ELEN 689: Special Topics
Advanced Mixed-Signal Interfaces

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A Lot of New Names for Future Broadband Communication Systems

- The Names
  - Software Defined Radios
  - Multi-Standard Radios
  - Cognitive Radios
  - Universal Radios

- Common Features
  - Very wideband systems, multiband channels, opportunistic frequency allocation, bandwidth reuse, intensely digital, scalable/reconfigurable RF/analog.

- Challenges
  - Conflicting requirements, large bandwidth/dynamic range but still want low power/small area.
Receiver Topologies
The Receiver Design Problem in Broadband Communications

- How much RF processing do I do before the ADC?
- How do I take advantage of technology scaling in this RF pre-processing?
- How do I make the front-end scalable and configurable to fit multiple standards?
Conventional Receivers

- Superheterodyne Receiver
- Single Conversion Receiver
- Upconversion
- Dual Conversion
- Image-Reject Receiver (Complex I&Q mixing)
- Direct Conversion Receiver
Image Rejection

- In high-IF RF receivers, RF LC or SAW filters are used to suppress the image before the down-conversion. Larger IFs are preferable to relax the filter Q factor.

- Ideally zero IF does not require the RF filter but still suffers from gain and phase mismatches.

Fig. 1. Conventional receiver with RF image filter.

Fig. 2. Direct-conversion receiver.


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Image Rejection Ratio

The identities
\[ \cos(x)\cos(y) = \frac{1}{2} [\cos(x - y) + \cos(x + y)] \]
\[ \cos(x)\sin(y) = \frac{1}{2} [\sin(x + y) - \sin(x - y)] \]

\[ I_{LO} = B \cos(\omega_{LO} t) \]
\[ Q_{LO} = A \sin(\omega_{LO} t) \]
\[ x_{RF} = \cos(\omega_{RF} t) \]
\[ I_{LO} * x_{RF} = \frac{B}{2} [\cos((\omega_{RF} + \omega_{LO}) t) + \cos((\omega_{RF} - \omega_{LO}) t)] \rightarrow \text{LPF} \rightarrow \frac{B}{2} \cos(\omega_{IF} t) \rightarrow \frac{B}{2} \cos(\omega_{IF} \tau) + \frac{A}{2} \cos(\omega_{IF} \tau) \]
\[ Q_{LO} * x_{RF} = \frac{A}{2} [\sin((\omega_{RF} + \omega_{LO}) t) - \sin((\omega_{RF} - \omega_{LO}) t)] \rightarrow \text{LPF} \rightarrow -\frac{A}{2} \sin(\omega_{IF} t) \rightarrow \frac{A}{2} \cos(\omega_{IF} t) \]
\[ \omega_{IF} = \omega_{RF} - \omega_{LO} \]
\[ x_{IMAGE} = \cos((\omega_{LO} - \omega_{IF}) t) \]
\[ I_{LO} * x_{IMAGE} = \frac{B}{2} [\cos((\omega_{LO} - \omega_{IF} + \omega_{LO}) t) + \cos((\omega_{LO} - \omega_{IF} - \omega_{LO}) t)] \rightarrow \text{LPF} \rightarrow \frac{B}{2} \cos(\omega_{IF} t) \rightarrow \frac{B}{2} \cos(\omega_{IF} t) - \frac{A}{2} \cos(\omega_{IF} t) \]
\[ Q_{LO} * x_{IMAGE} = \frac{A}{2} [\sin((\omega_{LO} - \omega_{IF} + \omega_{LO}) t) - \sin((\omega_{LO} - \omega_{IF} - \omega_{LO}) t)] \rightarrow \text{LPF} \rightarrow \frac{A}{2} \sin(\omega_{IF} t) \rightarrow \frac{A}{2} \cos(\omega_{IF} t) \]
\[ IRR_{gain} = \left[ \frac{A + B}{A - B} \right]^2 = \left[ \frac{1 + B/A}{1 - B/A} \right]^2 \approx \frac{4}{\varepsilon^2} \]
\[ IRR_{phase} = 1 + 4 \frac{\cot \Delta \phi}{(\Delta \phi)^2} = \frac{4}{(\Delta \phi)^2} \]
\[ IRR_{total} \approx \frac{4}{(\Delta \phi)^2 + \varepsilon^2} \]

0.1% gain error and 1º phase error leads to IRR of about 41 dB.

