Homework Assignment #6
Due Date – April 2, 2014 (Wed), 12:40PM in class.

Problem 1. The lifetime $X$ of an electronic device follows the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{C}{x^2}, & x > 10 \\ 0, & x \leq 10 \end{cases}$$

(a) Find the constant $C$.
(b) Find $P(X > 20)$.
(c) Find the cumulative distribution function $F(x)$ of this random variable $X$.
(d) What is the probability that among 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?

Problem 2. Nancy arrives at a bus stop at 10AM, knowing that the bus will arrive at some time uniformly distributed between 10AM and 10:30AM.
(a) What is the probability that Nancy will have to wait longer than 10 minutes?
(b) If, at 10:15AM, the bus has not yet arrived, what is the probability that Nancy will have to wait at least an additional 10 minutes?

Problem 3. If 65% of the population of a large community is in favor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain
(a) at least 50 who are in favor of the proposition;
(b) between 60 and 70 inclusive who are in favor;
(c) fewer than 75 in favor.

Problem 4. The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer are normally distributed with parameters $\mu = 1.4 \times 10^6$ hours and $\sigma = 3 \times 10^5$ hours. What is the approximate probability that a batch of 100 chips will contain at least 20 whose lifetimes are less than $1.8 \times 10^6$?

Problem 5. An image is partitioned into two regions, one white and the other black. A reading taken from a randomly chosen point in the white section will be normally distributed with $\mu_w = 4$ and $\sigma_w^2 = 4$, whereas one taken from a randomly chosen point in the black region will have a normally distributed reading with parameters $\mu_b = 6$ and $\sigma_b^2 = 9$. A point is randomly chosen on the image and has a reading of 5. If the fraction of the image that is black is $\alpha$, for what value of $\alpha$ would the probability of making an error be the same, regardless of whether one concluded that the point was in the black region or in the white region?