Homework Assignment #7

Due date – Nov. 29, 2012 (Thu) in class.

Problem 1. Frequency response of LTI system (10 points)
Solve problem P5.9 in the textbook.

Problem 2. Moving average system (10 points)
Consider a moving average system defined by the following difference equation:
\[ y(n) = \sum_{k=0}^{7} x(n-k) \]
(a) Find the impulse response \( h(n) \) and the transfer function \( H(z) \) of the system.
(b) Draw the pole-zero plot for \( H(z) \).
(c) Use Matlab to plot the magnitude response \( |H(\omega)| \) for the range \( 0 \leq \omega \leq \pi \).
   * Hint: >> help freqz
(d) Use Matlab to the phase response \( \Theta(\omega) = \angle H(\omega) \) for the range \( 0 \leq \omega \leq \pi \).
   * Hint: >> help angle
(e) Let \( H_3(z) = H(z^3) \). Use Matlab to plot \( |H_3(\omega)| \) and \( \Theta_3(\omega) = \angle H_3(\omega) \) for the range \( 0 \leq \omega \leq \pi \).

Problem 3. Digital resonator (10 points)
Consider the following digital resonator:
\[ H(z) = \frac{b_0}{(1-re^{j\omega_0 z}) (1-re^{-j\omega_0 z})} \]
(a) Let \( r = 0.75 \) and \( \omega_0 = \pi/3 \). Find \( b_0 \) such that \( |H(\omega_0)| = |H(-\omega_0)| = 1 \).
(b) Use Matlab to plot \( |H(\omega)| \) and \( \Theta(\omega) = \angle H(\omega) \) for the range \( -\pi \leq \omega \leq \pi \).
(c) Now, let \( r = 0.95 \) and \( \omega_0 = \pi/3 \). Find \( b_0 \) such that \( |H(\omega_0)| = |H(-\omega_0)| = 1 \).
(d) Use Matlab to plot \( |H(\omega)| \) and \( \Theta(\omega) = \angle H(\omega) \) for the range \( -\pi \leq \omega \leq \pi \).
Problem 4. Notch filter  (10 points)
Consider the following notch filter:

\[ H(z) = \frac{b_0(1 - e^{j\omega_0} z^{-1})(1 - e^{-j\omega_0} z^{-1})}{(1 - re^{j\pi} z^{-1})(1 - re^{-j\pi} z^{-1})} \]

(a) Let \( r = 0.75 \) and \( \omega_0 = \pi/3 \). Find \( b_0 \) such that \( |H(\pi)| = |H(-\pi)| = 1 \).

(b) Use Matlab to plot \( |H(\omega)| \) and \( \Theta(\omega) = \angle H(\omega) \) for the range \(-\pi \leq \omega \leq \pi\).

(c) Now, let \( r = 0.95 \) and \( \omega_0 = \pi/3 \). Find \( b_0 \) such that \( |H(\pi)| = |H(-\pi)| = 1 \).

(d) Use Matlab to plot \( |H(\omega)| \) and \( \Theta(\omega) = \angle H(\omega) \) for the range \(-\pi \leq \omega \leq \pi\).

Problem 5. Minimum and maximum phase systems  (15 points)
Consider a causal LTI system with the following transfer function:

\[ H(z) = \frac{(1 + 0.95z^{-1})(1 - 1.05e^{j\pi} z^{-1})(1 - 1.05e^{-j\pi} z^{-1})}{(1 - 0.9z^{-1})(1 - 0.9e^{j\pi} z^{-1})(1 - 0.9e^{-j\pi} z^{-1})} \]

(a) Draw the pole-zero plot for \( H(z) \).

(b) Find the minimum phase system \( H_{\text{min}}(z) \) that has the same magnitude response and the same number of poles & zeros as \( H(z) \).

(c) Find the maximum phase system \( H_{\text{max}}(z) \) that has the same magnitude response and the same number of poles & zeros as \( H(z) \).

(d) Use Matlab to plot \( |H_{\text{min}}(\omega)| \) and \( \Theta_{\text{min}}(\omega) = \angle H_{\text{min}}(\omega) \) for the range \( 0 \leq \omega \leq \pi \).

(e) Use Matlab to plot \( |H_{\text{max}}(\omega)| \) and \( \Theta_{\text{max}}(\omega) = \angle H_{\text{max}}(\omega) \) for the range \( 0 \leq \omega \leq \pi \).

(f) Compare the plots in parts (d) and (e) and discuss the results.
Problem 6. Filter design (15 points)
Design a bandpass filter $H(z)$ with the following specs by choosing the pole-zero locations:

- Use no more than 10 poles and 10 zeros
- The filter should be causal, real, and stable.
- $|H(w)| \approx 1$ for $\frac{\pi}{3} \leq |\omega| \leq \frac{2\pi}{3}$ and $|H(w)| \approx 0$, otherwise.

(a) Draw the pole-zero plot for $H(z)$.

(b) Use Matlab to plot $|H(\omega)|$ and $\Theta(\omega) = \angle H(\omega)$ for the range $-\pi \leq \omega \leq \pi$.

(c) Use Matlab to plot the group delay $\tau_g(\omega)$ for the range $-\pi \leq \omega \leq \pi$.
   * Hint: >> help grpdelay

(d) Let $x(n) = \cos \left( \frac{\pi n}{2} \right) u(n)$ and let $y(n) = h(n) * x(n)$. Use Matlab to plot the first 50 samples of $x(n)$ and $y(n)$ and discuss whether your filter is working as intended.
   * Hint: >> help filter

(e) Let $x(n) = \cos(\pi n) u(n)$ and let $y(n) = h(n) * x(n)$. Use Matlab to plot the first 50 samples of $x(n)$ and $y(n)$ and discuss whether your filter is working as intended.