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Image Rejection Ratio

- HW #4: Find the IRR for the case the input comes with a quadrature component as well, i.e., \( x_{RF} = I_D \cos(\omega_{RF} t) + Q_D \sin(\omega_{RF} t) \) and a direct zero-IF receiver is used.
Basic Equations of Image Rejection

- Gain mismatch $\alpha$ and phase mismatch $\theta$:

$$I' = \left(1 + \frac{\alpha}{2}\right) \cos(\omega t + \frac{\theta}{2})$$

$$= \left(1 + \frac{\alpha}{2}\right) \left\{ \frac{e^{j(\omega t + \frac{\theta}{2})} + e^{-j(\omega t + \frac{\theta}{2})}}{2} \right\}$$

$$Q' = \left(1 - \frac{\alpha}{2}\right) \sin \left(\omega t - \frac{\theta}{2}\right)$$

$$= \left(1 - \frac{\alpha}{2}\right) \left\{ \frac{e^{j(\omega t - \frac{\theta}{2})} - e^{-j(\omega t - \frac{\theta}{2})}}{2j} \right\}.$$  (2)

Then, assuming $\alpha$ and $\theta$ are small, the non-ideal complex signal $I' + jQ'$ and the non-ideal complex image $I' - jQ'$ can be approximated using Taylor series.

$$I' + jQ' \approx e^{j\omega t} + \left(\frac{\alpha - j\theta}{2}\right) e^{-j\omega t}$$  (3)

$$I' - jQ' \approx \left(\frac{\alpha + j\theta}{2}\right) e^{j\omega t} + e^{-j\omega t}.$$  (4)
Matrix Formulation

Matrix formulation of the non-ideal mixing:

\[
\begin{bmatrix}
I' \\
Q'
\end{bmatrix} = \begin{bmatrix}
1 + \frac{\alpha}{2} & -\frac{\theta}{2} \\
-\frac{\theta}{2} & 1 - \frac{\alpha}{2}
\end{bmatrix} \begin{bmatrix}
I \\
Q
\end{bmatrix}.
\]

Matrix formulation of the non-idealities correction:

\[
\begin{bmatrix}
I'' \\
Q''
\end{bmatrix} = \begin{bmatrix}
1 - \frac{\alpha}{2} & \frac{\theta}{2} \\
\theta/2 & 1 + \frac{\alpha}{2}
\end{bmatrix} \begin{bmatrix}
I' \\
Q'
\end{bmatrix}.
\]


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LMS algorithm for the estimation of $\alpha$ and $\theta$

\[
\begin{align*}
\alpha'[k + 1] &= \alpha'[k] + \mu_\alpha \text{sgn}[\text{LPF}\{\text{sgn}(I + Q) \\
&\quad \times \text{sgn}(I - Q)\}] \\
\theta'[k + 1] &= \theta'[k] - \mu_\theta \text{sgn}[\text{LPF}\{\text{sgn}(I) \cdot \text{sgn}(Q)\}]
\end{align*}
\]

\[
\text{LPF}(I^2 - Q^2) = \frac{(1 + \frac{\alpha}{2})^2}{2} - \frac{(1 - \frac{\alpha}{2})^2}{2} \approx \alpha.
\]

\[
\text{LPF}(IQ) = -\theta \frac{\left(1 - \frac{\alpha^2}{4}\right)}{2} \approx -\frac{\theta}{2}.
\]

$2^{20}$ iterations needed!!

➢ Fully-digital implementation

Fig. 12. Image rejection system.


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Analog Implementation

Fig. 13. (a) Complex baseband S/H. (b) Gain-boosted telescopic cascoded operational amplifier.

Fig. 14. Nine-bit trim capacitor.

Fig. 15. (a) Analog comparator. (b) Preamplifier. (c) Latch.

Fig. 17. Chip die photo.


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256 QAM Spectra Before and After Image Rejection

Fig. 18. Measured 256-QAM spectra before and after image rejection.


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256 QAM Constellation Before and After Image Rejection

Fig. 19. Effect of ADC resolution on IRR for 256-QAM.

Effect of IRR on Error Probability

Fig. 22. Effect of IRR on error probability of 64-QAM and 256-QAM.
Non-linearities
Improvement of Mixer Nonlinearities (IIP2) for Active Mixers

- Detailed circuit analysis of nonlinearities in diff pairs and doubled balanced mixers.
- The proposed approach is analog, trimming of current bias.

