Quiz on Basic Probability & Random Variables

Date – Aug 26, 2008 (Tue), 5:30PM in class.

Name: 
Email: 

Problem 1. Definitions

Define the following terms:

(a) Sample space:

(b) Event:

(c) Random variable:

(d) Random process:

Problem 2. Bayes’ Formula

Consider the events $F_1, \ldots, F_n$, of which one and only one must occur. How can we compute the conditional probability $P(F_i|E)$, for a given event $E$, using Bayes’ formula?

Problem 3. Binomial Random Variable

(a) Let $X$ be the number of success for $n$ independent trials, each with a success rate $p$. Find $P\{X = k\}$.

(b) What is the expectation $E[X]$ of the random variable $X$?

(c) What is the variance of $X$?
**Problem 4. Compound Random Variable**

Let $N$ be the number we get by rolling a fair die. We toss a fair coin $N$ times, and count the number of heads and call it $X$. What is the expectation of $X$? Justify your answer.

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**Problem 5. Geometric Random Variable**

We have a coin that has a probability $p$ of turning up heads. We successively toss this coin until the first head appears. Let $N$ be the number of coin tosses. Now, let $Y$ be a random variable that shows the result of the first coin toss ($Y = 1$ for head, $Y = 0$ for tail).

(a) How can we compute $E[N]$ in terms of $E[N|Y = 1]$ and $E[N|Y = 0]$?

(b) Compute $E[N|Y = 1]$.

(c) Express $E[N|Y = 0]$ using $E[N]$ and $p$.

(d) Based on above results, compute the expectation $E[N]$. 