ECEN 303: Assignment 3

Reading Assignment:

1. Chapter 3: Conditional Probability, pages 26–33;

Problems:

1. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is \( i, i = 2, 3, \ldots, 12 \)?

2. An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black?

3. Consider 3 urns. Urn A contains 2 white and 4 red balls; urn B contains 8 white and 4 red balls; and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from A was white, given that exactly 2 white balls were selected?

4. In a certain community, 36 percent of the families own a dog, and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is
   (a) the probability that a randomly selected family owns both a dog and a cat;
   (b) the conditional probability that a randomly selected family owns a dog given that it owns a cat?

5. A total of 48 percent of the women and 37 percent of the men that took a certain “quit smoking” class remained nonsmokers for at least one year after completing the class. These people then attended a success party at the end of a year. If 62 percent of the original class were male,
   (a) what percentage of those attending the party were women?
   (b) what percentage of the original class attended the party?

6. Each of 2 cabinets identical in appearance has 2 drawers. Cabinet A contains a silver coin in each drawer, and cabinet B contains a silver coin in one of its drawers and a gold coin in the other. A cabinet is randomly selected, one of its drawers is opened, and a silver coin is found. What is the probability that there is a silver coin in the other drawer?

Optional Problems:

1. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32 percent of women of childbearing age are smokers, what percentage of woman having ectopic pregnancies are smokers?
2. Two players take turns removing a ball from a jar that initially contains $m$ white and $n$ black balls. The first player to remove a white ball wins. Develop a recursive formula that allows the convenient computation of the probability that the starting player wins.

3. We have two jars, each initially containing an equal number of balls. We perform four successive ball exchanges. In each exchange, we pick simultaneously and at random a ball from each jar and move it to the other jar. What is the probability that at the end of the four exchanges all the balls will be in the jar where they started?

4. **Using a biased coin to make an unbiased decision.** Alice and Bob want to choose between the opera and the movies by tossing a fair coin. Unfortunately, the only available coin is biased and lands heads with probability $p$. Since they both know $p$, whomever calls the toss in the air can bias the decision towards their preference. Design a coin-toss experiment, which does not depend on $p$, so that they can use the biased coin to make a decision so that either option (opera or movies) is equally likely?

**Programming Challenges:**

1. Write a computer simulation with repeated trials to provide supporting evidence for the strategy you developed for the biased coin problem above. Use a biased coin with $p = 0.25$. As the number of trials becomes large, what should the empirical probability of selecting opera go to?