Improvement of Mixer Nonlinearities (IIP2) for Active Mixers

Uses PN sequences and correlation to estimate the nonlinearities. The LO bias is tuned to minimize distortion.

Fig. 8. Proposed even-order distortion cancellation scheme of the double-balanced mixer.

Liwei Sheng; Larson, L.E.; “An Si-SiGe BiCMOS direct-conversion mixer with second-order and third-order nonlinearity cancellation for WCDMA applications,” Microwave Theory and Techniques, IEEE Transactions on Volume 51, Issue 11, Nov. 2003 Page(s):2211 - 2220
Some of the New Approaches to Broadband Receivers

- **A high-frequency software defined radio**

- **Frequency channelizers**

- **Selectable RF filters and downconversion**

- **Subsampling and undersampling**

- **Analog decimation**
Some of the New Approaches to Broadband Receivers (cont…)

- **Sampling with built-in anti-aliasing**
  

- **Sample rate, downsampling and filtering**
  

- **A discrete-time RF sampling receiver**
  

- **UCLA SDR receiver**
  
  

- **Frequency-domain-sampling receivers**
  
  
  
Sampling with built-in anti-aliasing

- Sinc(x) anti-aliasing provided by windowing and integration. The sidelobes decay at 20 dB/decade with zeros at fs, 2fs, ..

- More general mixing waveforms can be used, although complexity goes up.

A simple integrator

- Assume a low noise transconductance amplifier
- LO=2.4 GHz
- Pseudo-differential architecture (b) is preferable.
- This is just mixing followed by integration which provides down-conversion and filtering in a single stage.
- How do you read the voltage out of the caps?
Cyclic Read-Out

- Cyclic change and discharge of caps. every N cycles.
- This can be modeled as a moving average

\[ W_i = \sum_{l=0}^{N-1} u_{i-l} \]

- If modeled as MA, the filter has a sinc frequency response whose lobes width and nulls positions depend on N.
High-Rate IIR Filtering

- History capacitor $C_H$ added
- LNTA sees constant capacitance $C_S$
- Let $a_1 = C_H / (C_H + C_R)$
- At switching time, $C_H$ retains $a_1$ portion of its total charge and shares $(1 - a_1)$ to the discharged $C_R$ cap. At sampling time $j$, the system charge $s_j$ is:
  
  $$s_j = a_1 s_{j-1} + w_j$$

  - The output charge $x_j$ is
  
  $$x_j = (1 - a_1) s_{j-1}$$

  This is a IIR filter with sampling frequency of $f_0 / N$ and single pole at

  $$f_c = \frac{1}{2\pi} \frac{f_0}{N} (1 - a_1) = \frac{1}{2\pi} \frac{f_0}{N} \frac{C_R}{C_H + C_R}$$
Example

- $C_R = 0.5 \text{pF}, C_H = 15.425 \text{pF}, a_1 = 0.9686$
- $f_0/N = 2.4 \text{GHz}/4 = 300 \text{MHz}$
- Additional zeros with $M = 4$

Additional IIR filtering during read-out process can also be introduced.
Adding more zeros to the FIR

- Redundant switched caps. Introduce more zeros in the transfer function when adding up their charges during read out:

\[ y_k = \sum_{l=0}^{M-1} u_{k-l} \]

- Illustrated is the case with M=4
A discrete-time RF sampling receiver

- Bluetooth and GSM receivers from TI use integrate and dump sampling followed by down sampling and filtering.
- Programmable filtering and decimation to achieve the anti-aliasing needed.

Direct conversion with tunable LO in the freq. range 800 MHz to 6 GHz.

Cascade of sinc^N filters followed by decimation to achieve the initializing needed.

Good for narrowband signals as a single ADC can handle the bandwidth. But SDR should also be good for wideband and ultra-wideband signals. Need parallel ADC to sample at a fraction of Nyquist rate. Parallelization of the front-end will be needed if want to keep the ADC sampling rate down.
SDR for narrowband, wideband and ultra-wideband signals

- Assume we have a tunable front-end that provides the downconversion and the antialiasing filtering needed for a wide range of standards.

- The problem now is that the signal bandwidth will have > 10X range. Example: 802.11g (ΣΔ ADC @ 50 Ms/s and 8 bits), UWB (ADC @ 500 Ms/s and 5 bits). Say you can run the ΣΔ ADC @ 100Ms/s and 5 bits, i.e. exchange OSR by DR). Can we use 5 of these ΣΔ ADCs to cope with UWB?

