops arising from non-zero impedance between the supply and ground.

- **10 \( \mu \text{F} \) capacitance** provides board-level bypass.

- Using wide ground traces is recommended.

- \( V_{CC} \) and \( V_{EE} \) can be dual +15V, -15V supplies (analog systems), or single 5 V and zero supply (mixed-mode applications)
Op Amp Fundamentals

Op Amp Powering:

- $V_{CC}$ and $V_{EE}$ set upper and lower bounds on the output swing capacity

1) Linear Region; $a = 0.2 \, \text{V/}\mu\text{V}$
   Model: dependent source

2) Positive Saturation Region;
   $V_{oH}$ remains fixed.

3) Negative Saturation Region;
   $V_{oL}$ remains fixed.

- Bipolar op amps: $V_{oH} \sim V_{CC} - 2 \, \text{V}, V_{oL} \sim V_{EE} - 2 \, \text{V}$
- CMOS op amps: $V_{oH} \sim V_{CC}, V_{oL} \sim V_{EE}$

- Common characteristic of saturating amplifier: Clipped output voltage
- Undesirable in many cases.
- Application: POP MUSIC FUZZ BOXES