- Note that the same ΣΔ ADC could operate @ 200 KHz and 14 bits for GSM and @ 1MHz and 12 bits for Bluetooth.

- How do you parallelize the ADCs and even the RF front-end to create an SDR for narrowband, wideband and ultra-wideband signals?
Motivation

Digital intensive RF receivers -> ADCs with wide bandwidths and large dynamic range. Solution? -> Parallelization

Parallelized ADCs

Time-interleaved ADC

- SHA has stringent tracking bandwidth requirements
- Each ADC sees full input signal bandwidth (non-linearity and aliasing)

Drawbacks

Filter-bank ADC

- Filters with very tough specs (aliasing)
- Signal reconstruction increases complexity

Drawbacks

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Parallel-Path Sampling

\[ r(t) \]

\[ e^{-j 2 \pi F_0 t} \]

\[ e^{-j 2 \pi F_{N-1} t} \]

**Salient Features**

- Simple mixers and integrators in the front end
- Windowed integration provides inherent anti-aliasing. Relaxed Filter design.
- Relaxed sample and hold requirements
- No signal reconstruction. Direct digital processing of Frequency samples.
- Relaxed ADC design with lower speeds

**Drawback**

- Area overhead associated with parallelization
Parallel-Path Receiver Architecture

Basis Functions

RF broadband signal

LNA

Mixing and Integration

Basis Co-efficients

ADC

Digital Post Processing

Recovered Symbols

Basis Functions

Gm

Gm

Gm

F₁ – I and Q

F₂ – I and Q

Fₙ – I and Q

∫

(\cdot)dt

(\cdot)dt

(\cdot)dt

m = 0 to M, M - no. of segments

Tc - Actual integration time

Ts - Integration time – Overlap time

Windowed Integration

Sinc filter

Inherent anti-aliasing

Circuit Implementation of the Gm Stage

1. High Linear Gm stage

- Flicker noise is removed by the degeneration resistor
- IIP3 is boost to almost 30 dBm
- Large Vdd is required.
- Used in 10 bits full scale input receiver.
Circuit Implementation of the Gm Stage

2. Noise Cancelling Gm stage
- Noise of the first stage is eliminated
- Gain is boosted
- Noise Figure is improved
- Used in wireless receiver while input signal is small.

Circuit Implementation of the Mixer

Double Balanced Passive Mixer
- Minimum signal and clock feed-through
- Even order harmonics are cancelled
- Almost noise free

![Double Balanced Passive Mixer Diagram]
Charge Sampling

- Windowed integration of $I_{in}$ on C1 and C2
- Inherent anti-aliasing $sinc$ filter
Overlap in windows

[Diagram showing electrical circuit and waveforms]
1 Path Circuit

Shooting for 10 bits 2.5 Gs/s ADC

- High linear Gm stage to accommodate fullscale input
- Overlapping windowing
- 200 Ms/s path sampling rate
- 55 dB SNDR

The whole front-end: 10 path (5 lo frequencies I/Q)
Chip Layout

- Area: 2.5mm*2.5mm
- Core Power: 320 mW
- 64 mW / Path (I and Q)

Overall Power consumption of the ADC:
320 mW + N * P_path,adc

45 nm (TI technology)
**System level issues of FD receiver**

**Noise Amplification**
- Out-of-band noise folds back creating dips in performance
- Overlap improves the filter
- Overlap results in over-sampling which reduces aliasing.
- Additional carriers can be detected.

**Effect of Jitter**
- Jitter sources: LO signal, Sampling clocks
- Jitter from LO signal dominant
- Filter mitigates noise from LO jitter.
- Long integration windows reduces jitter from sampling clocks.
Least Squares Data Estimation

Input symbols modulated on carriers
Output sampled basis coefficients

\[ \vec{a} = [a_i(0), a_q(0), a_i(1), a_q(1), \ldots a_i(S - 1), a_q(S)] \]
\[ \vec{r} = [R_{0,0}, R_{0,1} \ldots R_{0,N-1}, R_{1,0}, R_{1,1}, \ldots R_{M-1,N-1}]^T \]

Entire system represented as a linear transformation from data symbols (a) to output samples of multi-path receiver (r)

\[ G \cdot \vec{a} = \vec{r} \]

Least Squares (LS) solution for the system ->
\[ H = (G^H G)^{-1} G^H \]
\[ \hat{a} = H \cdot \vec{r} \]

Need for Calibration?

- H is sensitive to mismatches, offsets and imperfections in the system
- H must match the circuit implementation accurately for good SNR
Mismatches in the system

Offset in the LO frequency at transmitter and receiver.

Multi-carrier signal Transmitter

Gain and phase mismatch for each carrier. Flat gain model is used for channel.

Gain and Phase mismatch between multiple channels due to process variations and environment conditions.

Sampling clocks are synchronized with LO signals in the receiver

Analog samples

Mismatches in capacitors introduces gain error and distortion.

Finite bandwidth of circuits alters LO waveform shape

Phase offset in LO signals

Actual LO signal

Ideal LO signal

Charge sampling Integrator & Sampler

Mixer

Finite bandwidth of circuits alters LO waveform shape

Phase offset in LO signals

Actual LO signal

Ideal LO signal
Complete System Calibration

Multi-carrier signal
Transmitter

IFF
T

Multi-carrier signal
Modulator

LO

Wireless Channel

noise

Multi-carrier signal
Receiver

Multi-channel
FD Receiver
LNA + Gm stage +
Charge sampling circuit

Least Squares Estimation of data with LMS calibration

\[ \tilde{a} = (G^H G)^{-1} G^H \tilde{r} \]

Estimated Data

\[ \tilde{a} \]

\[ \hat{a} \]

Forward Problem calibration

\[ \hat{a} \]

\[ \tilde{a} \]

Reverse Problem calibration

\[ H = (G^H G)^{-1} G^H \]

Generate G matrix

\[ \hat{a}_{ref} \]

\[ \tilde{a}_{ref} \]

\[ \hat{a}_{ref} \]

\[ \tilde{a} \]

\[ e^{-j 2 \pi (f_o + \Delta F_c) T} \]

\[ e^{-j 2 \pi \Delta \hat{F}_c (L-1) T} \]

Frequency estimator

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LMS Calibration

Initialization of G matrix

\[ G \cdot \vec{a} = \vec{r} \]

Input -> \( a_1 \ a_2 \ a_3 \ldots \ a_S \)

Output -> \( r_1 \ r_2 \ r_3 \ldots \ r_S \)

- \( a_1 \rightarrow [1 \ 0 \ 0 \ldots 0]^T \)
- \( a_2 \rightarrow [0 \ 1 \ 0 \ldots 0]^T \)
- \( a_3 \rightarrow [0 \ 0 \ 1 \ldots 0]^T \)
- \( a_S \rightarrow [0 \ 0 \ 0 \ldots 1]^T \)

- \( r_1 \) forms 1\(^{st}\) row of G matrix
- \( r_2 \) forms 2\(^{nd}\) row of G matrix
- \( r_3 \) forms 3\(^{rd}\) row of G matrix
- \( r_S \) forms S\(^{th}\) row of G matrix

LMS calibration

Two methods:

1. Forward Problem Calibration
2. Reverse Problem Calibration
Forward Problem Calibration

\[ \hat{a} = (G^H G)^{-1} G^H \tilde{r} \]

\[ \hat{a} \]

Estimated Data \( \tilde{r} \)

\[ G \]

\[ \hat{a}_{\text{ref}} \]

\[ \tilde{r} \]

\[ \hat{a}_{\text{ref}} \]

Forward Problem update equation

\[ \hat{G}(L + 1) = \hat{G}(L) + \frac{e_{\text{r}}(L) \ast \hat{a}}{\|\hat{a}\|^2} \]

Reverse Problem Calibration

\[ H = (G^H G)^{-1} G^H \]

\[ \hat{a}_{\text{ref}} \]

\[ \hat{a} \]

\[ \tilde{r} \]

\[ \hat{a}_{\text{ref}} \]

\[ \tilde{r} \]

\[ H \]

\[ \hat{a}_{\text{ref}} \]

\[ \tilde{r} \]

Reverse Problem update equation

\[ \hat{H}(L + 1) = \hat{H}(L) + \frac{e_{\alpha}(L) \ast \tilde{r}}{\|\tilde{r}\|^2} \]

Forward Problem

\[ G \cdot \hat{a} = \tilde{r} \]

(samples from receiver)

Reverse Problem

\[ \hat{a} = H \cdot \tilde{r} \]

(samples from receiver)
Simulations

Mean squared error convergence

1. With arbitrary H matrix

2. With H matrix initialized from ‘r’ vector

Very slow convergence

Fast convergence
Simulations

SNDR post calibration

Input SNR = 100 dB
Frequency offset in carriers is not included

1. With arbitrary H matrix

2. With H matrix initialized from ‘r’ vector

All static mismatches are calibrated in both cases.
Frequency offset Estimation

The sampled basis coefficients in block L,

\[ R_{m,n,L} = \int_{mT_s+\Delta T}^{mT_s+T_c+\Delta T} x_L(t) \Phi_n^*(t) dt \]

where,

\[ \Phi_n(t) = e^{-j[2\pi f_{LO}(n)t + \phi_{LO}(n)]} - \frac{1}{3}e^{j[3 \cdot 2\pi f_{LO}(n)t + 3 \cdot \phi_{LO}(n)]} + \frac{1}{5}e^{-j[5 \cdot 2\pi f_{LO}(n)t + 5 \cdot \phi_{LO}(n)]} - \ldots \]

\[ x_L(t) = \sum_{s=1}^{S} \left[ a_i(s) \cos \left( 2\pi F'_c(s)t + \phi_c(s) + 2\pi \Delta F_c(L-1)T \right) \right. \]

\[ + \left. a_q(s) \sin \left( 2\pi F'_c(s)t + \phi_c(s) + 2\pi \Delta F_c(L-1)T \right) \right] \]

\[ F'_c(s) = F_c(s) + \Delta F_c \]

Frequency offset in carriers
**Frequency offset Estimation**

After a few steps of simplification,

$$ R_{m,n,L} = e^{2\pi j \Delta F_c (L - 1) T} \int_{mT_s}^{mT_s + T_c} \sum_{s=1}^{S} A_n e^{j\theta_n} \times \left[ \frac{a_i(s)}{2} e^{j[2\pi F'_c(s) t + \phi_c(s) + 2\pi F'_c(s) \Delta T - 2\pi f_{LO}(n)t + \phi'_{LO}(n)]} + \frac{a_q(s)}{2} e^{j[2\pi F'_c(s) t + \phi_c(s) + 2\pi F'_c(s) \Delta T - 2\pi f_{LO}(n)t + \phi'_{LO}(n)]} \right] dt $$

If $$ R_{m,n,L} = \alpha_{m,n} e^{j\beta_{m,n}} $$ then $$ R_{m,n,L+1} = e^{2\pi j \Delta F_c T} \times \alpha_{m,n} e^{j\beta_{m,n}} $$

Including noise in these terms, the samples in the $$ L^{th} $$ and $$ (L+1)^{th} $$ block are,

$$ R_{m,n,L} = \alpha_{m,n} e^{j\beta_{m,n}} + W_{m,n,L} $$

$$ R_{m,n,L+1} = e^{2\pi j \Delta F_c T} \times \alpha_{m,n} e^{j\beta_{m,n}} + W_{m,n,L+1} $$

**Maximum-Likelihood Estimation of Frequency offset,**

$$ \hat{\Delta F_c} = \frac{1}{2\pi T} \tan^{-1} \left[ \sum_{L=1}^{K} \frac{I\bar{m}(R_{m,n,L+1} R^*_{m,n,L})}{\sum_{L=1}^{K} R\bar{e}(R_{m,n,L+1} R^*_{m,n,L})} \right] $$

**Correction to the ‘r’ vector**

$$ \vec{r}_L^{(update)} = \vec{r}_L \cdot e^{-j2\pi \Delta F_c (L-1) T} $$
Simulations

Frequency Offset Estimation – $\Delta F_c$ vs. Number of blocks (L)

- $\Delta F_c = 10$ Hz
  - Estimate: 8.9 Hz
  - Error: 11%

- $\Delta F_c = 50$ Hz
  - Estimate: 48.9 Hz
  - Error: 2.2%

- $\Delta F_c = 100$ Hz
  - Estimate: 95.7 Hz
  - Error: 4.3%

- $\Delta F_c = 500$ Hz
  - Estimate: 476.8 Hz
  - Error: 4.6%

- $\Delta F_c = 1000$ Hz
  - Estimate: 987.9 Hz
  - Error: 1.22%

- $\Delta F_c = 5000$ Hz
  - Estimate: 4635 Hz
  - Error: 7.3%
Simulations

- About 20 dB improvement in performance with frequency offset estimation.
- Performance limited by the accuracy of the estimate.
Multi-channel Sinc Filter Bank

Multi-channel Analog Filter Bank

Charge sampling sinc filter

Continuous integrator + S&H

From mixer

To ADC

Gm

ADC

Digital Post Processing

Gm

ADC

Digital Post Processing

Gm

ADC

Digital Post Processing

Gm

ADC

Digital Post Processing

From mixer

Rf

Cf

Cs

To ADC
# Analog Complexity

<table>
<thead>
<tr>
<th>Sinc Filter Bank</th>
<th>Continuous integrator filter bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{-3dB} \approx 0.44 / Ts )</td>
<td>( f_{-3dB} \approx 1 / 2\pi R_f C_f )</td>
</tr>
<tr>
<td>DC gain = ( G_m ) Ts / C</td>
<td>DC gain = ( G_m R_f )</td>
</tr>
<tr>
<td>Int. noise = ( KT/C ) (2GmTs/C) + KT/C</td>
<td>Int. noise = ( G_m R_f .KT/C_f + KT/C_f + KT/C_s )</td>
</tr>
</tbody>
</table>
| GBW (op-amp 1,2) >> \( 1 / 2\pi R_o C \)  
GBW (1,2) > 7/(settling time) (\( \beta \sim 1 \)) (10bits) | GBW (1) >> \( 1 / 2\pi R_f C_f \)  
GBW (2) > 7/(settling time) (\( \beta \sim 1 \)) (10bits) |

**Example:** Assuming \( G_m = 1mA/V \), \( Ts = 4ns \),

- DC gain = 4, \( f_{-3dB} \approx 110MHz \)
- Noise = 9KT/C
- GBW (1,2) \( \approx 1.75 GHz \)

- For DC gain = 4, \( R_f = 4K \)
  - For \( f_{-3dB} \approx 110MHz \), \( C_f \approx C/3 \)
  - Noise = 13KT/C + KT/C_s
  - GBW (1) \( \approx 1.5 GHz \) (as \( C_f \approx C/3 \))  
  - GBW (2) \( \approx 3.5 GHz \) (for settling time = 2ns)
Digital Complexity

\[ \hat{a} = H \cdot \tilde{r} = (G^H G)^{-1} G^H \cdot \tilde{r} \]

Step 1:
\[ \tilde{p} = G^H \cdot \tilde{r} \]

Step 2:
\[ \hat{a} = (G^H G)^{-1} \tilde{p} \]

Each element in G is given by,
\[ G_{m,n,s} = \int_{mT_s}^{mT_s+T_c} e^{-j2\pi F_c(s)t} \Phi_n(t)dt \]
\[ = e^{-j2\pi F_c(s) mT_s} \int_{0}^{T_c} e^{-j2\pi F_c(s) t} \Phi_{m,n}(t)dt \]

\( f_{LO}(n) \cdot T_s \) is an integer. So, \( \Phi_{m,n}(t) \) is periodic repetition of \( \Phi_{0,n}(t) \)

\[ G_{m,n,s} = e^{-j2\pi F_c(s) mT_s} \int_{0}^{T_c} e^{-j2\pi F_c(s) t} \Phi_{0,n}(t)dt \]
\[ = e^{-j2\pi F_c(s) mT_s} Q_{s,n} \]

\( F_c(s) \cdot T_s = s/M + \text{integer} \)

\[ G_{m,n,s} = e^{-j2\pi s m/M} Q_{s,n} \]

Complexity of computation of \( p \): \( o(4NM \log M) + o(4NS) \approx o(4S(\log M + N)) \)
Digital Complexity

Sparsity of \((G^H G)^{-1}\)

\(G^H G\) is denoted by \(X = [X_{i,j}]_{S \times S}\)

\[
X_{i,j} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{-j2\pi(i-j)m/M} Q_{i,n} Q^*_{j,n}
= \sum_{n=0}^{N-1} Q_{i,n} Q^*_{j,n} \sum_{m=0}^{M-1} e^{-j2\pi(i-j)m/M}
\]

\[
X_{i,j} = \begin{cases} 
M \sum_{n=0}^{N-1} Q_{i,n} Q^*_{j,n} & (i - j) \mod M = 0 \\
0 & \text{otherwise}
\end{cases}
\]

\(X_{i,j}\) is non-zero only when \((i-j) \mod M = 0\)

• \(G^H G\) has only 2N non-zero elements in each row
• Inverse of \(G^H G\) also has the same sparsity.

Complexity of computation of
- \(G^H G\) \(-\) \(o(2N \times 2N \times 2S) = o(8N^2S)\)
- \((G^H G)^{-1}\) \(-\) \(o(2N \times 2N \times 2S) = o(8N^2S)\)
Digital Complexity

Step 1: \( \hat{p} = G^H \cdot \vec{r} \)  
   Complexity: \( o(4S( \log M + N)) \)
Step 2: \( \hat{a} = (G^H G)^{-1} \hat{p} \)  
   Complexity: \( o(4NS) \)

Total Complexity of LS estimation: \( o(4S( \log M + N)) + o(4NS) \)

Example:  \( S = 128, M = 32, N = 5 \)

Complexity of FFT: \( o(4S \log S) = o(28S) \)

Complexity of LS estimate, 

Sinc filter bank: \( o(4S(\log M + N)) + o(4NS) = o(60S) \)

Analog filter bank: \( o(4NMS) = o(640S) \)

Complexity of estimation during LMS calibration

Forward Problem: 
\[
\hat{a} = H \cdot \vec{r} \\
= \left( G^H G \right)^{-1} G^H \cdot \vec{r} \\
\underbrace{o(16N^2S)}_{o(4NMS)} \underbrace{o(4S(\log M+N))}_{o(4NS)}
\]

Reverse Problem: 
\[
\hat{a} = H \cdot \vec{r} \\
\underbrace{o(16N^2S) + o(4S(\log M+N)) + o(4NS)}_{o(460S)}
\]
<table>
<thead>
<tr>
<th>Comparative Study</th>
<th>Sinc Filter Bank</th>
<th>Analog Filter Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Front end</td>
<td>Larger capacitors</td>
<td>Smaller capacitors</td>
</tr>
<tr>
<td>complexity</td>
<td>No resistor required. Reset ensures finite DC gain.</td>
<td>Resistor required for finite DC gain.</td>
</tr>
<tr>
<td></td>
<td>Lesser noise</td>
<td>Noise is high.</td>
</tr>
<tr>
<td></td>
<td>Smaller GBW for op-amps.</td>
<td>Larger GBW for op-amps.</td>
</tr>
<tr>
<td>Analog Power consumption</td>
<td>Less</td>
<td>4 times higher</td>
</tr>
<tr>
<td>Digital complexity</td>
<td>$o(4S(N + \log M)) + o(4NS)$</td>
<td>$o(4NMS)$</td>
</tr>
<tr>
<td>(Estimation)</td>
<td>Example: $o(60S)$</td>
<td>Example: $o(640S)$</td>
</tr>
<tr>
<td>Digital complexity</td>
<td>$o(16N^2S) + o(4S(\log M+N)) + o(4NS)$</td>
<td>$o(4NMS)$</td>
</tr>
<tr>
<td>(Estimation @ calibration)</td>
<td>Example: $o(460S)$</td>
<td>Example: $o(640S)$</td>
</tr>
<tr>
<td>Digital power consumption</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Example: About 10% of $S$</td>
<td>Example: 10 times more $S$</td>
</tr>
</tbody>
</table>
Multi-Standard reconfigurable FD receiver

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bandwidth</th>
<th>Bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>UWB</td>
<td>500 MHz</td>
<td>5</td>
</tr>
<tr>
<td>WiMax</td>
<td>25 MHz</td>
<td>7</td>
</tr>
<tr>
<td>Wi-Fi</td>
<td>20 MHz</td>
<td>8</td>
</tr>
<tr>
<td>Bluetooth</td>
<td>1 MHz</td>
<td>12</td>
</tr>
<tr>
<td>GSM</td>
<td>200 KHz</td>
<td>14</td>
</tr>
</tbody>
</table>

RF broadband signal

LNA

Mixer output

GSM

Bluetooth

Wi-Fi

WiMax

UWB

Mixing Frequency allocation

0.8 GHz - 11 GHz

F1, F2, F3, F4 (UWB)

Nth order Decimation Filter

sinc² anti-aliasing filter

Mixer output to ADC

Mixer output to ADC

Mixer output to ADC

Mixer output to ADC

Gm

Mixer

Anti-aliasing filter & Sampler

ΣΔ ADC

Digital Post Processing

Recovered Symbols

Spring 2009

S. Hoyos - Advanced Mixed-Signal Interfaces
Decentralized TD Sensor Network

Data sampled @ sub-Nyquist rate by sensor nodes S1-S4 is transmitted to Fusion Center for processing.

h₁(t) is the equivalent channel impulse response for sensor node Si.